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Mixed H-infinity/D-stability control for linear repetitive processes with external disturbances

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Abstract— **This paper has as objective to study the problem of H-infinity control with D-stability constraint for uncertain continuous-time repetitive systems with external disturbances and design a control law, such that the closed-loop poles are placed within a particular region of the complex plane for all admissible uncertainties. Firstly, an equivalence between a two-dimensional control system and a repetitive control scheme such that study of convergence and stability properties have been proved. By analyzing these properties, all of the obtained conditions are formulated in the form of linear matrix inequalities and solutions gives the agreed controller gains. Finally, the performances of the proposed control laws were tested and simulated on an example and results are competitive in term of robustness and convergence.**

Keywords— **Repetitive control, uncertain linear systems, robust control, H-infinity and D-stability.**

I. INTRODUCTION

In engineering practice, repetitive processes are very common and are usually encountered in many industrial applications such as power supply systems [1-2], robotic manipulators [3], CD tracking [4], computer disk drives [5-6], etc. In those applications, the control systems are usually desired tracking or rejecting periodic exogenous signals with high control precision.

The repetitive control was first presented by Inoue et al. and applied to the control of a contouring servo system and a power supply for a proton synchrotron [7-8]. After that, it has been applied to many problems. The repetitive control affords a successfully practicable solution and that is a control scheme applied to systems that must cancel error, track periodic reference signals or reject periodic disturbances.

Referring to offered Wu et al. [9-13], some design methods of repetitive control system for a class of linear system based on two-dimensional continuous/discrete hybrid model are presented. The problem for the design repetitive controller is converted in a problem for a continuous-discrete two-dimensional system. After that, this problem is solved by combing two-dimensional Lyapunov theory with linear matrix inequalities approach.

In practice, the influence of external disturbances and uncertainties in the plant must be strictly considered when the repetitive controller is applied to real systems. In many cases, those parameters cause instability in the control system. The stability problem with the uncertainties is named robust stability problem. Yamada and al. [14-18] were proposed some design methods for repetitive control systems with considering disturbances and uncertainties.

In robust control system, stability of the closed-loop system makes the minimum specification. Sometimes, owing to bad transient responses in many applications or real physical systems, the system dynamic features do not make the desired goals such as transient oscillations, the rise time, the settling time, etc.

A satisfying performances can be achieved by placing the closed-loop pole to in an appropriate region of the complex plane. Enforcing all poles of a system in a specified region is named D-stability problem.

The main contribution of this paper is to study the problem of H-infinity control with D-stability constraint for uncertain continuous-time repetitive systems with external disturbances for all admissible uncertainties. In the first part, we will prove an equivalence between a two-dimensional control system and a repetitive control scheme such that study of convergence and stability properties. In the second part, all of the obtained conditions are formulated in the form of linear matrix inequalities and solutions gives the agreed controller gains. Finally, an example shows the efficiency of the proposed approach will be presented.

II. PROBLEM FORMULATION

Consider the uncertain linear system defined by the following state-space equation

$$\begin{cases} \dot{x}(t) = (A+\Delta A)x(t) + (B_u + \Delta B_u)\,u(t) + (B_d + \Delta B_d)\,d(t) \\ y(t) = C\,x(t) + D_u\,u(t) \end{cases} \tag{1}$$

where $x(t)$ is the state vector, $u(t)$ is the input control, $y(t)$ is the output of the system and $d(t)$ is an external disturbance. A , B_u , B_d , C and D_u are real matrices. ΔA , ΔB_u and ΔB_d denote real matrix functions representing norm-bounded time varying parametric uncertainties in the system model.

We consider the following assumptions:

(i) The pair (A, B_u) is stabilizable

(ii) $d(t)$ is an external disturbance signal with finite energy in the space $L_2[0,+\infty)$

(iii) Uncertainties under consideration have the following form

$$\begin{cases} A_\Delta = A + \Delta A = A + HF(t)E_A \\ B_{u\Delta} = B_u + \Delta B_u = B_u + HF(t)E_{Bu} \\ B_{d\Delta} = B_d + \Delta B_d = B_d + HF(t)E_{Bd} \\ F^T(t).F(t) < I \end{cases} \tag{2}$$

where I is the identity matrix of appropriate dimensions. $F(t)$ is unknown real time varying matrix contain uncertain parameters and H , E_A , E_{Bu} and E_{Bd} are known constant real matrices of appropriate dimensions denote how the uncertain parameters $F(t)$ affect the system (1).

The nominal plant is considered when $\Delta A = \Delta B_u = \Delta B_d = 0$. We can obtain:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_u\, u(t) + B_d\, d(t) \\ y(t) = C\, x(t) + D_u\, u(t) \end{cases} \tag{3}$$

The output error is defined by $e(t) = r(t) - y(t)$ where $r(t) = r(t+T)$ is the periodic reference and T is the fundamental period.

Fig. 1 represents the basic repetitive control system where G is the plant model.

