

# Iterative learning tracking control for a class of linear discrete-time switched systems

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**Abstract—** In this paper, we propose a new numerical approach for the iterative learning control of linear switched system. It is assumed that the considered switched systems are operated during a finite time interval repetitively, and then the iterative learning control scheme can be introduced. The objective of learning law is to guarantee the asymptotic convergence of the output error between the desired output and the actual output for the entire time interval through the iterative learning process. All the results are presented in terms of linear matrix inequalities (LMIs). A numerical simulation example is established shown the effectiveness of the proposed method.

**Keywords—** Switched systems; Iterative learning control (ILC); Hinf control; asymptotic stability; Linear matrix inequality.

## I. INTRODUCTION

The stability analysis of switched linear systems is one of the active over the past research due to their wide applications in many areas such (the automotive industry, aircraft and air-traffic control...) [1]. A switched system is a hybrid dynamical system, which consists of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching between them [2]. However, the stabilization problem of discrete switched systems with classic control law has been studied in [3], [4], [5], and [6]. Specifying the asymptotic stability for arbitrary switching systems over time systems is an especially attracting problem in this paper. Compared with the different results for synthesis issue by applying the classic control laws, relatively few efforts are made for designing a controller to achieve the strong stability for switched repetitive systems. This paper is addressed for the design feedback controllers (ILC) such that the system is asymptotically stable [7], [8]. Motivated by human learning, the basic idea of (ILC) is to use information from previous executions of the task in order to improve performance from pass-to-pass in the sense that the tracking error is sequentially reduced, and guaranteed the asymptotic stability of system. Control objectives can be achieved iteratively through

updating the control input in the iteration domain. However, to the best of our knowledge, the tracking error problem of discrete-time switched linear systems operating in a repetitive manner shows good results in practice, especially for the asymptotic stability problem with (ILC) control of switched systems, which motivates our present study. In this paper, the problem of iterative learning control for a class of linear discrete-time switched system is studied. The obtained formulation by applying ILC control is transformed into a synthesis problem of a special 2D repetitive switched system [9], [12]. However, the basic control problem like stability and stabilization can be relatively easily extended to involve robustness analysis and more control performance specifications using norms  $H_\infty$  [5], [10]. In particular, the main contribution of the paper is to provide the LMI characterization for asymptotic stability and then for solving the tracking error problem of linear discrete-time switched systems operating in a repetitive manner. In this context, the ILC tracking problem has been transformed into a stability analysis concept of 2D repetitive switched systems. In this sense, a necessary condition ensuring the stability under switching has been developed. This new formulation can be viewed as an interpretation of the  $H_\infty$  problem in the case of 2D repetitive switched systems.

This paper is organized as follows: In section 2, based on the problem formulation and the idea for transform the switched systems in to repetitive switched systems. In section 3, the control law design (ILC) by constructing a sequence of control inputs to a discrete switched system produces 2D repetitive system, and sufficient conditions for the existence of a stabilizing controller (ILC) are derived in terms of a set of matrix inequalities. In section 4, Numerical examples are presented to illustrate the feasibility and the effectiveness of the proposed design algorithms in this paper. Finally, the conclusion of this paper is given in Section 5.

II. PROBLEM FORMULATION

Consider the following discrete linear switched system described by the following form:

$$\begin{cases} x(t+1) = A_{\alpha(t)}x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad x(0) = 0, t \geq 0 \quad (1)$$

Where  $x(t) \in IR^n$  is the state vector in  $IR^n$ ,  $u(t) \in IR^m$  is the control vector in  $IR^m$ ,  $y(t) \in IR^m$  is the output vector  $\alpha(t): \{1,2,\dots\} \rightarrow M: \{1,2,\dots\}$  is a random switching rule, this means that the matrices  $(A_{\alpha}, B, C)$  are allowed to take values, at an arbitrary discrete time, in the finite set  $\{A_1, \dots, A_n\}$ .

Note that  $\alpha(t)$  is an arbitrary switching rule during the finite time interval  $[0, T]$ , we can assume the arbitrary switching rule is given as:

$$\alpha(t) : \begin{cases} 1, t \in [0, t_1] \\ 2, t \in [t_1, t_2] \\ \dots, t \in [t_{m-1}, T] \end{cases} \quad (2)$$

Where  $\alpha(t)$  denotes the switching sequence between three subsystem.

