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# New Multivariate Polynomial Interpolation-Based MPPT Applied to Battery Storage Photovoltaic System

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***Abstract*— This paper deals with the impact of many parameters on the behaviour of the battery-storage photovoltaic systems, particularly the nonlinearity of the load current as well as the atmospheric variable conditions. In order to optimize the energy transfer between source and load, it is necessary to instantaneously exploiting the maximum power point which can be generated by the photovoltaic generator.**

**This paper propose a new numerical analysis method based on multivariate polynomial interpolation applied to maximum power point tracking in energy storage photovoltaic system under unbalanced atmospherics condition. For the simulations setup of that battery-storage photovoltaic system, we used two real experimental measurements during a day, the first concern the irradiation, the second concern the temperature.**

**These long-term simulations results using by Matlab/simulink software, confirm the effectiveness of this proposed multivariate polynomial method both in terms of efficiency and time response.**

***Keywords*— Photovoltaic panel, MPPT, Multivariate polynomial method, Average DC/DC buck converter, Lead Acid Battery.**

## I. INTRODUCTION

The design of a model for an appropriate system is extremely complex. One of the research objectives is to simulate a real system using computer programs. Figure1 shows the process of approximating through simulation of any real system. It gives an overview of how the real system is approximated based on mathematical models which are implemented using various algorithms on a computer. However, the accuracy of models and simulations are essential in order to save considerable time and money in research process and development.

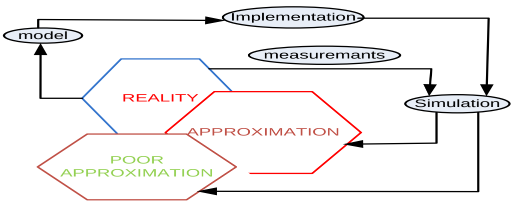


Fig.1. The process of approximating the real any system using numerical methods

In the literature a few results concerning the interaction optimization between a photovoltaic generator and a load. In particular, Essefi et al [1], have carried out a tracking maximum power controller using neural networks method for a stand-alone photovoltaic system, this method confirm her effectiveness both in terms of efficiency and fast response time, also it present negligible oscillations around the maximum power point, but it not accurate an especially it's ignores the intermediate inputs values of irradiation and temperature.

In [2], the author has presented a scheme for transferring power from the photovoltaic module to a battery using a solar charge controller based on a cuk DC/DC converter. In this work, a long-term irradiation is employed, but again the temperature is considered constant throughout a day.

In [3], the authors investigates the impact of the affects of load for the best choice of topology also the implementation issues of Incremental Conductance method with buck and Boost Converters are investigated. But the used load in this above work is linear, and the MPPT method presents the oscillations around the maximum power point.

Consequently these methods are not precise for real studies to the storage photovoltaic systems.

Thus, the goal of this research is to develop a new numerical approach MPPT for optimization of the power transfer to a photovoltaic energy storage system during the instantaneous changes of the climatic conditions.

In order to overcome this problem, a new bivariate polynomial method called the Lagrange's method is used to improve the capability of MPPT control strategy.

In this work, a topology of power interface and a dynamic battery model were used. We present a comparative study between two cases: firstly, we simulate our system under constant temperature and variable irradiation using the curve fitting technique method. In the second case, we simulate the same system under both variables temperature and irradiation using a new bivariate polynomial interpolation which is based on the Lagrange's theory.

## II. PHOTOVOLTAIC SYSTEM CONFIGURATION

The principle scheme of the studied photovoltaic battery storage system is shown in Fig. 2.

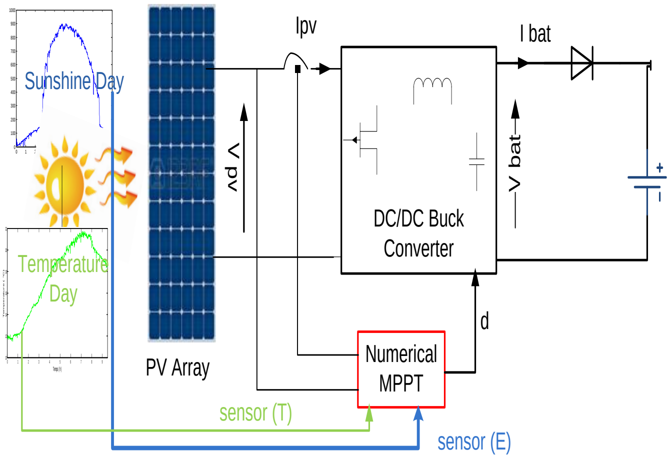


Fig 2. Diagram of the photovoltaic battery-storage system

This system consists of:

