

Robust Output Feedback Synchronization of Redundant Manipulators in Task Space with Time-varying Delays

Elif Cicek

Department of Control and Automation
Yildiz Technical University
Istanbul, Turkey
Email: ecicek@yildiz.edu.tr

Janset Dasdemir

Department of Control and Automation
Yildiz Technical University
Istanbul, Turkey
Email: janset@yildiz.edu.tr

Abstract—This study focuses on synchronization of kinematically redundant robot manipulators in task space under the effects of parametric uncertainty and time-varying communication delay. We consider as joint and/or end-effector velocities are not measurable and to overcome this problem we propose a linear-filter, which has a model-free structure. It is assumed that the position error and linear-filter output signals are shared over a directed communication topology under the effect of time-varying communication delay to guarantee the end-effector synchronization between robot manipulators. Control objectives are achieved by proposed output-feedback (OFB) robust controller and with the help of a combination of Lyapunov and Krasovskii stability analysis, we reach to a uniformly ultimately bounded result for both tracking and synchronization error signals. Finally, we present the simulation results on a system with 3 three-link planar robot manipulators to validate the feasibility of the suggested controller mechanism for synchronization.

I. Introduction

Design of general control structures for the use of robot manipulators in industrial applications has been an active and challenging research area in control engineering for many years. In parallel to the recent technological developments and appearance of new technologies, the size and scope of industrial applications are also drastically changing. Given the requirements in this area, it is clear that a single robot manipulator will not suffice for the required tasks such as assembly, welding, painting or transportation. For this reason, it is necessary to provide coordination of multiple robot manipulators with the use of communication structures so that they can perform a pre-planned task in a collaborative manner. The coordinated control structure to be designed must ensure that the end-effectors of the robot manipulators in operation are synchronized before reaching to a desired trajectory and track the trajectory simultaneously. Besides, manipulator trajectories for specific tasks to be performed are usually designed in task space by using the position that the end-effectors of the robot manipulators are to be located.

Also, redundancy in the manipulator structure, i.e. the structure that the dimension of link position variables n is greater than the dimension of operation space m , has an important role on dexterity [1].

In literature, most of the recommended control structures need velocity measurements of joints and/or end-effectors, beside their positions. The velocity sensors used for this purpose are generally expensive and give noisy measurements, that may affect the system, negatively. Another subject that needs to be discussed about synchronization is the time delay caused by communication during the information sharing between robot manipulators. There are several proposed controllers assuming that there exist constant and bounded time delays, in literature. Considering a real communication system, it can not be expected that the time delay in the whole system has the same value during the operation or the time delay value between each agent is the same [2].

In [3], the leader-follower synchronization was provided by the velocity signal obtained from an virtual agent with the assumption of exact model knowledge. In [4] and [5], global asymptotic synchronization was guaranteed by using an adaptive control structure. Also, in [5], time-varying communication delay and redundancy were taken into consideration. Under the assumption of measurable joint velocities in [6] and [7] adaptive based controllers were suggested for directed communication topology with constant time-delay. Recently, Phukan and Mahanta suggested in [8], a full-state feedback sliding mode controller for the synchronization of non-redundant robot manipulators.

In this work, we propose a comprehensive solution for OFB synchronization of robot manipulators in task space. For the replacement of velocity error, inspired by [9], a linear-filter is designed. With the help of the surrogate signal generated by linear-filter, trajectory tracking problem of each agent is solved without measuring joint and/or end-effector velocities. To overcome the parametric uncertainty of dynamic model, we suggested a robust controller. It is assumed that the position error and linear-filter output signals are shared over a directed communication topology under the effect of time-varying communication delay to guarantee the synchronization between robot manipulators. Also, redundancy problem is taken into consideration. Performing a combination of Lyapunov and Krasovskii stability analysis, we reach to a uniformly ultimately bounded result for both tracking and synchronization

error signals. Finally, we present the simulation results on a system with 3 three-link planar robot manipulators to validate the feasibility of the suggested robust OFB control structure for synchronization.