In the sequel, if  $k$  is defined as the switching number, Thus (1) is operated during a finite time interval repetitively, the system can be describes as:

$$\begin{cases} x_k(t+1) = A_k x_k(t) + Bu_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \quad x(0) = 0, t \geq 0 \quad (3)$$

Where,  $x_k(t)$  and  $u_k(t)$  are respectively the state and the control vector of system in switching  $k$ .  $t = 0, \dots, T, k \in IN$ .

According to the principle of ILC, there are two independent dynamic processes: system time  $t$  and learning iteration  $k$  during learning (process/ each mode) [8], [9], [10]. Every variable can be expressed as a 2-D repetitive switched function, such as  $x_k(t)$ , which represents  $x(t)$  in the  $k$ th-learning iteration ( $k$ th switching) [9]. This philosophy proposed to tackle the ILC problem for 2-D repetitive switched systems, with variable initial conditions. Basic assumption for the system is given as follow:

**Assumption:** the desired trajectory  $y_d(t)$  is iteration invariant.

III. MAIN RESULTS

In this section, we analyze the stability of switched system with three repetitive subsystems for the sake of convenience.

A. Stability analysis for linear switched system using ILC control

In this work, to solve the formulated problem we use the learning control described by the following updating ILC law:

$$u_{k+1}(t) = u_k(t) + K_1^p \eta_{k+1}(t+1) + K_2^p e_k(t+1) \quad (4)$$

And

$$\eta_{k+1}(t+1) = x_{k+1}(t) - x_k(t) \quad (5)$$

Where,  $e_k(t) = y_d(t) - y_k(t)$  is output tracking error,  $\eta_{k+1}$  denotes the state vector computed the cycle direction,  $K_1^p$  and  $K_2^p$  are gains appropriately dimensioned matrices to be designed, with  $p$  is number of subsystem.

Then clearly (4) and (5) can be written as, for a discrete switched system:

$$\begin{aligned} \eta_{k+1}(t+1) &= x_{k+1}(t) - x_k(t) \\ &= A_{k+1}x_{k+1}(t-1) + Bu_{k+1}(t-1) - A_kx_k(t-1) - Bu_k(t-1) \\ &= (A_{k+1} + BK_1^p)\eta_{k+1}(t) + (A_{k+1} - A_k)x_{k+1}(t-1) + BK_2^p e_k(t) \end{aligned} \quad (6)$$

And

$$\begin{aligned} e_{k+1}(t) &= y_d(t) - y_{k+1}(t) \\ &= y_d(t) - C(\eta_{k+1}(t+1) + x_k(t)) \\ &= e_k(t) - C\eta_{k+1}(t+1) \\ &= -C(A_{k+1} + BK_1^p)\eta_{k+1}(t) - C(A_{k+1} - A_k)x_{k+1}(t-1) + (I - CBK_2^p)e_k(t) \end{aligned} \quad (7)$$

$$x_{k+1}(t-1) = \eta_{k+1}(t) - x_k(t-1) \quad (8)$$

The obtained system in the closed-loop is given by the following form:

$$\begin{cases} \eta_{k+1}(t+1) = (A_{k+1} + BK_1^p)\eta_{k+1}(t) + [BK_2^p \quad (A_{k+1} - A_k)] \begin{bmatrix} e_k(t) \\ x_k(t-1) \end{bmatrix} \\ \begin{bmatrix} e_{k+1}(t) \\ x_{k+1}(t-1) \end{bmatrix} = \begin{bmatrix} -C(A_{k+1} + BK_1^p) \\ 1 \end{bmatrix} \eta_{k+1}(t) + \begin{bmatrix} (I - CBK_2^p) & -C(A_{k+1} - A_k) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_k(t) \\ x_k(t-1) \end{bmatrix} \end{cases} \quad (9)$$

The state-space model (9) is that of a discrete linear repetitive process of the form defined by the pass output and state vectors  $e_{k+1}(t), x_{k+1}(t-1)$  and  $\eta_{k+1}(t+1)$ , respectively.

The main contribution results in transforming the tracking error ILC problem into stability analysis concept of 2D repetitive switched systems.