- A photovoltaic generator which characteristics are illustrated in the following table

TABLE I
PHOTOVOLTAIC MODULE PARAMETERS

| | |
|---|---|
| $P_{opt}$ | 135 *Wc* |
| $I_{opt}$ | 7.32 *A* |
| $V_{opt}$ | 17.75 *V* |
| $V_{cot}$ | 21.92 *V* |
| $I_{cc}$ | 7.82 *A* |
| Type cells | polycrystalline |
| Number of cells | 72 cells |

- A DC/DC buck converter which is used to adjust the photovoltaic array output voltage to a value corresponding to the maximum power deliverable to the battery.
- A lead acid battery
- An MPPT control used for extract the maximum power from the PV array under simultaneous variations of irradiation and temperature.

The following sections present the modelling and the characteristic of each subsystem of the photovoltaic battery-storage system.

### *A. Solar array characteristic analysis*

The electrical model and the parameters of solar cell are illustrated on Fig. 3.

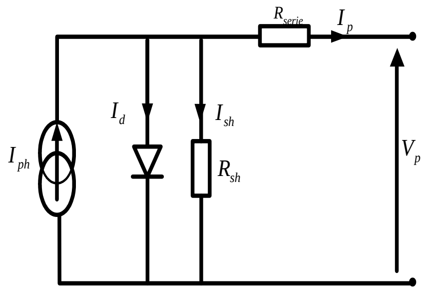


Fig 3. Photovoltaic cell of a equivalent circuit.

The relationship between the PV cell output current and voltage is given by [13]:

$$I_p = I_{ph} - I_s\left[\exp\left(\frac{q(V_p + R_s I_p)}{nK_B T}\right) - 1\right] - \left(\frac{V_p + R_s I_p}{R_{sh}}\right) .[4], [5] \quad (1)$$

The relationship between the PV array voltage and his output current is given by:

$$V_{PV} = \frac{N_s n T K_B}{q} Ln\left[\frac{I_{ph}}{I_s} - \frac{I_{pv}}{N_s I_s} - \frac{1}{R_{sh} N_p I_s}\left(\frac{V_{pv}}{N_s} + \frac{I_{pv} R_s}{N_p}\right) + 1\right] - \frac{I_{pv} R_s N_s}{N_p}. \quad (2)$$

Where $I_{pv} = Np * Ip$ and $Vpv = Ns * Vp$

The effects of solar irradiation levels and temperature are illustrated in typical (P-V) characteristics of GPV [3], [4].

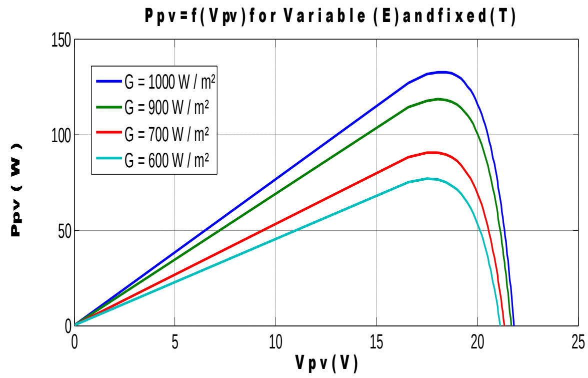


Fig.4. Variation of the GPV power and voltage, for different values of the illumination at fixed temperature

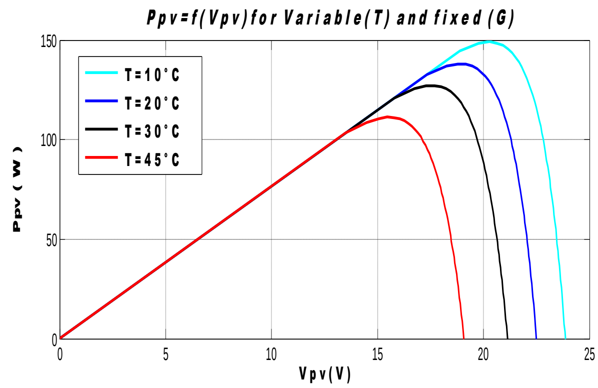


Fig.5. Variation of the GPV power and voltage, for different values of the temperature at fixed illumination