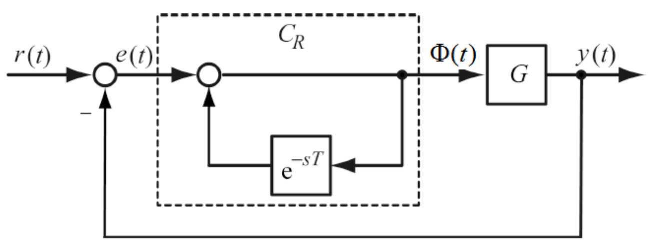

Fig. 1: Basic repetitive control system

The robust repetitive control law proposed of the system is

$$u(t) = G_{rob}\, x(t) + G_{rep}\Phi(t) \tag{4}$$

$$\Phi(t) = \begin{cases} e(t), & 0 \le t < T \\ \Phi(t-T) + e(t), & t \ge T \end{cases} \tag{5}$$

where $\Phi(t)$ defines the output signal of the repetitive controller and pair $\left(G_{rob}, G_{rep}\right)$ creates gain matrices to be determined. The first describes control action and the second describes the learning action.

Consider now the variables $k \in \mathbb{N}$ which used to describe learning between periods, $\tau \in [0,T[$ is a domain to depict control inside a period and $\psi(t)$ which is described in the time domain by

$$\begin{cases} \psi(t) = \psi(kT+\tau) := \psi_k(\tau) \\ \Delta\psi(t) = \psi(t) - \psi(t-T) := \Delta\psi_k(\tau) \end{cases} \tag{6}$$

Then, according to (1) - (6), we have

$$\Delta\dot{x}_k(\tau) = A_\Delta \Delta x_k(\tau) + B_{u\Delta}\, \Delta u_k(\tau) + B_{d\Delta}\, \Delta d_k(\tau) \tag{7}$$

$$e_k(\tau) = e_{k-1}(\tau) - C\,\Delta x_k(\tau) - D_u \Delta u_k(\tau) \tag{8}$$

Equations (7) and (8) creates a two-dimensional (2D) continuous-discrete hybrid model of the repetitive control system. Furthermore, (7) and (8) explicitly depict the robust control and learning actions, but the model (1) only characterizes the combined effect of those two actions.

Equation (7) represents the robust control action inside k-th period and second part describes the learning action between the k-th and (k-1)-th periods. As (7) doesn't includes the term $e_k(\tau)$, the control action during each period is explicitly independent of the learning action. In fact, (8) clarifies that learning is highly affected by the control action. For this reason, the convergence of the robust control action is fast, but learning is slower.

Next, the 2D control law can be written

$$\Delta u_k(\tau) = G_1 \Delta x_k(\tau) + G_2 e_{k-1}(\tau) \tag{9}$$

where

$$\begin{cases} G_1 = (I + G_{rep} D_u)^{-1}(G_{rob} - G_{rep} C) \\ G_2 = (I + G_{rep} D_u)^{-1} G_{rep} \end{cases} \tag{10}$$

Therefore, it is easy to conclude that the design of a 2D control law (9) is equally equivalent to design of a control law (4). Thus, the control system (4) is stable if a 2D stabilizing control law (9) is designed for the 2D system (10) and the matrices gains are given by

$$\begin{cases} G_{rob} = G_1 + G_2 (I - D_u G_2)^{-1}(D_u G_1 + C) \\ G_{rep} = G_2 (I - D_u G_2)^{-1} \end{cases} \tag{11}$$

Consequently, it is easy to adjust independently the robust control and learning actions using G_1 and G_2 respectively because the control action depends on both gains G_1 and G_2, while the learning action depends only on G_2. However, it is very difficult to do that using G_{rob} and G_{rep}.

III. PRELIMINARIES

In order to achieve the main results, some necessaries preliminaries will be introducing.

Lemma 1. [19]: Consider the following system:

$$\begin{cases} \dot{x} = Ax + B_d d \\ y = Cx \end{cases} \tag{12}$$

The system (12) is stable and satisfy H-infinity constraint if exists a Lyapunov function $V(x)$, for all $t > 0$ and given a scalar $\gamma > 0$ and a symmetric matrix $P > 0$, such that

$$\dot{V} + y^T y - \gamma^2 d^T d < 0 \tag{13}$$

Lemma 2. *(Shur complement)* [20]: For any symmetric matrix, Θ, of the form $\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{11}^T \\ \Theta_{12} & \Theta_{22} \end{bmatrix}$. If Θ_{22} is invertible then the following property hold:

$$\Theta < 0 \ \textit{if} \ \Theta_{22} < 0 \ \textit{and} \ \Theta_{11} - \Theta_{12}^T \Theta_{22}^{-1} \Theta_{12} < 0 \tag{14}$$

Lemma 3. [21]: Given matrices $K = K^T$, J, F and E of appropriate dimensions, then

$$K + JFE + (JFE)^T < 0 \tag{15}$$

for all F satisfying $F^T F \leq I$, if and only if there exists some $\varepsilon > 0$ such that

$$K + \varepsilon JJ^T + \varepsilon^{-1} E^T E < 0 \tag{16}$$

Definition 1. [22]: An LMI region is defined by a subset of the complex plane given by

$$D_z = \left\{ z \in \mathbb{C} : \Upsilon + \Lambda z + \Lambda^T \bar{z} < 0 \right\} \tag{17}$$

where $\Upsilon = \Upsilon^T$ and Λ are two real matrices.