B.  $H_{\infty}$  objectif problem

In the sequel we can rewrite the augmented system (9) as in [11].

Let  $G_{ibf}(t)$  is the transfer from  $\begin{bmatrix} e_k(t) \\ x_k(t-1) \end{bmatrix}$  to  $\begin{bmatrix} e_{k+1}(t) \\ x_{k+1}(t-1) \end{bmatrix}$ :

$$G_{ibf} = \begin{bmatrix} A_{bf} & B_{bfi} \\ C_{bfi} & D_{bfi} \end{bmatrix} = \quad (10)$$

$$\left[ \begin{array}{c} \overbrace{(A_{k+1} + BK_1^P)}^{A_{bf}} \\ \underbrace{[-C(A_{k+1} + BK_1^P) \quad 1]}_{[C_{bf1} \quad C_{bf2}]^T} \end{array} \right] \left[ \begin{array}{cc} \overbrace{[BK_2^P \quad (A_{k+1} - A_k)]}^{B_{bf1} \quad B_{bf2}} \\ \underbrace{[(I - CBK_2^P) \quad -C(A_{k+1} - A_k)]}_{[D_{bf1} \quad D_{bf2}]} \end{array} \right]$$

We consider the matrix  $G_{ibf}$  of the closed loop system (9), with the gain  $K_i^P$  and seek to minimize the norms  $H_\infty$  of the closed loop systems guaranteeing the convergence of error to zero. The objective problem is defined as follows:

$$\|G_{1e_{k+1}/e_k}\|_\infty < \gamma \tag{11}$$

**C. LMI approach to  $H_\infty$  synthesis**

First, we consider the increased system (9), if  $\eta_{k+j}(t+1)$  is the input signal and  $e_{k+j}(t)$  is the output signals the iterative learning control law can guarantee the asymptotic convergence of the output error between the desired output and the actual output for the entire time interval through the iterative learning process.

We have:

$$\begin{cases} \eta_{k+j}(t+1) = (A_{k+1} + BK_1^P)\eta_{k+j}(t) + BK_2^P e_k(t) \\ e_{k+1}(t) = -C(A_{k+1} + BK_1^P)\eta_{k+j}(t) + (I - CBK_2^P)e_k(t) \end{cases} \tag{12}$$

The design objectives for finding  $K_1^P, K_2^P$  is to minimize the  $H_\infty$  norm of the closed-loop transfer function  $G_{ibf}(t)$  for  $e_{k+1}(t)$  to  $e_k(t)$  i.e. Satisfies:

$$\|G_{1bf e_{k+1}/e_k}\|_\infty < \gamma \tag{13}$$

Theorem 2 [5]: Applying the theorem given in [5] to the closed-loop system (12), we see that our desired  $H_\infty$  controller exists if and only if there is symmetric positive matrix  $S_i$ , positive matrix  $G_i, N_1^P, K_2^P$ , with appropriate dimensions, and a scalar  $\alpha \in ]-1, 1[$ , such that we obtained the LMI (14) are feasible.

$$\begin{bmatrix} -\gamma^2 I & * & * & * \\ BK_2^P & \alpha^2 S_i - S_j & * & A_{k+1}G_i + BN_1^P + \alpha G_i - \alpha S_i \\ (I - CBK_2^P) & 0 & -I & -CA_{k+1}G_i - CBN_1^P \\ 0 & * & * & S_i - (G_i - G_i^T) \end{bmatrix} < 0 \tag{14}$$

In this case  $K_1^P$  are given by  $K_1^P = N_1^P(G_i)^{-1}$

**Proof:** Appling theorem in [5] with,

$$\begin{aligned} A_{ibf} &= (A_{k+1} + BK_1^P), B_{1bf} = BK_2^P \\ C_{1bf} &= -C(A_{k+1} + BK_1^P), D_{1bf} = (I - CBK_2^P) \end{aligned}$$

We obtained the following LMI condition (14).

**IV. NUMERICAL EXAMPLE**

In this section, a numerical example is given. Let us consider the linear discrete-time switched system (3), composed with three subsystems:

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.98 & 9.8 \\ 1.225 & -1.95 \end{bmatrix}, A_2 = \begin{bmatrix} -1.65 & 6.6 \\ 0 & -1.95 \end{bmatrix} \\ A_3 &= \begin{bmatrix} 8 & 0 \\ 1.6 & -0.8 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [-0.1 \quad 1] \end{aligned}$$

And the desired trajectory,

$$y_d(t) = \sin(8(T_j - 1)/25), j = \{1, \dots, n\}$$

The initial conditions  $x_k(0) = 0$ , and the control law  $u_k(t)$  are given in (4).