## B. Battery Model

In this paper we use the dynamic model most popular type of rechargeable battery used in [5-6-13]. The relationship governing the charging of this battery is given by (3).

$$f\,(it,i^*,i,Exp)=E_0-K\,\frac{Q}{it+0{,}1\times Q}\,i^*-K\,\frac{Q}{Q-it}\,it$$
$$+Laplace^{-1}\left[\frac{Exp(s)}{Sel(s)}\times\frac{1}{s}\right] \qquad (3)$$

Where:

$(i^* \prec 0)$, $E_0$ = Constant voltage $(V)$

$Exp(s)$ = Exponentiel zone Dynamics $(V)$

$Sel(s)$ =1, the battery mode during battery charging

$K$ = Polarization constant $(Ah^{-1})$

$i^*$ = Low frequency current dynamics $(A)$

$i$ = Battery current $(A)$, $it$ = Extracted capacity $(Ah)$

$Q$ = Maximum battery capacity $(Ah)$

$A$ = Exponential voltage (V)

$B$ = Exponential capacity $(Ah^{-1})$

In our case, we used a lead acid battery which are characteristics are illustrated in the following table.

TABLE 2
BATTERY PARAMETERS

| | |
|---|---|
| Nominal voltage battery | $V_n = 12V$ |
| Nominal capacity battery | $Q_{max} = 100Ah$ |
| Full charge voltage | $Vfull = 13V$ |
| internal resistance battery | $r_n = 0{,}0012\Omega$ |

## C. Average model of the DC/DC converter

In stand-alone photovoltaic power systems, the output voltage is effectively a constant DC bus due to the slow dynamics of the batteries [7-13].

To model the converter we choose two state variables including capacitor voltage $Vbat(t)$ and inductor current $iL(t)$

The system state space representation is

$$\dot{x} = A\,x + B\,u \qquad (4)$$
$$y = C\,x$$

Where

$$A == \begin{bmatrix} 0 & \dfrac{-d^2+d-1}{L} \\ \dfrac{d^2-d+1}{L} & 0 \end{bmatrix},\; B = \begin{bmatrix} \dfrac{d^2}{L} & 0 \\ 0 & \dfrac{d}{C} \end{bmatrix},\; C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T,$$

d is the switches control signal, $u = Vpv$ is the input vector, $y = Vbat$ is the output vector, and $x = [iL, Vbat]^T$ is the state variables vector.

The elaborated model is simulated in Matab/Simulink with the following parameters [13].

TABLE.3
SIMULATION PARAMETERS

| |
|---|
| $Vpv_{opt} = 17{,}6V$ for $E = 1000W/m^2$ and $T = 298\,°K$ |
| $C = 55\mu F$, $L = 120\mu H$, |

The dynamic response of the converter under variation of the duty cycle from d =1 to d = 0.68 at t = 0.02 s is the following:

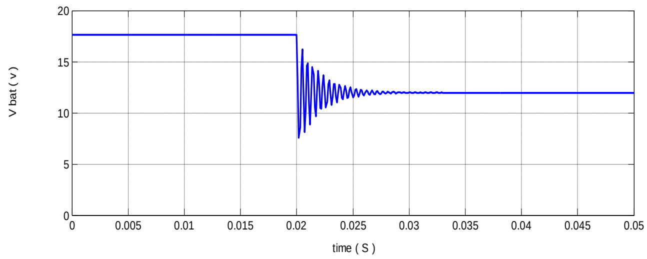


Fig. 6 Temporal response of the average back converter under duty cycle variations [13]

We notice that the voltage of battery requires 10 seconds to be stabilized. This result allows us to discern that this converter can provide high-precision representation.

## D. MPPT Algorithm

D. 1 Position of the problem

As shown in Fig. 5 the change in ambient temperature affects essentially the output voltage of the GPV and slightly her output current. In contrary, as shown in Fig. 4 the variation in solar irradiation affects considerably the output current of the GPV and slightly her output voltage. So, under unbalanced climatic conditions the output power of GPV is unstable.

Many maximum power point tracking techniques for photovoltaic system have been developed in the literature such as the P & O method or the IC method. Recently, the technique using neural networks is employed. These methods have advantage of being robust and relatively simple to design, but they have the following disadvantages:

- The load impacts is not considered.
- They present the simulations to particular points of temperature and irradiation.

Our goal is to simulate the photovoltaic energy storage system under long term atmospheric conditions, and under the affects of feedback current of battery.