II. Notations and Mathematical Preliminaries

The Euler-Lagrange based formulation of dynamic model for n -link, revolute, direct drive robot manipulator is given by the following form [10]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F_d\dot{q} + G(q) = \tau \quad (1)$$

Due to page limitation, readers are referred to see:

- [10], [11] for the properties of given model and signals in (1),
- [12], [13] for the properties of Jacobian matrix $J(q)$, pseudo-inverse Jacobian matrix $J^+(q)$ and their relation with task-space.

III. Output Feedback Task-Space Synchronization

A. Problem Definition

In this study we focused on the end-effector position synchronization of multiple robot manipulators, while each of them follows the given desired trajectory. To ensure the tracking goal for each robot manipulator in the system with the dynamical model given in (1), the position tracking error $e_i(t) \in \mathbb{R}^m$ ($i \in \mathcal{S} \triangleq \{1, \dots, N\}$) is defined as

$$e_i = x_{di} - x_i \quad (2)$$

where $x_i(t) \in \mathbb{R}^m$ stands for actual position and $x_{di}(t) \in \mathbb{R}^m$ symbolizes the desired trajectory given to the robot manipulators.

B. Linear-filter Design

Since we assumed that joint and/or end-effector velocities are not measurable, based on the subsequent stability analysis, we design a linear-filter $r_{fi} \in \mathbb{R}^m$ as

$$r_{fi} = p_i - (k_{1i} + 1)e_i \quad (3)$$

where $p_i(t) \in \mathbb{R}^n$ is an auxiliary variable with the dynamic equation

$$\dot{p}_i = -(k_{1i} + 1 + \alpha)p_i + ((k_{1i} + 1)^2 + \alpha(k_{1i} + 1) - (k_{1i} + 1) + k_{2i})e_i - e_{fi}. \quad (4)$$

In the given equation, $k_{1i}, k_{2i} \in \mathbb{R}^{n \times n}$, $\alpha \in \mathbb{R}^1$ are positive constant gains and α is same for all agents. $e_{fi}(t)$ is an auxiliary variable with the following dynamics

$$\dot{e}_{fi} = -k_{3i}e_{fi} + r_{fi} \quad (5)$$

with a positive constant gain $k_{3i} \in \mathbb{R}^{n \times n}$. Dynamics of (3) can be presented as,

$$\dot{r}_{fi} = -\alpha r_{fi} - (k_{1i} + 1)J_i(q_i)\eta_i + k_{2i}e_i - e_{fi} \quad (6)$$

where $\eta_i(t) \in \mathbb{R}^n$ is an auxiliary signal as follows

$$\eta_i = J_i^+(q_i)(r_{fi} + e_i + \dot{x}_{di}) + (I_n - J_i^+(q_i)J_i(q_i))g_i - \dot{q}_i. \quad (7)$$

Given vector function $g(\cdot) \in \mathbb{R}^n$ is the sub-task function, that is chosen according to pre-defined sub-task, e.g. joint limitation, manipulability, obstacle avoidance, etc. If we define sub-task error as $e_N = (I_n - J_i^+(q_i)J_i(q_i))(g_i - \dot{q}_i)$ it is easy to show that $e_N = (I_n - J_i^+(q_i)J_i(q_i))\eta_i$ which means we can regulate e_N as long as we can regulate η_i . For further information the reader is referred to [12], [13]. From this point, for ease of presentation we will use the notation

$$z_i = J_i^+(q_i)(r_{fi} + e_i + \dot{x}_{di}) + (I_n - J_i^+(q_i)J_i(q_i))g_i. \quad (8)$$

Also we can define the time derivative of position error as $\dot{e}_i = J_i(q_i)\eta_i - r_{fi} - e_i$ and the joint velocity as $\dot{q}_i = z_i - \eta_i$.

