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Cascade control for nonlinear non-minimum phase system based on backstepping approach and input output linearization

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Abstract—**The problem of tracking control of nonlinear systems whose the zero dynamics are unstable is addressed in this paper. It is shown that, using a novel cascade structure where the backstepping approach is given to stabilise the internal dynamics and the standard input output linearization to stabilise the external dynamics. Assuming high-gain feedback for the external dynamics, a stability analysis of the global system is provided based on singular perturbation theory. Simulations of an inverted cart-pendulum illustrate the theoretical results.**

***Keywords*— tracking control, Non-minimum phase system, integrator backstepping, input output linearization, singular perturbed system.**

I. INTRODUCTION

In the past few years, the problem of tracking control of nonlinear non-minimum phase system has attracted big attention, due to its applications in environment. The unstable zero dynamics problem has been treated in the literature by different approaches.

The first approaches based on input output linearization [1], [2]-[3] to enlarge the class of nonlinear systems where an input output linearization can be used [4]-[5]. In this contribution, Kravaris and Soroush have developed several results on the approximate linearization of non minimum phase systems [6], [7], [8]-[9]. For instance, in [8]-[9], the system output is differentiated as many times as the order of the system where the input derivatives that appear in the control law are set to zero when computing the state feedback input. In [10], the system input output feedback is first linearized. Then, the zero dynamics is factorized into stable and unstable parts. The unstable part is approximately linear and independent of the coordinates of the stable part. Moreover, an original technique of control based on an approximation of the method of exact input-output linearization was proposed in the works of Hauser and al [11]. In [12]-[13], the approximation presented in [11] is used to improve the desired control performance.

The second approaches based on a cascade structure involving feedback linearization and stabilization of the internal dynamics has been considered in the literature. The system is first input-output feedback linearized, and then the internal dynamics are stabilized. In [14], the internal dynamics are stabilized using output redefinition and repetitive learning control. [15] addresses the problem of swinging up an inverted pendulum and controlling it around the upright position. The internal dynamics are stabilized using elements of energy control and Lyapunov control. In [16], a control law is derived based on extended linearization and predictive control. [17] proposes a new control law that combines between the input output linearization and backstepping.

In this paper, we address the problem of tracking control of a single input single output of non minimum phase nonlinear systems. The idea here is to transform the given system into Brynes-Isidori normal form, then to use the singular perturbed theory in which a time-scale separation is artificially introduced through the use of a state feedback with a high-gain for the linearized part. The integrator backstepping approach is introduced to generate a reference trajectory for stabilizing the internal dynamics [18]. The stability analysis for the proposed approach is based on the results of the singular perturbation theory [19].

The present paper is organized as follows: in Section II some mathematical preliminaries are presented. The cascade control law design and stability analysis are given in section III. Section IV gives the inverted cart-pendulum to illustrate the effectiveness of the proposed approach. Finally, some concluding remarks are provided in Section V.

II. THEORETICAL BACKGROUND

In this paper, we consider a single input single output nonlinear system (SISO) of the form:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \tag{1}$$

where $x \in \Re^n$ is the n-dimensional state variables, $u \in \Re$ is a scalar manipulate input and $y \in \Re$ is a scalar output. $f(.)$, $g(.)$ and $h(.)$ are smooth functions describing the system dynamics.

A. Input output linearization

Consider the output $y = h(x)$ for system (1). The nonlinear system (1) has relative degree r at the point x_0 if:

$$\begin{cases} L_f^k h(x) = 0 \quad \forall x \neq x_0 \text{ and } \forall k \leq r-1 \\ L_g L_f^{(k-1)} h(x) \neq 0 \end{cases} \tag{2}$$

So, the relative degree r is the number of times we have to differentiate the output y with respect to time before the input appears [19].
If $y \le n$, then system (1) can be feedback linearized into Byrnes-Isidori normal form [20]:

$$z = \begin{bmatrix} h(x) & L_f h(x) & \dots & \eta_1 & \dots & \eta_{n-r} \end{bmatrix}^T \quad (3)$$

