P-TYPE ROBUST ITERATIVE LEARNING CONTROL ALGORITHM

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***Abstract***—**we study in this paper, the monotonic convergence of Proportional type robust iterative learning control algorithm. We want to control a class of two dimensional (2D) linear systems with parametric uncertainty and present external disturbances. The analysis and synthesis of this control law is based on H∞ setting and linear matrix inequality (LMI). Sufficient conditions for robust monotonic convergence of the proposed algorithm will be presented. A connected cart-inverse pendulum example is presented in the end of this paper to demonstrate the effectiveness of the proposed learning algorithm.**

 ***Keywords-* uncertain systems iterative learning control, robust control, linear matrix inequality, 2D systems, H∞ setting and robust stability.**

1 INTRODUCTION

The study of 2-D systems is a significant domain of control design, owing to their use in diverse applications such as repetitive processes, iterative learning controllers, and control synthesis.

Iterative processes are a distinct class of two dimensional 2D systems of both theoretical and practical interest. These systems cannot be controlled and studied by direct application of existing techniques from standard 1D systems theory. The key unique feature of 2D systems is that the process dynamics depend on two independent variables propagating information in two independent directions [1, 2]. The study of the 2D systems is motivated by many applications such as repetitive processes [3, 4, 5, 6], control synthesis and processes theoretic problems and iterative learning [7, 8, 9, 10].

Iterative learning control uses knowledge processes from previous iteration of repeated motion to generate a feedforward control law to use on subsequent iterations and thereby aims to improve performance from pass to pass. It is clear that iterative learning processes have two dimensional 2D structure, where information propagation occurs along a given finite time interval (first direction t) and from iteration to iteration (second direction k).

The study and analysis of stability and robust stability of two dimensional continuous-discrete systems were investigated by Busłowicz, [11, 12, 13, 14], Bistritz [15, 16] and Xiao [17], the problem of monotonic convergence of 2D processes is also studied in [18]. These problems are solved based on several stability study approaches like H∞ setting [18, 19], the performance weighting function [20] and the min-max method using the quadratic performance criterion [21].

The ILC approach was introduced by Arimoto, many approaches have been considered. Early research works on ILC approach were generally in their study, without precise synthesis or design procedures. However, the conditions of convergence presented in the literature are classically not sufficient for recent ILC tasks.

Robust iterative learning control represents an important topic for controlling systems with parameters uncertainties. The synthesis of this type of control law is based on different approaches. The H∞ approach based on resolution linear inequality matrix LMI problems offers the possibility to designing a new control law robust and effective used to improving the robust stability of 2D linear systems with considerable uncertainties in the parameters of matrix inputs. By using iterative learning controller the monotonic convergence and the systems stability are guaranteed and achieved after an esteemed number of iterations.

The robust monotonic convergence problem is presented, in this paper, for a class of uncertain process presented uncertainty in the parametric of the process.

The H∞ setting based on the LMI resolution technique is studied here to design a RILC capable to decrease the errors output from iteration to iteration and neglect the system uncertainty effect.

The rest of this paper is organized as follows. The ILC problem is defined and the class of 2D uncertain systems is described in section II. In section III, sufficient conditions for robust stability and robust monotonic convergence, based on H∞ setting with LMI techniques, are developed. A simulation results carried out on connected cart-inverse pendulum system are presented in section IV. Finally, a discussion of the results and a conclusion are presented.

2 PROBLEM SETUP

A class of two dimensional linear uncertain systems with parametric uncertainty in the system and nonzero constant initial error is studied here. The H∞ norm based on linear matrix inequality LMIs techniques is presented, in this paper, to design a new iterative algorithm to reduce the error from trial to trial and eliminate the uncertainty from the system. The monotonic convergence and the robust stability of 2D systems are guaranteed by using the proposed scheme. Our goal is to design and synthesis a new control law based on iterative learning control capable to drive the system described by (2) to follow the reference model described by (1) with zero error. The errors trajectory must decreases from iteration to iteration until becomes zero.

Let us consider the reference model defined by a state space model:

 (1)

Where ,  and  represent respectively the reference state vector, the reference control input and the reference output.

The resetting condition is satisfied at each trial i.e., where  is the initial state of the referenced model.

The systems considered in this paper are described by two dimensional uncertain linear state space models with nonzero constant initial error and parametric uncertainty in the system:

 (2)

Where is the state vector,  is the output, is the control input signal, is the disturbance,  is the constant matrix,  is the gain matrix of control input,  is the gain matrix of output,  is the gain matrix of disturbance input and ΔA, ΔB and ΔH represent admissible uncertainties. *k≥0* denotes the number of iteration and . The boundary condition defined by.