C. Error System Development

To obtain the dynamic formulation for error system, we take the time derivative of (7), multiply both sides with $M_i(q_i)$, substitute (6), \dot{e}_i and use \ddot{q}_i from E-L model given in (1), we yield to the open loop dynamics as

$$\begin{aligned} M_i\dot{\eta}_i &= C_i(q_i, \dot{q}_i)\dot{q}_i + F_{di}\dot{q}_i + G_i(q_i) - \tau_i + W_i(q_i)\dot{q}_i \\ &+ M_i(q_i)J_i^+(q_i) \left(-(\alpha + 1)r_{fi} - k_{1i}J_i(q_i)\eta_i \right. \\ &\quad \left. + (k_{2i} - 1)e_i - e_{fi} + \ddot{x}_{di} \right) \end{aligned} \quad (9)$$

with $W_i(q_i) \in \mathbb{R}^{n \times n}$

$$\begin{aligned} W_i(q_i)\dot{q}_i &= M_i(q_i) \left(J_i^+(q_i)(r_{fi} + e_i + \dot{x}_{di}) \right. \\ &\quad \left. + (-\dot{J}_i^+(q_i)J_i(q_i) - J_i^+(q_i)\dot{J}_i(q_i))g_i \right. \\ &\quad \left. + (I_n - J_i^+(q_i)J_i(q_i))\dot{g}_i \right). \end{aligned} \quad (10)$$

By using the definition given for \dot{q}_i and Property $C_i(q_i, \varsigma_i)\nu = C_i(q_i, \nu)\varsigma_i$, we reformulate the open-loop error dynamics as

$$\begin{aligned} M_i(q_i)\dot{\eta}_i &= -M_i(q_i)J_i^+(q_i)k_{1i}J_i(q_i)\eta_i - C_i(q_i, \dot{q}_i)\eta_i \\ &- F_{di}\eta_i - W_i(q_i)\eta_i - C_i(q_i, \eta_i)z_i - \tau_i + Y_i\theta_i \end{aligned} \quad (11)$$

where $Y_i\theta_i \in \mathbb{R}^n$ is a linear parameterization of

$$\begin{aligned} Y_i\theta_i &= C_i(q_i, z_i)z_i + F_{di}z_i + G_i(q_i) + W_i(q_i)z_i \\ &+ M_i(q_i)J_i^+(q_i) \left(-(\alpha + 1)r_{fi} \right. \\ &\quad \left. + (k_{2i} - 1)e_i - e_{fi} + \ddot{x}_{di} \right). \end{aligned} \quad (12)$$

denoting $Y_i(x_{di}, \dot{x}_{di}, \ddot{x}_{di}, x_i, q_i, e_i, e_{fi}, r_{fi}, g_i) \in \mathbb{R}^{n \times r}$ the regression matrix and $\theta_i \in \mathbb{R}^r$ system parameters, e.g. mass, inertia, friction coefficients. Based on the subsequent stability analysis, we design the controller as follows

$$\begin{aligned} \tau_i &= Y_i\hat{\theta}_i + J_i^T(q_i) \left(-\gamma_i(k_{1i} + 1)r_{fi} + \gamma_i k_{2i}e_i \right. \\ &\quad \left. + \Gamma_i \|Y_i\|^2 (r_{fi} + e_i) + \sum_{j \in \mathcal{S}_i} \delta_{ij} \tau_{syn} \right) \end{aligned} \quad (13)$$

with synchronization part τ_{syn}

$$\begin{aligned} \tau_{syn} &= k_{4ij}(e_i(t) - e_j(t - T_{ij}(t))) \\ &- (k_{1i} + 1)k_{5ij}(r_{fi}(t) - r_{fj}(t - T_{ij}(t))) \end{aligned} \quad (14)$$

In (13), $\hat{\theta}_i$ stands for the constant estimated values of system parameters. First three terms in pharantesis are for tracking problem of each robot manipulator, with the nonlinear damping term $\|Y_i\|^2(r_{fi} + e_i)$. δ_{ij} given in (13), symbolizes the communication among agents in the system, i.e. $\delta_{ij} = 1$ if agent i receives information from agent j , otherwise $\delta_{ij} = 0$, and S_i is a subset of S containing all agents that send information to agent i . Terms in τ_{syn} ensure the position synchronization as well as the velocity synchronization between agents, respectively. The signals $e_j(t - T_{ij}(t))$ and $r_{fj}(t - T_{ij}(t))$ represent time-varying delayed position error signals and surrogate velocity error signals transferred over communication links. Based on the subsequent Lyapunov-Krasovskii [14] stability analysis, the time-varying delay $T_{ij}(t)$ is a continuously differentiable function and $\dot{T}_{ij}(t) = d_{ij} < 1$. Given $\Gamma_i, \gamma_i, k_{2i}, k_{4ij}, k_{5ij} \in \mathbb{R}^{m \times m}$ are the positive and diagonal control gain matrices.