The resulting system with the transformed variables (1) can be written as:

$$\begin{cases} \dot{z}_i = z_{i+1} \quad i = 1, \dots, r-1 \\ \dot{z}_r = v = L_f^r h(x) + L_g L_f^{r-1} h(x) u \\ \dot{\eta} = q(z, \eta) \\ y = z_1 \end{cases} \quad (4)$$

where v is the new control law
Thus, the control law can be written as:

$$u(x) = \frac{v - L_f^r h(x)}{L_g L_f^{r-1} h(x)} \quad (5)$$

B. Integrator backstepping approach

In this section, we consider the nonlinear system (1) which is written in the strict-feedback form given by [21], [22]-[23]:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i) x_{i+1}, \ \forall i \in \{1, 2, \dots, n-1\} \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n) u \\ y = h(x_1) \end{cases} \quad (6)$$

where $\bar{x}_i = \begin{bmatrix} x_1 & x_2 & \dots & x_i \end{bmatrix}^T \ \forall i = 1, 2, \dots, n$, u and y are the i^{th} state variables, the system input and output are all assumed to be available for measurement; $f_i(.)$ and $g_i(.)$, $i = 0, \dots, n$, are smooth nonlinear functions and $g_i \ne 0$.
The aim of the control is the trajectory tracking of the output y of the system (6), an error base $e_i, \forall i \in \{1, \dots, n\}$ is created as the difference between all the system states and their reference states $x_{i\,ref}$.

$$e_i = x_i - x_{i\,ref} \quad \forall i \in \{1, 2, \dots, n\} \quad (7)$$

The most popular method for nonlinear systems in the strict feedback form of (6) is the integrator backstepping approach developed in [21]-[24]. If this approach were applied to (6), then it would be possible to create a generator of trajectory for the state vector x in the form:

$$\begin{cases} x_{1ref} = h^{-1}(y_{ref}) \\ x_{2ref} = \frac{1}{g_1}\left(-f_1 + \dot{x}_{1ref} - \lambda_1 e_1\right) \\ x_{(i+1)ref} = \frac{1}{g_i}\left(-f_i + \dot{x}_{i\,ref} - g_{i-1} e_{i-1} - \lambda_i e_i\right), \ \forall i \in \{1, 2, \dots, n-1\} \end{cases} \quad (8)$$

and the control law is as follow:

$$u = \frac{1}{g_n}\left(-f_n + \dot{x}_{n\,ref} - g_{(n-1)} e_{(n-1)} - \lambda_n e_n\right) \quad (9)$$

where $y_{ref}(t)$ is a reference trajectory at least C^n, h is a bijective function and h^{-1} is C^n.
Using (8) and (9), then the error dynamics equations are as follows:

$$\begin{cases} \dot{e}_1 = g_1 e_2 - \lambda_1 e_1 \\ \dot{e}_i = g_i e_{i+1} - g_{i-1} e_{i-1} - \lambda_i e_i, \ \forall i \in \{2, \dots, n-1\} \\ \dot{e}_n = g_{n-1} e_{n-1} - \lambda_n e_n \end{cases} \quad (10)$$

It is easy to determine that the equilibrium point $e = 0$ is the unique solution of (10).

In order to illustrate the stability of the origin $e = 0$ of system (10), the following exponential stability theorem is introduced.

Theorem 1 [19]: Given system (1), if there exists a Lyapunov function $V(x)$ and positive constants χ_1, χ_2 and χ_3 such that $\chi_1 \|x\|^2 \le V(x) \le \chi_2 \|x\|^2$ and $\dot{V}(x) \le -\chi_3 \|x\|^2$, then the origin is exponentially stable.
Consider the following Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{n} e_i^2 \quad (11)$$

Therefore, the Lyapunov derivative $\dot{V}$ is

$$\dot{V} = -\sum_{i=1}^{n} \lambda_i e_i^2 \qquad \text{with} \quad \lambda_i > 0, \ \forall i \in \{1, 2, \dots, n\} \quad (12)$$