For this reason, we used the experimental works which illustrated the variations of illumination and temperature during a day in Tunis (Tunisia) on 05 April 2012 [8-13]. These signals are presented by the Figs. 7-8.

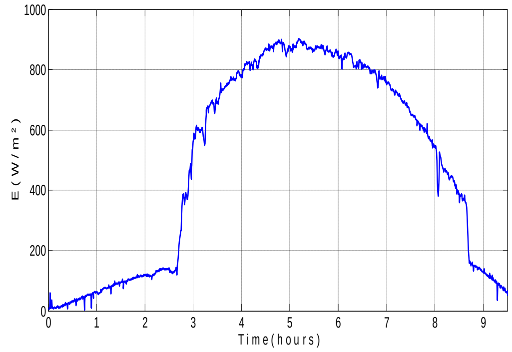

Fig. 7 Variation of the illumination during a day in Tunis (Tunisia) on 05 April 2012 [8-13]

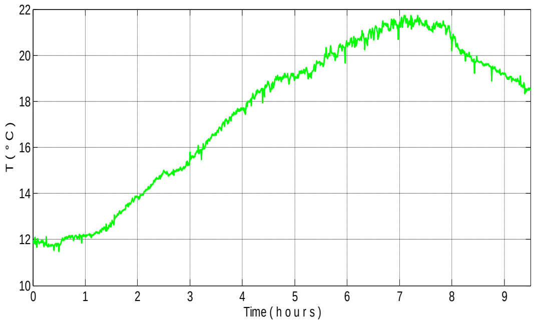

Fig. 8 Variation of the temperature during a day in Tunis (Tunisia) on 05 April 2012 [8]

D.2 Numerical MPPT Method

Often in engineering, interpolation is one tool which can be used to estimate the values of points between those points which were sampled [11].

Among the polynomial interpolation techniques we find:

- The Vandermonde method (easy to calculate, easy to generalize).
- The Lagrange polynomials (easy to find by hand).
- The Newton polynomials (efficient to implement when using Horner's rule).

We will look on the Lagrange's method for determined the interpolating polynomial whose coefficients are determined by the Vandermonde matrix.

Mathematically [9-10-11]:

Let $f$ is a continuous function in $R$, $x_0 \prec x_1 \prec \cdots \prec x_n$, $(n+1)$ are pair wise distinct dots in the interval $[a, b]$.

Consider the polynomials of degree (n) which are called Lagrange polynomials defined by:

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)} \quad , \quad 0 \leq j \leq n \qquad (5)$$

By posing: $\pi_n(x) = \prod_{j=0}^{n} (x - x_j)$

The Lagrange polynomials are written in a simpler way, of the form:

$$\forall x \neq x_i , \quad L_i(x) = \frac{\pi_n(x)}{(x - x_i)\pi_n'(x_i)} \qquad (6)$$

We demonstrate the following result: every continuous function on a bounded interval and known as $(n+1)$ distinct points can be approximated by a polynomial that coincides with this function in these $(n+1)$ dots.

Let $f : [a,b] \to R$ is a continuous function and if $(n+1)$ are pair wise distinct dots $x_0 \prec x_1 \prec \cdots \prec x_n$ in the interval $[a, b]$, then there is a single polynomial $P_n$ of degree n called polynomial interpolation of Lagrange, whose value coincides f with the dots $x_i$ , i.e. checking

$$P_n(x_i) = f(x_i) \qquad (7)$$

is which is given by the formula

$$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i) \qquad (8)$$

We prove the existence of the Lagrange Interpolation polynomial; one can also look analytically a polynomial of the form:

$$P_n(x) = a_n x^n + \quad .........a_1 x + a_0 \qquad (9)$$

Satisfactory the relations $P_n(x_i) = f(x_i)$

That is to solve the linear system

$$\begin{bmatrix} 1 & x_0 & x_0^1 & x_0^2 \cdots x_0^n \\ 1 & x_1 & x_1^1 & x_1^2 \cdots x_1^n \\ \vdots & & & \\ 1 & x_n & x_n^1 & x_n^2 \cdots x_n^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} \qquad (10)$$

This system has a unique solution because its determinant is that of Vandermonde is nonzero [9-10].