By substituting the control law (13) into (11), we obtain the closed loop dynamics for $\eta_i(t)$ as

$$\begin{aligned} M_i(q_i)\dot{\eta}_i = & -M_i(q_i)J_i^+(q_i)k_{1i}J_i(q_i)\eta_i - C_i(q_i, \dot{q}_i)\eta_i \\ & - F_{di}\eta_i - W_i(q_i)\eta_i - C_i(q_i, \eta_i)z_i + Y_i\tilde{\theta}_i \\ & - J_i^T(q_i) \left(-\gamma_i(k_{1i} + 1)r_{fi} + \gamma_i k_{2i}e_i \right. \\ & \left. + \Gamma_i \|Y_i\|^2(r_{fi} + e_i) + \sum_{j \in S_i} \delta_{ij} \tau_{syn} \right) \end{aligned} \quad (15)$$

D. Stability Analysis

Theorem 1: Considering a system, that consists of N robot manipulators with the dynamic structure in (1), the filter structure of (3), (4), (5) and the OFB robust controller given by (13), guarantee uniform ultimate boundedness of tracking and synchronization errors in the sense that

$$\|y(t)\| \leq \sqrt{\frac{\lambda_{max}}{\lambda_{min}} \|y(0)\|^2 \exp(-Kt) + \frac{\lambda_{max}}{4\Gamma K \lambda_{min}} \|\tilde{\theta}\|^2 (1 - \exp(-Kt))}$$

where

$$y_i(t) = \begin{bmatrix} \eta_i(t)^T & r_{fi}(t)^T & e_{fi}(t)^T & e_i(t)^T & e_{syn}(t)^T \end{bmatrix}^T$$

and

$$e_{syn}(t) = \begin{bmatrix} \delta_{ij}(e_i(t) - e_j(t - T_{ij}(t)))^T \\ \delta_{ij}(r_{fi}(t) - r_{fj}(t - T_{ij}(t)))^T \end{bmatrix}$$

with

$$\begin{aligned} \lambda_{1i} = & \frac{1}{2} \min \left\{ m_{1i}, \lambda_{\min} \{ \gamma_i \}, \lambda_{\min} \{ k_{2i} \}, \right. \\ & \left. \lambda_{\min} \{ k_{4ij} \}, \lambda_{\min} \{ k_{5ij} \} \right\} \\ \lambda_{2i} = & \frac{1}{2} \max \left\{ m_{2i}, \lambda_{\max} \{ \gamma_i \}, \lambda_{\max} \{ k_{2i} \}, \right. \\ & \left. \lambda_{\max} \{ k_{4ij} \}, \lambda_{\max} \{ k_{5ij} \} \right\}. \end{aligned} \quad (16)$$

In (16), m_{1i} and m_{2i} symbolize the lower and upper bounds of $\|M_i(q_i)\|$, respectively. All closed-loop signals remain

TABLE I: Robot Parameters and Controller Gains

Parameters & Gains	Robot 1	Robot 2	Robot 3
p_1	0.4752 [kg.m ²]	0.32 [kg.m ²]	0.5666 [kg.m ²]
p_2	0.12 [kg.m ²]	0.0819 [kg.m ²]	0.1594 [kg.m ²]
p_3	0.108 [kg.m ²]	0.0725 [kg.m ²]	0.1442 [kg.m ²]
β_1	1.2684 [kg.m ²]	0.8528 [kg.m ²]	1.4993 [kg.m ²]
β_2	0.3884 [kg.m ²]	0.2587 [kg.m ²]	0.485 [kg.m ²]
β_3	0.045 [kg.m ²]	0.0304 [kg.m ²]	0.0626 [kg.m ²]
f_{d1}	5.3 [Nm.s]	4.6 [Nm.s]	4.8 [Nm.s]
f_{d2}	2.4 [Nm.s]	1.9 [Nm.s]	1.6 [Nm.s]
f_{d3}	1.1 [Nm.s]	0.8 [Nm.s]	1.2 [Nm.s]
l_1	0.2 [m]	0.3 [m]	0.25 [m]
l_2	0.25 [m]	0.3 [m]	0.35 [m]
l_3	0.25 [m]	0.3 [m]	0.25 [m]
k_{1i}	0.5,0.5	0.5,0.5	0.5,0.5
k_{2i}	2,2	2,2	2,2
k_{3i}	50,50	50,50	50,50
k_{4ij}	35,35	35,35	35,35
k_{5ij}	60,60	60,60	60,60
γ_i	3,3	3,3	3,3
Γ_i	0.01,0.01	0.01,0.01	0.01,0.01
α	15	15	15