So, the origin $e = 0$ of system (10) is globally exponential stable

C. Singular perturbed system

A singularly perturbed system is one that exhibits a two-timescale behavior, i.e. it has a slow and fast dynamics and is modeled as follows [25]-[26]:

$$\dot{\eta} = F_1(\eta, z, u, \varepsilon) \quad \eta(0) = \eta_0 \quad (13a)$$

$$\varepsilon \dot{z} = F_2(\eta, z, u, \varepsilon), \quad z(0) = z_0 \quad (13b)$$

$$y = h(x) \quad (13c)$$

where $\eta \in \Re^m$ and $z \in \Re^p$ are respectively the slow and fast variables and $\varepsilon > 0$ is a small positive parameter. The functions F_1 and F_2 are assumed to be continuously differentiable. η_0 and z_0 are respectively the initial conditions of the vectors η and ξ. If $\varepsilon \to 0$, the dynamics of z acts quickly and leads to a time-scale separation. Such a separation can either represent the physics of the system or can be artificially created by the use of high-gain controllers.
As $\varepsilon \to 0$, z can be approximated by its quasi-steady state $\bar{\xi} = \psi(\eta, u)$ obtained by solving $f_1(\eta, \xi, 0) + g_1(\eta, \xi, 0) u = 0$.
So, the reduced (slow) system is given by:

$$\dot{\eta} = f_2\left(\eta, \psi(\eta, u), 0\right) + g_2\left(\eta, \psi(\eta, u), 0\right)u = \bar{F}_2(\eta, u) \quad (14)$$

Note that the reduced system (14) is not necessarily affine in input.

In the next theorem we establish the exponential stability of the singular perturbed system (13).

Theorem 2 [19]: Assume that the following conditions are satisfied:

- The origin is an equilibrium point for (13),
- $\psi(\eta, u)$ has a unique solution,
- The functions f_1, f_2, g_1, g_2, ψ and their partial derivatives up to order 2 are bounded for z in the neighborhood of $\bar{z}$,
- The origin of the boundary-layer system (13) is exponentially stable for all η,
- The origin of the reduced system (14) is exponentially stable.

Then, there exists $\varepsilon^* > 0$ such that, for all $\varepsilon < \varepsilon^*$, the origin of (13) is exponentially stable.

III. CASCADE CONTROL LAW DESIGN

In this section, an approach to the tracking control problem of the nonlinear non minimum phase system is proposed based on a singular perturbed theory and a combination of backstepping and input output linearization. In particular, it is shown that the closed-loop system can be described as an interconnection of two subsystems: the reduced subsystem and the boundary-layer subsystem. The stability analysis of the proposed approach is provided using the results of the singular perturbation theory [19].

A. Boundary Layer subsystem

Consider the nonlinear system described by (1), then we apply the control law (3) which is given by:

$$v = y^{(r)} = y_{ref}^{(r)} + \sum_{i=0}^{r-1} \bar{k}_{i+1}\left(y_{ref} - y\right)^{(i)} \quad \text{with} \quad \bar{k}_{i+1} = \frac{k_{i+1}}{\varepsilon^{r-i}} \quad (15)$$

where $y_{ref}(t)$ is the reference trajectory for the output, $\varepsilon \to 0$ a small positive parameter, and $k_i > 0, \forall i \in \{1, 2, \ldots, n-1\}$ are the coefficients of a Hurwitz polynomial [19].

and the internal dynamics are given by:

$$\dot{\eta} = Q\left(\eta, y, \dot{y}, \ldots, y^{(r-1)}\right) \quad (16)$$

Under the assumption that the gains $\bar{k}_i$ are chosen large, such as for any choice of $\varepsilon > 0$, the closed loop is stable and ε can be used as a single tuning parameter, the system (15)-(16) can be written in the form of a singular perturbed system (13) . So the fast state can be defined by:

$$z_i = \varepsilon^{i-1} y^{(i-1)}, \quad i = 1, \ldots, r \quad (17)$$

If we replace (17) by (16), we obtain

$$\dot{\eta} = Q(\eta, z, \varepsilon), \quad \eta(0) = \eta_0 \quad (18)$$

and also by (15), such that

$$\varepsilon \dot{z}_r = \varepsilon^r y_{ref}^{(r)} + \sum_{i=0}^{r-1} k_{i+1}\left(z_{(i+1)ref} - z_{i+1}\right) \quad (19)$$