Another script, this system can be written as:

$$V * a = y \qquad (11)$$

Where

$\forall i = 0,......,n,$

The coefficients $a = (a_j)_{j=0,.......,n}$ are solutions of the linear system

$y = (f(a_j))_{j=0,.......,n}$

And V= $\begin{bmatrix} 1 & x_0 & x_0^1 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^1 & x_1^2 & \cdots & x_1^n \\ \vdots \\ 1 & x_n & x_n^1 & x_n^2 & \cdots & x_n^n \end{bmatrix}$ is the Vandermonde matrix.

We can generalize the Lagrange's method to interpolate multivariate real-valued functions. We will focus on bivariate polynomials which has the following form:

$$P(x,y) = \sum_{i,j} a_{i,j} x^i y^j \quad (12)$$

In our case, irradiation (G) and temperature (T) are the two variable parameters affecting the power of the photovoltaic generator. The Matlab software allows us to have the $(n * n)$ Vandermonde matrix (V) by evaluating the n terms at each of the n points. It also allows determining the coefficients of the polynomial which are written in the following table.

TABLE 4

COEFFICKENTS OF THE VANDERMANDE'S POLYNOMIAL INTERPOLATION

| | | |
|---|---|---|
| $C_1 = 10^{-4}$ | $C_2 = -52*10^{-4}$ | $C_3 = 845*10^{-4}$ |
| $C_4 = -9*10^{-5}$ | $C_5 = 21*10^{-4}$ | $C_6 = -1092*10^{-4}$ |
| $C_7 = 6*10^{-5}$ | $C_8 = 38*10^{-4}$ | $C_9 = 6002*10^{-4}$ |

The resultant polynomial interpolation is:

$$d(GT) = C_1 * G^2 * T^2 - C_2 * G^2 * T^1 + C_3 * G^2 - C_4 * G * T^2 + C_5 * G * T - C_6 * G + C_7 * T^2 + C_8 * T + C_9 \quad (13)$$

## III. LONG TERM SIMULATIONS SYSTEM AND INTERPRETATIONS

These tests aim to investigate the efficiency our new MPPT method for solar charger application during the simultaneous changes in temperature and irradiation during a day. Its are done with a signal of experimental measurement of irradiation illustrated in Fig 7 and a signal of temperature illustrated in Fig 8. These two signals are processed under Matlab with of samples 15 second.

In our simulations comparative two techniques are presented:

- The first technique is this least Square method; she

is implemented under long-term variations of irradiation and a constant temperature. This MPPT method is managed by relation between illumination G and the optimal duty-cycle as the following:

$$D_{opt} = k_6 G^6 + k_5 G^5 + k_4 G^4 + k_3 G^3 + k_2 G^2 + k_1 G + k_0 \quad (14)$$

The coefficients used in the above equation are shown in the table 5.

TABLE.5

COEFFICIENTS OF THE EQUATION (14)

| | |
|---|---|
| $k_0 = 0{,}9113$ | $k_1 = -0{,}0019$ |
| $k_2 = k_3 = 0$ | $k_4 = k_5 = k_6 = 0$ |

The relation between $P_{;qx}$ and $V_{opt}$ is given by

$P_{;qx}$ = $a_6 V_{opt}^6 + a_5 V_{opt}^5 + a_4 V_{opt}^4 + a_3 V_{opt}^3 + a_2 V_{opt}^2 + a_1 V_{opt} + a_0$

(15)

The coefficients used in the above equation are shown in the table 6.

TABLE.6

COEFFICIENTS OF THE EQUATION (15)

| | | |
|---|---|---|
| $a_0 = 69345$ | $a_1 = -21788$ | $a_2 = 2776$ |
| $a_3 = -182$ | $a_4 = 6$ | $a_5 = a_6 = 0$ |

The following figure shows the MPP trajectory for different irradiation values at constant temperature (25C°) [13].

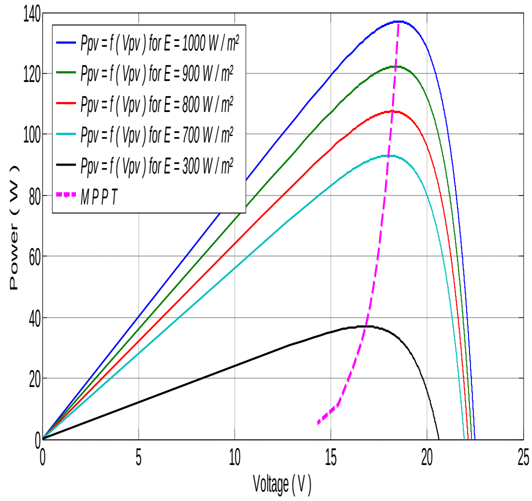


Fig. 11 Trajectory of MPPT of the photovoltaic Generator [13]

- The second technique is the Lagrange's method.