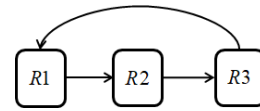


Fig. 1: Communication Topology.

bounded, as long as the positive gains satisfy following conditions

$$\begin{aligned} \alpha = & \frac{k_{6ij}}{\lambda_{\min} \{ k_{5ij} \}} + \frac{\lambda_{\max}^2 \{ k_{1j} + 1 \} (1 - d_{ij})}{4\lambda_{\min} \{ k_{5ij} \}} \\ & + \frac{\lambda_{\max}^2 \{ k_{2i} \}}{\lambda_{\min} \{ k_{5ij} \}} + \frac{\lambda_{\max}^2 \{ k_{2j} \} (1 - d_{ij})}{4\lambda_{\min} \{ k_{5ij} \} d_{ij}^2} \\ & + \frac{\lambda_{\max}^2 \{ k_{5ij} \} (1 - d_{ij})}{4\lambda_{\min} \{ k_{5ij} \}} \end{aligned}$$

$$\lambda_{\min} \{ k_{1i} \} > \zeta_{C_{2i}} \|z_i(0)\| + \Gamma_i (\|Y_i\|^2 + \|Y_i\|^4) + \sum_{i \in S_i} \delta_{li} \vartheta_{3li} d_{li}^2$$

$$\gamma_i = \frac{k_{7i}}{\alpha} + \frac{\lambda_{\max}^2 \{ k_{4ij} \}}{4\alpha} + \frac{\Gamma_i}{4\alpha}$$

$$k_{2i} = \frac{k_{9i}}{\gamma_i} + \frac{\lambda_{\max}^2 \{ k_{5ij} \}}{4\gamma_i} + \frac{\Gamma_i}{4\gamma_i}, \quad k_{3i} = \frac{k_{8i}}{\gamma_i} + \frac{\lambda_{\max}^2 \{ k_{5ij} \}}{4\gamma_i}$$

$$\lambda_{\min} \{ k_{4ij} \} > 1 + \frac{1 - d_{ij}}{4} + \frac{(1 - d_{ij})}{4d_{ij}^2} + \frac{1}{4(1 - d_{ij})}$$

$$k_{6ij} > 1 + \frac{1}{4(1 - d_{ij})}, \quad k_{7i} > \sum_{i \in S_i} \delta_{li} \vartheta_{3li} d_{li}^2$$

$$k_{8i} > \sum_{i \in S_i} \delta_{li} I_{n \times n}, \quad k_{9i} > \sum_{i \in S_i} \delta_{li} \vartheta_{2li} d_{li}^2$$

where $\vartheta_{1ij} = j_{2j}^2 (\lambda_{\max}^2 \{ k_{4ij} \} + \lambda_{\max}^2 \{ k_{5ij} \})$, $\vartheta_{2ij} = \lambda_{\max}^2 \{ k_{4ij} \} + \lambda_{\max}^2 \{ k_{5ij} \}$, $\vartheta_{3ij} = \lambda_{\max}^2 \{ k_{4ij} \} + \alpha^2 \lambda_{\max}^2 \{ k_{5ij} \}$ with the upper bound of $\|J_j(q_j)\| \geq j_{2j}$ and the subindice l stands for other agents in the system, that receive information from agent i .