In this paper, LMI region chosen is the intersection of three regions given in Fig. 1 by

D_1: conical sector : $\begin{cases} a\,\mathrm{Re}(z) + \left| b\,\mathrm{Im}(z) \right| < 0 \\ h = \arctan\left(-\dfrac{a}{b} \right) \end{cases}$

D_2: disk of radius r centered at $(q,0)$

D_3: $\alpha -$stability : $\mathrm{Re}(z) < -\alpha$

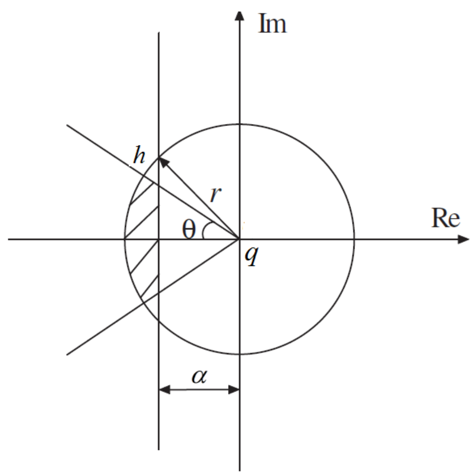

Fig. 2: LMI region

Theorem 1. [22]: Let Π a real matrix and $D_z = D_1 \cap D_2 \cap D_3$ be an LMI region. All the eigenvalues of Π are in LMI region D_z if exists a symmetric matrix Ψ such that we have the followings LMI

$$\begin{bmatrix} h(\Pi\Psi + \Psi\Pi^T) & (*) \\ \Psi\Pi^T - \Pi\Psi & h(\Pi\Psi + \Psi\Pi^T) \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} -r\Psi & (*) \\ -q\Psi + \Psi\Pi^T & -r\Psi \end{bmatrix} < 0 \tag{19}$$

$$2\alpha\Psi + \Pi\Psi + \Psi\Pi^T < 0 \tag{20}$$

In the next section, we will study the problem of H_∞ control with D-stability constraint for uncertain continuous-time repetitive systems which is used to analyze the system stability and to prove the convergence of the tracking error. The synthesis of this control law will be based on the optimization problem under LMI constraints.

IV. MAIN RESULTS

This section is devoted to developing the robust H_∞ control based on a repetitive control for uncertain system (1) and it is desired that the poles of the closed-loop system remain in region D_z of the complex plane.

Consider the following two-dimensional Lyapunov function

$$V_k(\tau) = V_{1,k}(\tau) + V_{2,k}(\tau) = \Delta x_k^T(\tau) P \Delta x_k(\tau) + e_k^T(\tau) Q e_k(\tau) \quad (21)$$

$$\begin{cases} V_{1,k}(\tau) = \left[\Delta x_k(\tau)\right]^T .P.\left[\Delta x_k(\tau)\right] \\ V_{2,k}(\tau) = \left[e_k(\tau)\right]^T .Q.\left[e_k(\tau)\right] \end{cases} \quad (22)$$

where $P > 0$ and $Q > 0$ are a symmetrical matrices.

The associated increment with Lyapunov function is defined by

$$\Delta V_k(\tau) = \dot{V}_{1,k}(\tau) + \Delta V_{2,k}(\tau) \quad (23)$$

$$\begin{cases} \dot{V}_{1,k}(\tau) = \Delta \dot{x}_k^T(\tau) P \Delta x_k(\tau) + \Delta x_k^T(\tau) P \Delta \dot{x}_k(\tau) \\ \Delta V_{2,k}(\tau) = e_{k-1}^T(\tau) Q e_k(\tau) - e_{k-1}^T(\tau) Q e_{k-1}(\tau) \end{cases} \quad (24)$$

Using Lemma 1, it easy shown that the H_∞ disturbance attenuation holds if there exist a scalar $\gamma > 0$ such that the Hamiltonian satisfies

$$\mathbb{H}_k(\tau) = \Delta V_k(\tau) + e_{k-1}^T(\tau) e_{k-1}(\tau) - \gamma^2 \Delta d_k^T(\tau) \Delta d_k(\tau) < 0 \quad (25)$$

Equation (25) can be set in the following form

$$\mathbb{H}_k(\tau) = \begin{bmatrix} \Delta x_k(\tau) \\ e_{k-1}(\tau) \\ \Delta d_k(\tau) \end{bmatrix}^T [\mathrm{X}] \begin{bmatrix} \Delta x_k(\tau) \\ e_{k-1}(\tau) \\ \Delta d_k(\tau) \end{bmatrix} < 0 \quad (26)$$

A. Design of Robust Control for Nominal Plant

In this section, we will develop an H-infinity norm based on a repetitive control for the nominal plant which is used to analyze the system stability and to prove the convergence of the tracking error. It is in addition desired that the poles of the closed-loop system remain in regions of the complex plane to ensure certain performances on the transient response. The problem of control law synthesis must be satisfied with sufficient existence conditions solutions solved by the following theorem:

Theorem 2.: Suppose the linear repetitive processes described by model (3). There exists a control law of the form (4) such that the closed-loop system is stable and satisfies the H-infinity constraint if and only if exist two symmetric matrices $\Pi_1 > 0$, $\Pi_2 > 0$ and two matrices Γ_1, Γ_2 such that we have the following LMI

$$\begin{bmatrix} -\Pi_2 & (*) & (*) & (*) & (*) & (*) & (*) \\ \alpha_1 & \alpha_2 & (*) & (*) & (*) & (*) & (*) \\ \alpha_3 & \Gamma_2^T B_u^T & -\Pi_2 & (*) & (*) & (*) & (*) \\ 0 & B_d^T & 0 & -\gamma^2 I & (*) & (*) & (*) \\ 0 & \alpha_4 & 0 & 0 & -\Pi_2 & (*) & (*) \\ 0 & 0 & 0 & 0 & 0 & -\Pi_2 & (*) \\ 0 & 0 & I & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (27)$$

$$\begin{bmatrix} h\alpha_2 & (*) \\ \Pi_1 A^T - A\Pi_1 + \Gamma_1^T B_u^T - B_u \Gamma_1 & h\alpha_2 \end{bmatrix} < 0 \quad (28)$$

$$\begin{bmatrix} -r\Pi_1 & (*) \\ -q\Pi_1 + \Pi_1 A^T + \Gamma_1^T B_u^T & -r\Pi_1 \end{bmatrix} < 0 \quad (29)$$

$$2\alpha\Pi_1 + \alpha_2 < 0 \quad (30)$$

where

$$\begin{cases} \alpha_1 = \Pi_1 C^T + \Gamma_1^T D_u^T \\ \alpha_2 = \Pi_1 A^T + A\Pi_1 + \Gamma_1^T B_u^T + B_u \Gamma_1 \\ \alpha_3 = \Gamma_2^T D_u^T - \Pi_2 \\ \alpha_4 = C\Pi_1 + D_u \Gamma_1 \end{cases} \quad (31)$$

then the system is generalized quadratically D-stable with H_∞ performance γ. After resolution of the LMI (27-30), the stabilization gains are given by

$$\begin{cases} G_1 = \Gamma_1 .\Pi_1^{-1} \\ G_2 = \Gamma_2 .\Pi_2^{-1} \end{cases} \quad (32)$$

Proof. :

By using (26), we can be set X in the following form

$$\mathrm{X} = \begin{bmatrix} \mathrm{X}_{11} & (*) & (*) \\ \mathrm{X}_{21} & \mathrm{X}_{22} & (*) \\ B_d^T P & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (33)$$

where

$$\begin{cases} X_{11} = (A + B_u G_1)^T P (A + B_u G_1) + (C + D_u G_1)^T Q (C + D_u G_1) \\ X_{21} = (B_u G_2)^T P + (D_u G_2 - I)^T Q (C + D_u G_1) \\ X_{22} = (D_u G_2 - I)^T Q (D_u G_2 - I) + I - Q \end{cases} \quad (34)$$

The inequality (33) becomes in the following expression

$$X = \begin{bmatrix} W^T V + VW + T^T RT + Y^T RY + Z^T Z - S & (*) \\ U^T V & -\gamma^2 I \end{bmatrix} < 0 \quad (35)$$

where

$$W = \begin{bmatrix} A + B_u G_1 & B_u G_2 \\ 0 & 0 \end{bmatrix}, \; T = \begin{bmatrix} 0 & 0 \\ C + D_u G_1 & D_u G_2 - I \end{bmatrix},$$

$$Y = \begin{bmatrix} C + D_u G_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & I \end{bmatrix}, \; U = \begin{bmatrix} B_d \\ 0 \end{bmatrix}.$$

$$V = \begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix}, \; R = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}, \; S = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix}$$

For eliminate bilinearities, lemma 2 is used as many times as necessary and (35) can be rewritten by

$$X = \begin{bmatrix} -R & (*) & (*) & (*) & (*) \\ T^T R & W^T V + VW - S & (*) & (*) & (*) \\ 0 & U^T V & -\gamma^2 I & (*) & (*) \\ 0 & RY & 0 & -R & (*) \\ 0 & Z & 0 & 0 & -I \end{bmatrix} \quad (36)$$

After replacing the variables with their expressions in (36), we get the following LMI

$$X = \begin{bmatrix} -Q & (*) & (*) & (*) & (*) & (*) & (*) \\ \lambda_1 & \lambda_2 & (*) & (*) & (*) & (*) & (*) \\ \lambda_3 & \lambda_4 & -Q & (*) & (*) & (*) & (*) \\ 0 & B_d^T P_1 & 0 & -\gamma^2 I & (*) & (*) & (*) \\ 0 & \lambda_5 & 0 & 0 & -Q & (*) & (*) \\ 0 & 0 & 0 & 0 & 0 & -Q & (*) \\ 0 & 0 & I & 0 & 0 & 0 & -I \end{bmatrix} \quad (37)$$

where

$$\begin{cases} \lambda_1 = (C + D_u G_1)^T Q \\ \lambda_2 = (A + B_u G_1)^T P + P(A + B_u G_1) \\ \lambda_3 = (D_u G_2 - I)^T Q \\ \lambda_4 = (B_u G_2)^T P \\ \lambda_5 = Q(C + D_u G_1) \end{cases} \quad (38)$$