with $z_{ref} = \begin{bmatrix} y_{ref} & \varepsilon \dot{y}_{ref} & \varepsilon^2 \ddot{y}_{ref} & \ldots & \varepsilon^r y_{ref}^{(r)} \end{bmatrix}^T$

thus, (16) can be written as follows:

$$\begin{cases} \varepsilon \dot{z}_i = z_{i+1}, & i = 1, \ldots, r-1 \\ \varepsilon \dot{z}_r = \varepsilon^r y_{ref}^{(r)} + \sum_{i=0}^{r-1} k_{i+1}\left(z_{(i+1)ref} - z_{i+1}\right) \end{cases} \quad (20)$$

B. Reduced subsystem

As the tuning parameter ε is small, so the quasi-steady-state assumption can be introduced. Thus, the reduced subsystem (QSS subsystem) when setting to zero in (18).

Letting $\varepsilon \to 0$ in (17)

$$\begin{cases} z_1 = y \\ z_2 = \ldots = z_r = 0 \end{cases} \quad (21)$$

we use this result and let $\varepsilon \to 0$ in the last equation of (20), we obtain

$$\begin{aligned} \varepsilon \dot{z}_r &= \varepsilon^r y_{ref}^{(r)} + \sum_{i=0}^{r-1} k_{i+1}\left(z_{(i+1)ref} - z_{i+1}\right) = 0 \\ &\Rightarrow k_1\left(z_{1ref} - z_1\right) = 0 \\ &\Rightarrow z_{1ref} = z_1 \end{aligned} \quad (22)$$

Therefore, when $\varepsilon \to 0$:

$$z = \bar{z} = \begin{bmatrix} y_{ref} & 0 & \ldots & 0 \end{bmatrix}^T \quad (23)$$

The vector $\bar{z}$ is the quasi-steady-state value of z .

The internal dynamics depends on the output y , its derivatives $\dot{y}, \ldots, y^{(r-1)}$ and the small parameter ε , such as:

$$\begin{aligned} \dot{\eta} &= Q(\eta, z, \varepsilon) \\ &= Q\left(\eta, y, \ldots, y^{(r-1)}, \varepsilon\right), \quad \eta(0) = \eta_0 \end{aligned} \quad (24)$$

Under the quasi-steady-state QSS assumption, the output y tends to y_{ref} and the derivatives $\dot{y}, \ldots, y^{(r-1)}$ tend to their references $\dot{y}_{ref}, \ldots, y_{ref}^{(r-1)}$. Then, the internal dynamics is written by

$$\dot{\eta} = Q\left(\eta, y_{ref}, \ldots, y_{ref}^{(r-1)}, \varepsilon\right), \quad \eta(0) = \eta_0 \quad (25)$$

Thus, the reference trajectory $\left(y_{ref}, \dot{y}_{ref}, \ldots, y_{ref}^{(r-1)}\right)$ will be used for the stabilization of the internal dynamics.

we define a novel state vector $\bar{\eta} = \begin{bmatrix} \eta_1 & \ldots & \eta_{n-r} & y_{ref} & \ldots & y_{ref}^{(r-1)} \end{bmatrix}^T$ such as the reduced subsystem (25) can be written by

$$\dot{\bar{\eta}} = \bar{Q}\left(\bar{\eta}, u_{QSS}\right) \quad \text{with} \quad u_{QSS} = y_{ref}^{(r)} \tag{26}$$

Note that it is important to include additional states $y_{ref}, \dot{y}_{ref}, \ldots, y_{ref}^{(r-1)}$ since they are considered as independent variables, and the last derivative $u_{QSS} = y_{ref}^{(r)}$ is considered as the control law for (26).