Proof 1: To begin the stability analysis, we define a non-negative function as

$$\begin{aligned}
 V_1(t) &= \sum_{i \in \mathcal{S}} V_{1i}(t) \\
 &= \frac{1}{2} \sum_{i \in \mathcal{S}} \left\{ \eta_i^T M_i \eta_i + e_i^T \gamma_i k_{2i} e_i + e_{f_i}^T \gamma_i e_{f_i} \right. \\
 &\quad + r_{f_i}^T \gamma_i r_{f_i} \\
 &\quad + \sum_{j \in \mathcal{S}_i} \delta_{ij} (e_i(t) - e_j(t - T_{ij}(t)))^T \\
 &\quad \times k_{4ij} (e_i(t) - e_j(t - T_{ij}(t))) \\
 &\quad + \sum_{j \in \mathcal{S}_i} \delta_{ij} (e_{f_i}(t) - e_{f_j}(t - T_{ij}(t)))^T \\
 &\quad \left. \times k_{5ij} (r_{f_i}(t) - r_{f_j}(t - T_{ij}(t))) \right\} \quad (17)
 \end{aligned}$$

that can be upper and lower bounded as $\lambda_{1i} \|y_i\|^2 \leq V_{1i} \leq \lambda_{2i} \|y_i\|^2$ Differentiating (17) with respect to time and substituting (6), (5), \dot{e}_i , (15) with some algebraic manipulations we obtain

$$\begin{aligned}
 \dot{V}_1(t) &\leq \sum_{i=1}^N \left\{ - \left[m_{1i} j_{1i} j_{1i}^+ \lambda_{\min} \{k_{1i}\} \right] \|\eta_i\|^2 \right. \\
 &\quad + \left[\|z_i(t) + \Gamma_i (\|Y_i\|^2 + \|Y_i\|^4) \right] \|\eta_i\|^2 \\
 &\quad + \frac{1}{4\Gamma_i} \|\tilde{\theta}_i\|^2 - k_{7i} \|r_{f_i}\|^2 - k_{8i} \|e_{f_i}\|^2 - k_{9i} \|e_i\|^2 \\
 &\quad - \sum_{j \in \mathcal{S}_i} \delta_{ij} \left[\lambda_{\min} \{k_{4ij}\} - \left(1 + \frac{1-d_{ij}}{4} + \frac{(1-d_{ij})}{4d_{ij}^2} \right) \right. \\
 &\quad \left. + \frac{1}{4(1-d_{ij})} \right] \|e_i(t) - e_j(t - T_{ij}(t))\|^2 \\
 &\quad - \sum_{j \in \mathcal{S}_i} \delta_{ij} \left[k_{6ij} - \left(1 + \frac{1}{4(1-d_{ij})} \right) \right] \\
 &\quad \times \|r_{f_i}(t) - r_{f_j}(t - T_{ij}(t))\|^2 \\
 &\quad + \sum_{j \in \mathcal{S}_i} \delta_{ij} (1 - d_{ij}) \vartheta_{1ij} \|\eta_j(t - T_{ij}(t))\|^2 \\
 &\quad + \sum_{j \in \mathcal{S}_i} \delta_{ij} d_{ij}^2 (1 - d_{ij}) \vartheta_{2ij} \|e_j(t - T_{ij}(t))\|^2 \\
 &\quad + \sum_{j \in \mathcal{S}_i} \delta_{ij} (1 - d_{ij}) \|e_{f_j}(t - T_{ij}(t))\|^2 \\
 &\quad \left. + \sum_{j \in \mathcal{S}_i} \delta_{ij} d_{ij}^2 (1 - d_{ij}) \vartheta_{3ij} \|r_{f_j}(t - T_{ij}(t))\|^2 \right\}. \quad (18)
 \end{aligned}$$

where j_{1i} and j_{1i}^+ are lower bounds of $\|J_i(q_i)\|$ and $\|J_i^+(q_i)\|$, respectively. To analyze the stability of last four terms indicating time-delay, we define a Lyapunov-Krasovskii functional as