Equation (37) will be pre-multiplying and post-multiplying, respectively, by $diag\left\{Q^{-1}, Q^{-1}, P^{-1}, Q^{-1}, I, Q^{-1}, Q^{-1}, I\right\}$ and its transpose. Therefore, the LMI becomes

$$X = \begin{bmatrix} -Q^{-1} & (*) & (*) & (*) & (*) & (*) & (*) \\ \delta_1 & \delta_2 & (*) & (*) & (*) & (*) & (*) \\ \delta_3 & \delta_4 & -Q^{-1} & (*) & (*) & (*) & (*) \\ 0 & B_d^T & 0 & -\gamma^2 I & (*) & (*) & (*) \\ 0 & \delta_5 & 0 & 0 & -Q^{-1} & (*) & (*) \\ 0 & 0 & 0 & 0 & 0 & -Q^{-1} & (*) \\ 0 & 0 & I & 0 & 0 & 0 & -I \end{bmatrix} \quad (39)$$

where

$$\begin{cases} \delta_1 = P^{-1}(C + D_u G_1)^T \\ \delta_2 = P^{-1}(A + B_u G_1)^T + (A + B_u G_1)P^{-1} \\ \delta_3 = Q^{-1}(D_u G_2 - I)^T \\ \delta_4 = Q^{-1}(B_u G_2)^T \\ \delta_5 = (C + D_u G_1)P^{-1} \end{cases} \quad (40)$$

Let

$$\begin{cases} \Pi_1 = P^{-1} \\ \Pi_2 = Q^{-1} \\ \Gamma_1 = G_1 \Pi_1 \\ \Gamma_2 = G_2 \Pi_2 \end{cases} \quad (41)$$

After replace and rearrange correspondent's terms, we obtain (27).

Now, it suffices to apply theorem 1 and replace the matrix Ψ by $A_{cl} = A + B_u G_1$ and choose $\Pi = \Pi_1$. After rearranging the terms, we can get LMI (28-30). Therefore, resolving the control law problem is to find two gains matrices G_{rob} and G_{rep} minimizing the H_∞ cost under D-stability constraint. This completes the proof.

B. Design of Robust Control for Plant with Uncertainties

Now, let consider the plant with uncertainties under consideration have the following form

$$\begin{cases} A_\Delta = A + HF(t)E_A \\ B_{u\Delta} = B_u + HF(t)E_{Bu} \\ B_{d\Delta} = B_d + HF(t)E_{Bd} \end{cases} \quad (42)$$

A sufficient existence conditions solutions must be satisfied and the problem of control law synthesis is solved by the following theorem:

Theorem 3. : For the uncertain system (1) and a given constant $\gamma > 0$, if there exists two symmetric matrices $\Pi_1 > 0$,

$\Pi_2 > 0$ and two matrices Γ_1, Γ_2 and a scalar $\varepsilon > 0$ satisfying the followings LMI:

$$\begin{bmatrix} -\Pi_2 & (*) & (*) & (*) & (*) & (*) & (*) & (*) & (*) \\ \alpha_1 & \alpha_2 & (*) & (*) & (*) & (*) & (*) & (*) & (*) \\ \alpha_3 & \Gamma_2^T B_u^T & -\Pi_2 & (*) & (*) & (*) & (*) & (*) & (*) \\ 0 & B_d^T & 0 & -\gamma^2 I & (*) & (*) & (*) & (*) & (*) \\ 0 & \alpha_4 & 0 & 0 & -\Pi_2 & (*) & (*) & (*) & (*) \\ 0 & 0 & 0 & 0 & 0 & -\Pi_2 & (*) & (*) & (*) \\ 0 & 0 & I & 0 & 0 & 0 & -I & (*) & (*) \\ 0 & \varepsilon H & 0 & 0 & 0 & 0 & 0 & -\varepsilon I & (*) \\ 0 & \alpha_5 & E_{Bu}\Gamma_2 & E_{Bd} & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \quad (43)$$

$$\begin{bmatrix} h\alpha_2 & (*) & (*) & (*) & (*) & (*) \\ \alpha_6 & h\alpha_2 & (*) & (*) & (*) & (*) \\ hH^T & -H^T & -\varepsilon I & (*) & (*) & (*) \\ H^T & hH^T & 0 & -\varepsilon I & (*) & (*) \\ \alpha_5 & 0 & 0 & 0 & -\varepsilon I & (*) \\ 0 & \alpha_5 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \quad (44)$$

$$\begin{bmatrix} -r\Pi_1 & (*) & (*) & (*) \\ -q\Pi_1 + \Pi_1 A^T + \Gamma_1 B_u^T & -r\Pi_1 & (*) & (*) \\ H^T & 0 & -\varepsilon I & (*) \\ 0 & \alpha_5 & 0 & -\varepsilon I \end{bmatrix} < 0 \quad (45)$$

$$\begin{bmatrix} 2\alpha\Pi_1 + \alpha_2 & (*) & (*) \\ \varepsilon H^T & -\varepsilon I & (*) \\ \alpha_5 & 0 & -\varepsilon I \end{bmatrix} < 0 \quad (46)$$

where

$$\begin{cases} \alpha_1 = \Pi_1 C^T + \Gamma_1^T D_u^T \\ \alpha_2 = \Pi_1 A^T + A\Pi_1 + \Gamma_1^T B_u^T + B_u \Gamma_1 \\ \alpha_3 = \Gamma_2^T D_u^T - \Pi_2 \\ \alpha_4 = C\Pi_1 + D_u \Gamma_1 \\ \alpha_5 = E_A \Pi_1 + E_{Bu}\Gamma_1 \\ \alpha_6 = \Pi_1 A^T - A\Pi_1 + \Gamma_1^T B_u^T - B_u \Gamma_1 \end{cases} \quad (47)$$

then the system is generalized quadratically D-stable with H_∞ performance γ. After resolution of the LMI (43-46), the stabilization gains are given by

$$G_1 = \Gamma_1.\Pi_1^{-1} \text{ and } G_2 = \Gamma_2.\Pi_2^{-1} \quad (48)$$

Proof. :