In this case, we assume the switching sequence is:

$$\alpha(t) : \begin{cases} 1, t \in [0,5] \\ 2, t \in [6,11] \\ 3, t \in [12,17] \end{cases}$$

Appling the of Theorem 2, and by varying values of  $\gamma$  and  $\alpha$ , the corresponding control law gains are obtained.

TABLE I. COMPARISON BETWEEN CONTROL LAW GAINS FOR DIFFERENT VALUE OF,  $\gamma$  AND  $\alpha$ .

$\gamma = 0.85, \alpha = 0.95$	$\gamma = 0.01, \alpha = 0.01$
$K_1^1 = [-1.317 \quad 2.9314]$ ,	$K_1^1 = [-1.323 \quad 2.94]$
$K_2^1 = 0.8233$	$K_2^1 = 1.0001$
$K_1^2 = [-0.161 \quad 2.568]$	$K_1^2 = [-0.165 \quad 2.64]$
$K_2^2 = -0.2735$	$K_2^2 = -7.354e^{-4}$
$K_1^3 = [-0.163 \quad 2.644]$ ,	$K_1^3 = [-0.165 \quad 2.64]$ ,
$K_2^3 = 0.5563$	$K_2^3 = 1$

Thus, the resulting ILC process can be guaranteed with its tracking error converging to zero along the iteration and the time. Figure 1 show the time evolution of the reference trajectory  $y_d(t)$  and the output  $y_k(t)$  at different value  $\gamma, \alpha$ . It is worth noting that the given conditions in Theorem 2 are obtained by using the lyapunov approach over the time switching and the relaxed parameters  $\alpha$  and  $\gamma$ , play a key role to guarantee the asymptotic convergence of the error

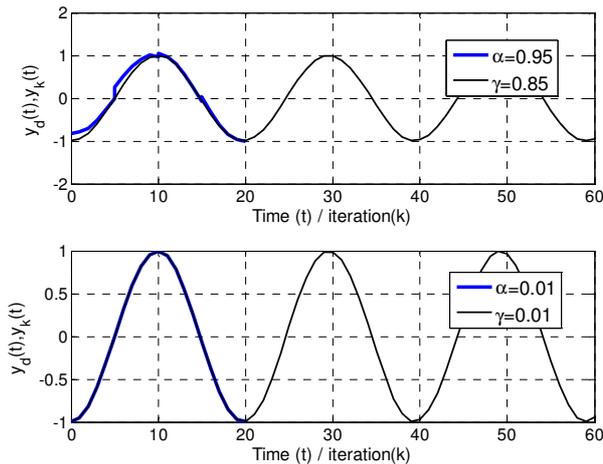


Figure1. The time and iteration evolution of the desired trajectory  $y_d(t)$  and the output  $y_k(t)$

Figure.2 shows the time and iteration evolution of the output error  $e_k(t) = y_d(t) - y_k(t)$ . As shown in this figure, the tracking error converging to zero along the time/iteration and more accurate as the iteration number increases.

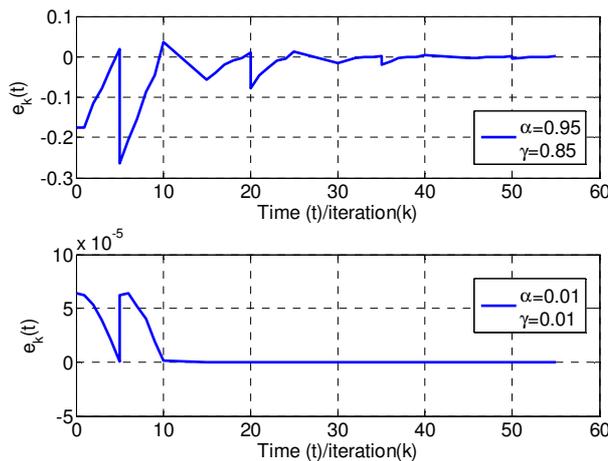


Figure2. Decreasing of the output tracking error as iteration number increased, for different value of  $\gamma$  and  $\alpha$

The previous results show the effectiveness of the repetitive control ILC compared to other conventional control [4]. The (ILC) control achieves the asymptotically stability of the studied closed-loop switched system, and guaranteed a small value of  $\gamma$ .