To determine u_{QSS}, we need the assumption that the internal dynamics (26) is written in the following strict feedback form:

$$\dot{\bar{\eta}} = \begin{cases} \dot{\eta}_1 = f_1(\eta_1) + g_1(\eta_1)\eta_2 \\ \dot{\eta}_2 = f_2(\bar{\eta}_2) + g_2(\bar{\eta}_2)\eta_3 \\ \vdots \\ \dot{\eta}_{n-r} = f_{n-r}(\bar{\eta}_{n-r}) + g_{n-r}(\bar{\eta}_{n-r})\eta_{n-r+1} \\ \dot{\eta}_{n-r+1} = f_{n-r+1}(\bar{\eta}_{n-r+1}) + g_{n-r+1}(\bar{\eta}_{n-r+1})\eta_{n-r+2} \\ \vdots \\ \dot{\eta}_n = f_n(\bar{\eta}_n) + g_n(\bar{\eta}_n)u_{QSS} \end{cases} \tag{27}$$

Then, we use the integrator backstepping approach to determine the reference trajectory y_{ref} and the control law u_{QSS}. So, we define $y_{QSS} = h(\eta_1)$ as a virtual output for the subsystem (26) and $y_{QSS\,ref}$ are the reference trajectory for the output y_{QSS}. By referring to the equations (8) and (9), we obtain the following trajectory generator:

$$\eta_{ref} = \begin{cases} \eta_{1ref} = h^{-1}\left(y_{sref}\right) \\ \eta_{2ref} = \dfrac{1}{g_1}\left(-f_1 + \dot{\eta}_{1ref} - \lambda_1 \boldsymbol{\eta}_1\right) \\ \eta_{(i+1)ref} = \dfrac{1}{g_i}\left(-f_i + \dot{\eta}_{(i-1)ref} - g_{i-1}\boldsymbol{\eta}_{i-1} - \lambda_i \boldsymbol{\eta}_i\right), \quad i = 3, \ldots, n-1 \end{cases} \tag{28}$$

where $\boldsymbol{\eta}_i = \eta_i - \eta_{iref} \quad \forall i = 1, \ldots, n$

and the control law is given by

$$u_{QSS} = \frac{1}{g_n}\left(-f_n + \dot{\eta}_{n-1ref} - g_{n-1}\boldsymbol{\eta}_{n-1} - \lambda_n \boldsymbol{\eta}_n\right) \tag{29}$$

C. Stability analysis

In this section, we use the theorem 2 of exponential stability of singular perturbed system to analyze the stability of the closed loop system. If both the reduced and the boundary layer subsystems are exponentially stable, then the combination is also exponentially stable. The following steps will be used to prove the stability of the proposed approach:

1) Exponential stability of the Boundary Layer subsystem

Let us consider the error vector given by

$$\boldsymbol{z} = z - z_{ref} \tag{30}$$

Then, the boundary layer subsystem (20) becomes:

$$\varepsilon \dot{\boldsymbol{z}} = \left[\boldsymbol{z}_1 \quad \boldsymbol{z}_2 \quad \ldots \quad \sum_{i=0}^{r-1} k_{i+1}\boldsymbol{z}_{i+1} \right]^T \tag{31}$$

Letting $\tau = \dfrac{t}{\varepsilon}$ yield:

$$\frac{d\boldsymbol{z}}{d\tau} = A\boldsymbol{z} \tag{32}$$

with A is defined by

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -k_1 & -k_2 & -k_3 & \ldots & -k_{n-r} \end{bmatrix}$$

Using the theorem1, the origin $\boldsymbol{z} = 0$ is exponentially stable, and the Lyapunov function is

$$V = \frac{1}{2}\boldsymbol{z}^T \Psi \boldsymbol{z} \tag{33}$$

where $A^T\Psi + \Psi A = -Q$ and Q is a matrix defined positive

2) Exponential stability of the reduced subsystem

The stability of the reduced subsystem is provided by using the integrator backstepping approach. So the Lyapunov function is given by:

$$V = \frac{1}{2}\sum_{i=1}^{n-1} \boldsymbol{\eta}_i^2 \tag{34}$$

3) Global exponential stability

Using the theorem 2, we can conclude that the origin of (1) is exponentially stable. Although all the conditions of the theorem 1 are satisfied such that

- The origin $\left(\eta = 0, z = 0 \text{ and } y_{ref} = 0\right)$ is an equilibrium point for the subsystems (20) and (26)
- $\bar{Q}\left(\bar{\eta}, u_{QSS}\right)$ has a unique solution $\eta^* = \begin{bmatrix} 0 & \ldots & 0 & y_{ref} & 0 & \ldots & 0 \end{bmatrix}$ Furthermore, as a result of the integrator backstepping approach, y_{ref} is a function of η
- Q and its partial derivatives up to order 2 are bounded for η in the neighborhood of $\bar{\eta}$
- The origin of the boundary layer system (20) is exponentially stable $\forall \eta$
- The origin of the reduced system (26) is exponentially stable

IV. ILLUSTRATIVE EXAMPLE

In this section, the effectiveness of the proposed cascade control law is demonstrated using the example of inverted cart-pendulum.