By using (24), we can be set X in the following form

$$\mathrm{X} = \begin{bmatrix} \mathrm{X}_{11} & (*) & (*) \\ \mathrm{X}_{21} & \mathrm{X}_{22} & (*) \\ B_{d\Delta}^T P & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (49)$$

where

$$\begin{cases} \mathrm{X}_{11} = (A_\Delta + B_{u\Delta}G_1)^T P (A_\Delta + B_{u\Delta}G_1) + (C + D_u G_1)^T Q (C + D_u G_1) \\ \mathrm{X}_{21} = (B_{u\Delta}G_2)^T P + (D_u G_2 - I)^T Q (C + D_u G_1) \\ \mathrm{X}_{22} = (D_u G_2 - I)^T Q (D_u G_2 - I) + I - Q \end{cases} \quad (50)$$

Let

$$W = \begin{bmatrix} A_\Delta + B_{u\Delta}G_1 & B_{u\Delta}G_2 \\ 0 & 0 \end{bmatrix},\ T = \begin{bmatrix} 0 & 0 \\ C + D_u G_1 & D_u G_2 - I \end{bmatrix},$$

$$Y = \begin{bmatrix} C + D_u G_1 & 0 \\ 0 & 0 \end{bmatrix},\ Z = \begin{bmatrix} 0 & I \end{bmatrix},\ U = \begin{bmatrix} B_d \\ 0 \end{bmatrix}.$$

$$V = \begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix}, R = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix},\ S = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix}$$

The inequality (49) becomes in the following expression

$$\mathrm{X} = \begin{bmatrix} W^T V + VW + T^T RT + Y^T RY + Z^T Z - S & (*) \\ U^T V & -\gamma^2 I \end{bmatrix} < 0 \quad (51)$$

Lemma 2 is used as many times as necessary and (51) can be rewritten by

$$\mathrm{X} = \begin{bmatrix} -R & (*) & (*) & (*) & (*) \\ T^T R & W^T V + VW - S & (*) & (*) & (*) \\ 0 & U^T V & -\gamma^2 I & (*) & (*) \\ 0 & RY & 0 & -R & (*) \\ 0 & Z & 0 & 0 & -I \end{bmatrix} < 0 \quad (52)$$

After replacing the variables with their expressions in (52), we get the following LMI

$$\mathrm{X} = \begin{bmatrix} -Q & (*) & (*) & (*) & (*) & (*) & (*) \\ \lambda_1 & \lambda_2 & (*) & (*) & (*) & (*) & (*) \\ \lambda_3 & \lambda_4 & -Q & (*) & (*) & (*) & (*) \\ 0 & B_{d\Delta}^T P & 0 & -\gamma^2 I & (*) & (*) & (*) \\ 0 & \lambda_5 & 0 & 0 & -Q & (*) & (*) \\ 0 & 0 & 0 & 0 & 0 & -Q & (*) \\ 0 & 0 & I & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (53)$$

where

$$\begin{cases}\lambda_1 = (C + D_u G_1)^T Q \\ \lambda_2 = (A_\Delta + B_{u\Delta} G_1)^T P + P(A_\Delta + B_{u\Delta} G_1) \\ \lambda_3 = (D_u G_2 - I)^T Q \\ \lambda_4 = (B_{u\Delta} G_2)^T P \\ \lambda_5 = Q(C + D_u G_1)\end{cases} \tag{54}$$

Pre-multiply and post-multiply (53), respectively, by $diag\left\{Q^{-1}, Q^{-1}, P^{-1}, Q^{-1}, I, Q^{-1}, Q^{-1}, I\right\}$ and its transpose. Thus, the LMI becomes

$$X = \begin{bmatrix} -Q^{-1} & (*) & (*) & (*) & (*) & (*) & (*) \\ \delta_1 & \delta_2 & (*) & (*) & (*) & (*) & (*) \\ \delta_3 & \delta_4 & -Q^{-1} & (*) & (*) & (*) & (*) \\ 0 & B_{d\Delta}^T & 0 & -\gamma^2 I & (*) & (*) & (*) \\ 0 & \delta_5 & 0 & 0 & -Q^{-1} & (*) & (*) \\ 0 & 0 & 0 & 0 & 0 & -Q^{-1} & (*) \\ 0 & 0 & I & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{55}$$

where

$$\begin{cases}\delta_1 = P^{-1}(C + D_u G_1)^T \\ \delta_2 = P^{-1}(A_\Delta + B_{u\Delta} G_1)^T + (A_\Delta + B_{u\Delta} G_1)P^{-1} \\ \delta_3 = Q^{-1}(D_u G_2 - I)^T \\ \delta_4 = Q^{-1}(B_{u\Delta} G_2)^T \\ \delta_5 = (C + D_u G_1)P^{-1}\end{cases} \tag{56}$$