The uncertainties matrices ΔA, ΔB and ΔH are supposed verifying the following assumption:

 (3)

where*H1, E1, E2 and E3* are known constant matrices of compatible dimensions. *F* is unknown matrix with constant entries and satisfies

 (4)

**3 ROBUST STABILITY ANALYSIS**

For linear iterative processes of the form considered in the system (2), the general robust iterative learning control is described by the following structure:

 (5)

The learning rules and  represent respectively the robust control and the iterative learning control that is iteratively updated, where  and  represent the learning gains matrix and.

 (6)

The analysis and synthesis of RILC for 2D uncertain linear systems described by (2) will be presented here. Based on the state space model description of the systems dynamics, the sufficient conditions which guarantee the robust stability of the system and the robust monotonic convergence is developed in this section in terms of the feasibility of LMIs.

We define the tracking error model as follows:

 (7)

Let consider the following learning state variable:

**** (8)

With the help of the equality (2) and integrating the control law (6), we develop the new state variable described by the following expression:

**** (9)

Proof:

****

****

Where:







After substituting (2) into (1) and integrating the control law (6), the output error becomes:

**** (10)

Proof:

From the equality (10):

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From the equalities (9) and (10), we considered the new 2D uncertain linear system described by the following state representation:

 (11)

Where:

 , , , , , , ,,  and .

Based on (2) the induced uncertainties in the representation (11) verify the following condition:

 (12)

Where 

To show stability of systems described by (11), we will require a Lyapunov function interpretation where the variable function is taken to be:

**** (13)

With  and .

It is now routine to conclude that stability along the pass holds if .

It is clear that the 2D system dynamics represented in (11) are affected by disturbances and uncertainties. The principal goal in this approach is the design of a robust gain *K*rob and a P type iterative learning gain *KP*. These gains guarantee the system stability and the monotonic convergence while satisfying the H∞ constraint.

**Theorem 1:** Suppose that a robust control law described by (5) is applied to a 2D linear iterative system of the form (11), with uncertainties form modeled by (3) and (12). Then, the resulting system is stable along the pass for all tolerable uncertainties and has norm bound  if there exist matrices,, N1 and a scalar  such that the LMI presented in (14) holds:

 (14)

Where:









If (14) holds, the robust control law *Krob* is given by and the iterative control law *KP* are given directly from the resolution of the LMI.

Proof: introduced the associated Hamiltonian as:

**** (15)

And it is simple to show that H∞ disturbance attenuation is equivalent to:

We can write:

**** (16)

Where:



**** (17)

And











Applying a three successive modified Schur lemma to the equality (17) followed by replacing the variables by theirs appropriates expressions in the result then pre and post multiply the result by  to eliminate the bilinearity.

Then setting in the result. Finally, noting that the result doesn’t depend to and  leads to:

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**** (18)

 Where: 

The second term in the above inequality can be written as:

 (19)

Where:







Lemma 1: Let Ʃ1 and Ʃ2 be real matrices of appropriate dimensions. Then for any matrix *F* satisfying *FTF ≤ I* and a scalar ε ˃ 0 the following inequality holds [22]:

 (20)

An obvious application of lemma 1 followed by application of the Schur complement lemma and replacing the variables by theirs expression yields (14) and the proof is complete.

**4 SIMULATION EXAMPLE**

To prove the efficiency of our RILC approach we use the mechanical example represented by a connected cart-inverse pendulum (fig. 1).



Fig.1. Connected cart-inverse pendulum

The state variable x:



is the cart position and is angular position of the pendulum.

The desired input is the force practical to the cart and the disturbance is a friction:













and denote the cart-rail friction and the pendulum viscous friction respectively.

Suppose that: 

 For  we apply the decomposition procedure given by (15), we get



We propose, in this paper, three schemes, the first one is the design of a P RILC for 2D uncertain systems. A LMI solution of (16) is done by,  and .

Fig. 2, Fig. 3 and Fig.4 represent the result of the simulation of P Type scheme. Fig.5 presents the error result during the trials 300.



Fig.2. P Type scheme: Outputs and desired signal at the first iteration.



Fig.3. P Type scheme: Outputs and desired signal at the iteration 100.



Fig.4. P type scheme: Outputs and desired signal at the iteration number 300.



Fig.5. P type scheme: and  versus iteration k.

It is very clear that errors of the uncertain system decreases from iteration to iteration until becomes zero from the iteration number 20. The robust monotonic convergence is achieved and the stability of the system is demonstrated. The robust iterative learning control is designed well and it achieved the objective of the present approach. Our approach is fast comparing to others research work, in this example we can see that the convergence is achieved in the iteration 20.

**5 CONCLUSION**

A robust Monotonic convergence problem for a class of 2D linear systems with parametric uncertainty with non-zero constant initial error in the system is studied in this paper. Robust stability is successfully proved. Based on H infinity setting using the LMI techniques, a new robust iterative learning control is designed for uncertain linear systems with considerable disturbances. The sufficient conditions are given by the LMIs which can directly determine the learning gains of the proposed control law.

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