A. Description of the inverted cart-pendulum system

Consider the cart-inverted pendulum illustrated in fig. 1. The cart must be moved using the force $u(t)$ so that the pendulum remains in the upright position as the cart tracks varying positions at the desired time. The differential equations describing the motion are [27]:

$$\begin{cases}(M+m)y_p + ml\theta\cos\theta + ml\theta^2\sin\theta = u \\ l\theta - y_p\cos\theta - g\sin\theta = 0\end{cases} \quad (35)$$

where θ is the angle of the pendulum, y_p is the displacement of the cart, and u is the control force, parallel to the rail, applied to the cart.

The numerical parameters of the inverted pendulum system are $M = 0.455kg$, $m = 0.21kg$, $l = 0.355$ and g=9.81m/s^2

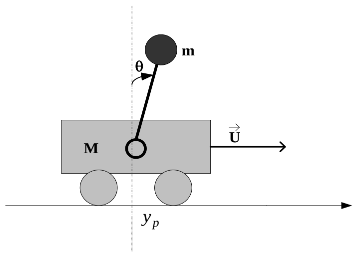

Fig. 1. inverted pendulum system

Consider θ as the output and let $x = \begin{bmatrix}\theta & \theta & y_p & y_p\end{bmatrix}^T$

The inverted cart-pendulum can be written as the system (1):

$$\begin{cases}x = f(x) + g(x)u \\ y = h(x)\end{cases} \quad (36)$$

Where θ represents the output, u is the input, x is the state-space vector. Hence, one has:

$$f(x) = \begin{bmatrix} x_2 \\ \frac{1}{l}\left(g\sin x_1 - \frac{m(lx_2^2 + g\cos x_1)\sin x_1}{M + m(\sin x_1)^2}\cos x_1\right) \\ x_3 \\ \frac{m(lx_2^2 + g\cos x_1)\sin x_1}{M + m(\sin x_1)^2}\end{bmatrix}, \; g(x) = \begin{bmatrix} 0 \\ \frac{\cos x_1}{M + m(\sin x_1)^2} \\ 0 \\ \frac{1}{M + m(\sin x_1)^2}\end{bmatrix}$$

and $h(x) = \theta$

B. cascade control law

Applying the proposed approach mentioned in section 4 to the system (36) of the inverted cart-pendulum, we obtain the following steps:

- **Step1**:

The relative degree of system (36) is $r = 2 < 4$. The input to be applied for input-output linearization is:

$$u = lm(lx_2^2 + g\cos x_1)\sin x_1 - \frac{l\left(M + m(\sin x_1)^2\right)}{\cos x_1}v \quad (37)$$

Thus, the dynamic system becomes:

$$\begin{cases}\theta = v \\ y_p == \frac{lv + g\sin\theta}{\cos\theta}\end{cases} \quad (38)$$

and the internal dynamics is given by:

$$\eta = \begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix} = \begin{bmatrix} y_p \\ y_p - \frac{\cos\theta}{l}\theta\end{bmatrix} \quad (39)$$

- **Step2**: The high-gain feedback is given by:

$$v = \theta_{ref}^{(2)} + \frac{k_2}{\varepsilon}(\theta_{ref} - \theta) + k_1(\theta_{ref} - \theta) \quad (40)$$

Under the QSS assumption that $\theta = \theta = v = 0$ and $\theta \to \theta_{ref} \cong 0$, $\sin\theta = \theta$ and $\cos\theta = 1$.