Let

$$\begin{cases}\Pi_1 = P^{-1} \\ \Pi_2 = Q^{-1} \\ \Gamma_1 = G_1 \Pi_1 \\ \Gamma_2 = G_2 \Pi_2 \\ \bar{H}_0 = \begin{bmatrix} 0 & H & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ \bar{E}_0 = \begin{bmatrix} 0 & E_A \Pi_1 + E_{Bu} \Gamma_1 & E_{Bu} \Gamma_2 & E_{Bd} & 0 & 0 & 0 \end{bmatrix}^T\end{cases} \tag{57}$$

Let Σ_0 be the matrix that we consider X for the nominal system.

Now, it suffices to apply theorem 1 and replace the matrix Ψ by $A_{cl} = A_\Delta + B_{u\Delta} G_1$ and choose $\Pi = \Pi_1$.

$$\begin{bmatrix} h\alpha_{2\Delta} & (*) \\ \alpha_{6\Delta} & h\alpha_{2\Delta} \end{bmatrix} < 0 \tag{58}$$

$$\begin{bmatrix} -r\Pi_1 & (*) \\ -q\Pi_1 + \Pi_1 A_\Delta^T + \Gamma_1^T B_{u\Delta}^T & -r\Pi_1 \end{bmatrix} < 0 \tag{59}$$

$$2\alpha\Pi_1 + \Pi_1 A_\Delta^T + A_\Delta \Pi_1 + \Gamma_1^T B_{u\Delta}^T + B_{u\Delta} \Gamma_1 < 0 \tag{60}$$

where

$$\begin{cases}\alpha_{2\Delta} = \Pi_1 A_\Delta^T + A_\Delta \Pi_1 + \Gamma_1^T B_{u\Delta}^T + B_{u\Delta} \Gamma_1 \\ \alpha_{6\Delta} = \Pi_1 A_\Delta^T - A_\Delta \Pi_1 + \Gamma_1^T B_{u\Delta}^T - B_{u\Delta} \Gamma_1\end{cases} \tag{61}$$

Let

$$\begin{cases}\Sigma_1 = \begin{bmatrix} h\alpha_2 & (*) \\ \alpha_6 & h\alpha_2 \end{bmatrix} \\ \bar{H}_1 = \begin{bmatrix} hH & H \\ -H & hH \end{bmatrix} \\ \bar{E}_1 = diag(E_A \Pi_1 + E_{Bu} {\Gamma_1}^T, E_A \Pi_1 + E_{Bu} {\Gamma_1}^T)\end{cases} \tag{62}$$

$$\begin{cases}\Sigma_2 = \begin{bmatrix} -r\Pi_1 & (*) \\ -q\Pi_1 + \Pi_1 A^T + \Gamma_1^T B_u^T & -r\Pi_1 \end{bmatrix} \\ \bar{H}_2 = \begin{bmatrix} H \\ 0 \end{bmatrix} \\ \bar{E}_2 = \begin{bmatrix} 0 & E_A \Pi_1 + E_{Bu} {\Gamma_1}^T \end{bmatrix}\end{cases} \tag{63}$$

$$\begin{cases}\Sigma_3 = 2\alpha\Pi_1 + \Pi_1 A^T + A\Pi_1 + \Gamma_1^T B_u^T + B_u \Gamma_1 \\ \bar{H}_3 = H \\ \bar{E}_3 = E_A \Pi_1 + E_{Bu} {\Gamma_1}^T\end{cases} \tag{64}$$

Applying Lemma 2 and Lemma 3, inequalities (55, 58-60) can be given in the form

$$\begin{bmatrix} \Sigma_x & (*) & (*) \\ \bar{H}_x^T & -\varepsilon^{-1} I & (*) \\ \bar{E}_x & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{65}$$

where index x=0,1,2,3.

Equation (65) will be pre-multiplying and post-multiplying, respectively, by $diag\{I, \varepsilon I, I\}$ and its transpose. Therefore, the LMI becomes

$$\begin{bmatrix} \Sigma_x & (*) & (*) \\ \varepsilon \bar{H}_x^T & -\varepsilon I & (*) \\ \bar{E}_x & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{66}$$

After replacing and rearrange correspondent's terms, we get LMI (43-46). This ends the proof.

V. ILLUSTRATIVE EXAMPLE

In this section, we give an example to demonstrate the effectiveness of the proposed approach. Consider the following nominal linear system:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -2 & 3 \\ 4 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) + \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix} d(t) \\ y(t) = \begin{bmatrix} 4 & 0 \end{bmatrix} x(t) + u(t) \end{cases} \tag{67}$$

The uncertain matrices are described by:

$$F(t) = diag\left(1-\exp(-t), \sin(2t)\right) \tag{68}$$

$$H = \begin{bmatrix} 0 & 0 \\ 1 & 0.1 \end{bmatrix}, E_A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_{Bu} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, E_{Bd} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

The periodic reference trajectory and the external disturbance applied to the system has been defined, respectively, by the following functions:

$$r(t) = \sin\left(\frac{2\pi}{10}t\right) + 0.5\sin\left(\frac{4\pi}{10}t\right) \tag{69}$$

$$d(t) = 0.5\sin\left(\frac{2\pi}{10}t\right) \tag{70}$$

Thus, subject to constraints D_z we can choose $h=2,\ q=0,\ r=5$ and $\alpha=1$.