## V. CONCLUSION

In this paper, the problem of ILC-tracking control for a class of discrete-time linear switched systems under arbitrary switching signals in time domain has been investigated. The control law design (ILC) by constructing a sequence of control inputs to a discrete switched system produces 2D switched

repetitive system. This can guarantee the asymptotic stability of the closed loop switched system and the convergence error along the time and the pass. Necessary conditions for the existence of such controller are formulated in terms of a set of LMI. A numerical example is given to illustrate the effectiveness of the proposed method.

## REFERENCES

- [1] Y. Liu, L.Y. Sun, B.C. Lu and M.Z. Dai, Feedback control of networked switched fuzzy time-delay systems based on observer, ICIC Express Letters, vol.4, no.6(B), pp.2369-2376, (2010).
- [2] S.B Attia, S. Salhi, M. Ksouri , Static Switched Output Feedback Stabilization For Linear Discrete-time Switched Systems, International Journal Of Innovative Computing Information and Control (IJICIC) , Volume 8, Number 5(A), pp. 3203-3213, May (2012).
- [3] S.Ben Attia, S.Salhi, M.Ksouri,LMI Formulation For Static Output Feedback Design Of Discrete-time Switched Systems, Journal of Control Science and Engineering (JSCE), volume : (2009), article ID 362920, 7 pages.
- [4] S.Ben Attia, S.Salhi, M.Ksouri, Mode-Independent state feedback design for discrete switched linear, European Conference on Modelling and Simulation, ECMS, Madrid, 6-9 june (2009).
- [5] S.Ben Attia, S.Salhi, J.Bernussou, M.Ksouri, Improved LMI formulation for robust dynamic output feedback controller design of discrete-time switched systems via switched Lyapunov function, the 3rd International Conference on Signals, Circuits and Systems (SCS'09), November 6-8 Djerba, Tunisia (2009).
- [6] S.Ben Attia, S.Salhi, M.Ksouri, Static Switched Output Feedback Stabilization for Linear Discrete-Time Switched Systems, 17th IEEE International Conference on Electronics, Circuits, and Systems (ICECS), Athens-Greece, December 12-15, (2010).
- [7] J. Dridi, S. Ben Attia , S. Salhi, and M. Ksouri ,Repetitive Processes Based Iterative Learning Control Designed by LMIs, International Scholarly Research Network ISRN Applied Mathematics Volume (2012), Article ID 365927, 18 pages doi:10.5402/2012/365927.
- [8] J.Dridi,H.Ouerfelli,S.Salhi,,"Stability Along the Pass of Differential Linear Repetitive Process Using an LMI" International Conference on Integent Automation and Robotics,IEEE-ICIAR'2013,October 04-06-2013 Hammamet,Tunisia
- [9] H.Ouerfelli , J.Dridi, Ben attia, S., & Salhi, S. (2014). Iterative learning control for discret time switched. International Conference on Control, Engineering & Information Technology (CEIT'14), 310–321.
- [10] W. Paszke, K. Galkowski, E. Rogers, (2012). Repetitive process based iterative learning control design using frequency domain analysis. In Proceedings of the the 2012 IEEE multi-conference on systems and control (MSC 2012), Dubrovnik, Croatia, 3–6 October 2012.
- [11] C. Scherer, P. Gahinet, and M.Chilali, Multi-objective Output-Feedback Control via LMI Optimization, IEEE transactions on automatic control, vol. 42, no. 7, (1997).
- [12] B. Xu-Hui, Y. Fa-Shan , H. Zhong-Sheng, W. F, Zhong, Iterative Learning Control for a Class of Linear Discrete-time Switched Systems, Acta Automatica Sinica, Vol. 39, No. 9,(2013).
- [13] Rogers, E., Galkowski, K., & Owens, D. H, "Control systems theory and applications for linear repetitive processes", volume 349 of Lecture notes in control and information sciences. Springer, 2007.