It interpolates two long-term variables (irradiation and temperature) by a bivariate polynomial given in (13).

The simulation results illustrated show that the difference which appeared in Figs. 12, 13, 14 and 15 is due to the irradiation drop that has not exceeded 22°C on the day of temperature signal levy.

However the unstable temperature affects all the characteristics of photovoltaic energy storage system, in particular the state of charge battery, the current of charge battery, the voltage of charge of battery. Thus, we confirm the performance of adapter and of new control numerical methods.

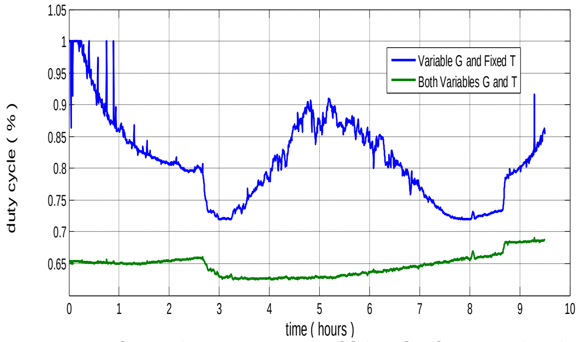


Fig. 12 Duty Cycles to (T = 25 C°, E variable) and to bivariate (T, E)

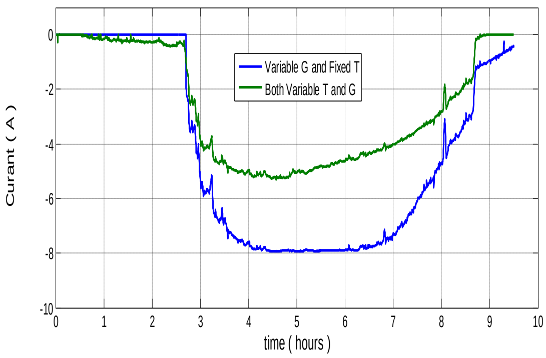


Fig. 13 current charge battery to (T =25 C°, E variable) and to bivariate (T, E)

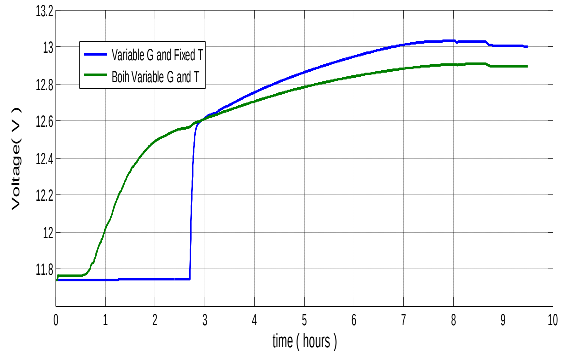


Fig. 14 voltage charge battery to (T =25 C°, E variable) and to bivariate (T, E)

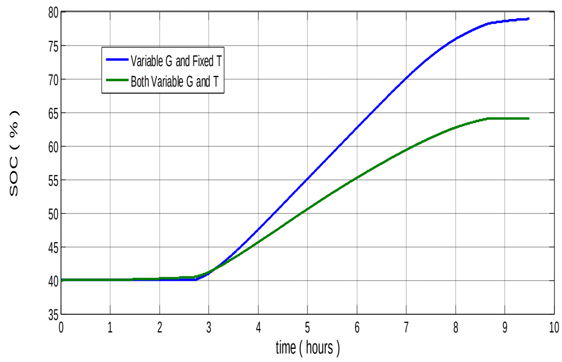


Fig. 15 States of charge to (T = 25 C°, E variable) and to bivariate (T, E)

## IV. CONCLUSION

In this paper, a new bivariate polynomial method has been investigated to maximum power point tracking applied to photovoltaic energy storage systems under variable atmospheric conditions The least Square technique has been also used to simulate this system without taking into account the variation of the temperature during a day.

It should be noted that during these simulations the retroation current of the load has been takes in consideration, as well as the added value of the use of a average buck converter DC/DC which present fast response time

These simulations results shows than the proposed polynomial interpolation method outperforms these conventional methods in term of the accuracy , particularly the least squares technique.

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