For the nominal system, by using Theorem 2, the gains of 2D controller and parameters of the robust repetitive control are:

$$\begin{cases} G_1 = [\text{-}2.9186 \quad 2.1421],\ G_2 = 0.2265 \\ G_{rob} = [\text{-}2.6019 \quad 2.7694],\ G_{rep} = 0.2928 \end{cases} \tag{71}$$

Simulation results (reference signal/output, tracking error and control input) in Fig. 3 show that the system is stable in closed-loop and enters the steady state in the third period.

For the uncertain system, by using Theorem 3, the gains of 2D controller and parameters of the robust repetitive control are:

$$\begin{cases} G_1 = [-2.5794 \ 1.3433],\ G_2 = 0.1053 \\ G_{rob} = [-2.4122 \ 1.5013],\ G_{rep} = 0.1177 \end{cases} \tag{72}$$

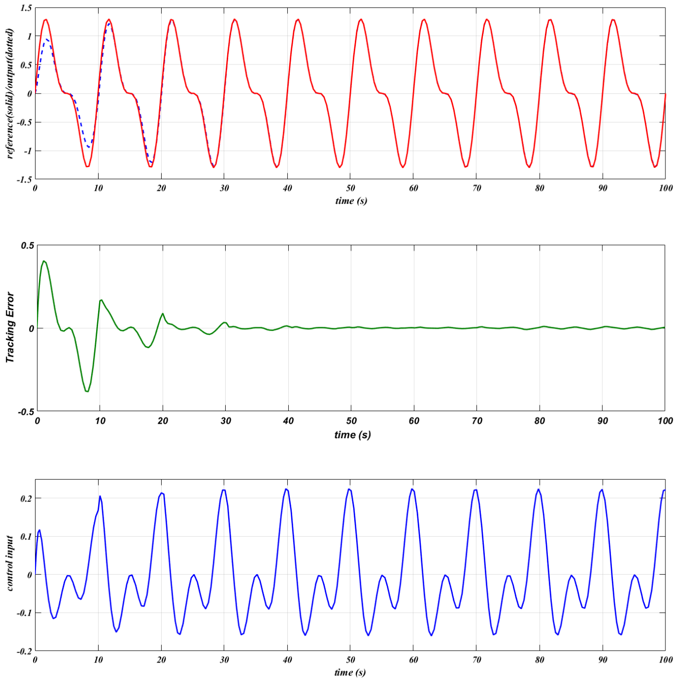

Fig. 3: Simulation results (r(t)/y(t), e(t), u(t)) for the nominal system

Simulation results are shown in Fig. 4. It easy to remark that the system is robustly stable for the periodic uncertainties and it enters into the steady state in the seventh period.

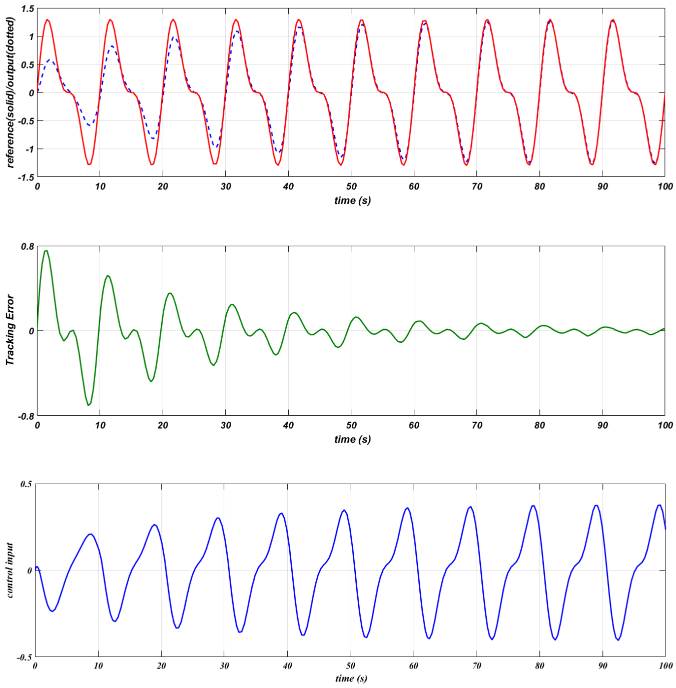

Fig. 4: Simulation results (r(t)/y(t), e(t), u(t)) for the uncertain system

VI. Conclusion

This paper is interested in the problem of H-infinity control with D-stability constraint for uncertain continuous-time repetitive systems with external disturbances. The main objective is the design of a control law, such that the system closed-loop poles are placed within a particular region of the complex plane for all admissible uncertainties. All of the obtained conditions are formulated in the form of linear matrix inequalities and solutions gives the agreed controller gains. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach. Repetitive control is no different from other control laws. It has its advantages, its disadvantages, its problems of robustness and applicability. However, it remains recommended for processes that work periodically or repetitively. Authors intend to continue research on this problem and the extension of obtained results to other class parameter uncertainty is actually under study.

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