$$\begin{aligned}
 V_2(t) &= \sum_{i \in \mathcal{S}} \left\{ V_{1i}(t) + \sum_{j \in \mathcal{S}_i} \delta_{ij} \vartheta_{1ij} \int_{t-T_{ij}}^t \eta_j^T(\omega) \eta_j(\omega) d\omega \right. \\
 &\quad + \sum_{j \in \mathcal{S}_i} \delta_{ij} \vartheta_{2ij} d_{ij}^2 \int_{t-T_{ij}}^t e_j^T(\omega) e_j(\omega) d\omega \\
 &\quad \left. + \sum_{j \in \mathcal{S}_i} \delta_{ij} \vartheta_{3ij} d_{ij}^2 \int_{t-T_{ij}}^t r_{f_j}^T(\omega) r_{f_j}(\omega) d\omega \right\}
 \end{aligned}$$

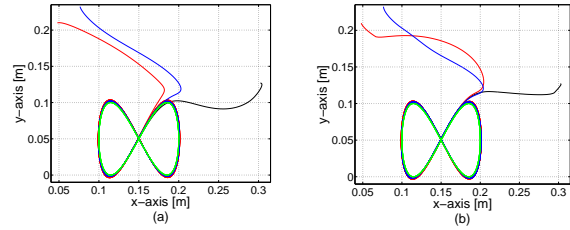


Fig. 2: End-effector trajectories (a) Without synchronization (b) With synchronization.

$$+ \sum_{j \in \mathcal{S}_i} \int_{t-T_{ij}}^t e_{f_j}^T(\omega) e_{f_j}(\omega) d\omega \quad (19)$$

that can be bounded similar as V_{1i} and we can write the upper bound for $\dot{V}_2(t)$ as

$$\begin{aligned}
 \dot{V}_2(t) &\leq \sum_{i=1}^N \left\{ - \left[m_{1i} j_{1i} j_{1i}^+ \lambda_{\min} \{k_{1i}\} \right] \|\eta_i\|^2 \right. \\
 &\quad + \left[\|z_i(t) + \Gamma_i (\|Y_i\|^2 + \|Y_i\|^4) + \sum_{i \in \mathcal{S}_i} \delta_{li} \vartheta_{1li} \right] \|\eta_i\|^2 \\
 &\quad + \frac{1}{4\Gamma_i} \|\tilde{\theta}_i\|^2 - \left[k_{7i} - \sum_{i \in \mathcal{S}_i} \delta_{li} \vartheta_{3li} d_{li}^2 \right] \|r_{f_i}\|^2 \\
 &\quad - \left[k_{8i} - \delta_{li} I_{n \times n} \right] \|e_{f_i}\|^2 \\
 &\quad - \left[k_{9i} - \sum_{i \in \mathcal{S}_i} \delta_{li} \vartheta_{2li} d_{li}^2 \right] \|e_i\|^2 \\
 &\quad - \sum_{j \in \mathcal{S}_i} \delta_{ij} \left[\lambda_{\min} \{k_{4ij}\} - \left(1 + \frac{1-d_{ij}}{4} + \frac{(1-d_{ij})}{4d_{ij}^2} \right) \right. \\
 &\quad \left. + \frac{1}{4(1-d_{ij})} \right] \|e_i(t) - e_j(t - T_{ij}(t))\|^2 \\
 &\quad - \sum_{j \in \mathcal{S}_i} \delta_{ij} \left[k_{6ij} - \left(1 + \frac{1}{4(1-d_{ij})} \right) \right] \\
 &\quad \left. \times \|r_{f_i}(t) - r_{f_j}(t - T_{ij}(t))\|^2 \right\}. \quad (20)
 \end{aligned}$$

For the general system, we can reform (20) as

$$\dot{V}_2(t) \leq -K \|y\|^2 + \frac{1}{4\Gamma} \|\tilde{\theta}\|^2 \quad (21)$$

and we can solve the differential inequality as [15]

$$\|y(t)\| \leq \sqrt{\frac{\lambda_{max}}{\lambda_{min}} \|y(0)\|^2 \exp(-Kt) + \frac{\lambda_{max}}{4\Gamma K \lambda_{min}} \|\tilde{\theta}\|^2 (1 - \exp(-Kt))}. \quad (22)$$

From (22), we can conclude that $y(t)$ is bounded as given in Theorem 1 (i.e. $\eta_i, r_{f_i}, e_i, e_{f_i}, e_{syn} \in \mathbb{L}_\infty$). Under the assumption that desired trajectory x_{di} and its derivatives, sub-task function $g(t)$ and $\dot{g}(t)$ are all bounded, by employing standard signal chasing arguments, we can say that all signals remain bounded.