Using the equation (26), the reduced subsystem can be written as:

$$\bar{\eta}(x) = \begin{bmatrix}\eta_1 = \eta_2 \\ \eta_2 = g\,\eta_3 \\ \eta_3 = \eta_4 \\ \eta_4 = u_{QSS}\end{bmatrix} = \begin{bmatrix} y_p \\ g\theta_{ref} \\ \theta_{ref} \\ \theta_{ref}\end{bmatrix} \quad (41)$$

The control objective is to make the output θ track a desired reference trajectory θ_{ref} given by the equation (29) and (30) at the same time that the displacement of the cart tracks the following trajectory:

$$y_{pref} : \begin{cases} 0 & t < 0 \\ (1 - \cos(t)) & 0 \le t \le 2\pi \\ 0 & 2\pi \le t \le 4\pi \\ -2e^{0.5(4\pi - t)} & t \ge 4\pi \end{cases} \quad (42)$$

The desired displacement (42) has smooth switching at $t = 0$, $t = 2\pi$ and non-smooth switching at $t = 4\pi$. The non-smooth switching is introduced to study the behavior of the driven dynamics solution in response to a sharp change in the desired output.

- **Step3:**

The input θ_{ref} and the reference trajectory θ_{ref} that stabilize the internal dynamics (38) is computed by the integrated backstepping approach

C. simulation results

The simulation results are presented by fig. 2-4, fig2 show the evolution of the angle pendulum θ compared to the desired one θ_{ref}. In this figure, indeed, there is a perfect agreement between the two trajectories. Fig 3 displays the cart displacement y_p and the reference signal y_{pref}. Figure 4 represents the evolution of the stabilizing control law. The dynamics of this control signal is quite satisfactory.

In fact, there is no unacceptable physical overshoot. One can also see the reduced response time in which the control law stabilizes the controlled variable. The tracking error between the reference and the trajectory is reduced. This shows the very interesting results given by the developed approach.

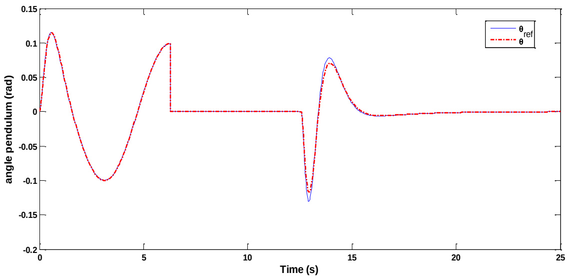

Fig. 2. Evolution of the cart displacement θ and the reference trajectory θ_{ref}

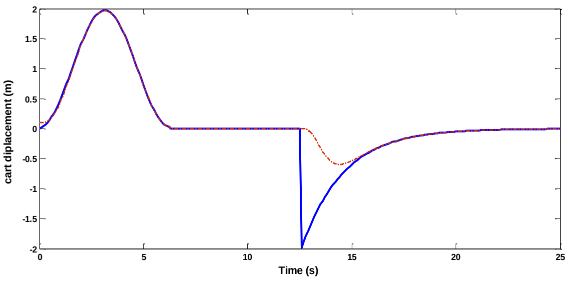

Fig. 3. Evolution of the cart displacement y_p and the reference trajectory y_{pref}

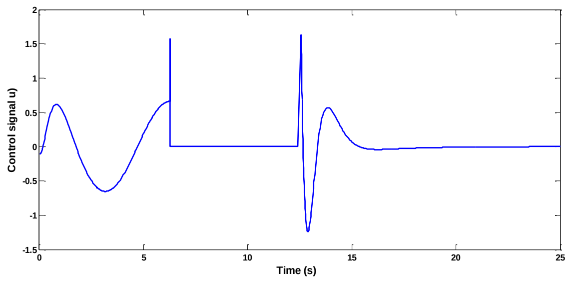

Fig. 4. Evolution of the control signal u

V. Conclusions

In this paper, the problem of tracking control has been addressed for a class of nonlinear non-minimum phase systems, based on approximation of the non-minimum phase system by another singular perturbed system. The proposed approach uses the input output linearization technique to cancel the nonlinearities of the external dynamics and to stabilize the internal dynamics by the integrator backstepping approach. A stability analysis of the proposed approach has been provided based on the singular perturbation theory. The efficacy and the validity of the proposed approach are illustrated through an example of inverted cart-pendulum.

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