IV. Numerical Studies

The suggested filter-based robust synchronization scheme was tested in Simulink™ of Matlab™, using three 3-link planar

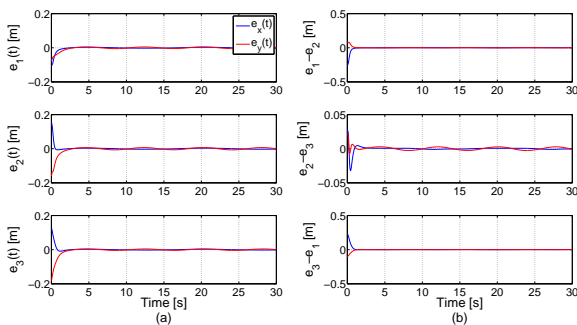


Fig. 3: (a) Position errors of each robots (b) Synchronization errors. Future work will be on extending this result to global asymptotic synchronization.

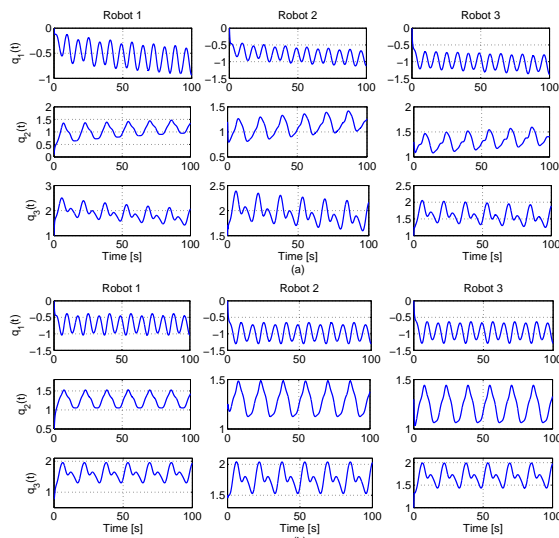


Fig. 4: Joint positions (a) Without subtask function (b) With subtask function.

robot manipulators with the dynamic model presented in [16] and parameters given in Table 1. It was assumed that link lengths are known and the other parameter values were estimated to be 20%, 30% and 15% for Robot 1, 2 and 3 incorrect, respectively. Time-delay values on communication graph were taken as $T_{1,2} = 0.1 + 0.06\sin(t)$, $T_{2,3} = 0.12 + 0.05\sin(0.5t)$, $T_{3,1} = 0.15 + 0.14\sin(0.3t)$. The desired trajectory of robot manipulators was defined as $x_{di}(t) = [0.6 + 0.1\cos(t) \quad 0.9 - 0.1\sin(t)]^T [m]$. The subtask function $g(t)$ was selected for all robots as $g(t) = -2(q_3 - q_2 + 0.5q_1) [1 \quad -1 \quad 1]^T$ as given in [13], to obtain the optimum link configuration is given by $(q_3 - 0.5q_2) = 0.5(q_2 - q_1)$.

End-effector positions of robot manipulator are presented in Figure 2(a) and (b). It is clear that under the effect of synchronization, end-effectors of robot manipulators meet before they track to the given desired trajectory. It can be seen from the Figure 3 that tracking errors of each robot and synchronization error between them stay in a bound around zero. Finally, Figure 4 shows that sub-task function $g(t)$ ensures the optimum link configuration of each robot

manipulator during synchronization and tracking.

V. Conclusion

In this work, we presented a complete solution for cooperative end-effector position synchronization of robot manipulators under the effects of time-varying delay and parametric uncertainties. We proposed a filter-based OFB robust control scheme to achieve aforementioned control objectives without velocity measurement. The proposed structure ensures the synchronization under a directed communication network, with a uniformly ultimately bounded tracking and synchronization errors.

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