

Bond Graph Based Modelling and SMC Control of a Tunnel Dryer

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Abstract—In this paper, a sliding mode control with a proportional integral (PI) sliding surface applied to the hybrid tunnel dryer has been assessed through simulations. A dynamic model of the process was developed specifically for this purpose. The pseudo-bond graph methodology was used in modelling the system, such methodology was very suitable for this thermal process and permits us to establish a mathematical model. The proposed sliding mode controller is used to control the thermal and mass variables in the dryer, such as temperature and product moisture content. Simulation results show that this control strategy can obtain excellent control performance with no chattering problem and a good tracking of trajectory.

Keywords—Pseudo bond graph modelling; sliding mode control; PI sliding surface; tunnel dryer; temperature; moisture content

I. INTRODUCTION

Drying is one of the oldest techniques used in chemical and food industries. Although many studies have been devoted to explore the different features and types of dryers, the literature is very poor in terms of optimization and control equipment. When we try to improve a drying process, different targets can be set according to the drying method, the type of product or application at industrial level.

Control of heat and mass transfer in drying process is important to enhance product quality such as colour and flavour [1]. The challenge for the engineering designer is now to define optimal dryers, which provide a product of constant good quality. Reducing the drying time allowing increased productivity, may be a priority. It can also act to improve the quality of the final product or to reduce manufacturing costs including restricting energy intake. Control of the drying process can also be effective in reducing the influence of external disturbances [2].

The first part of this work presented in this paper focuses on the development of a graphical model that describes the phenomena of heat and mass transfer in the dryer, for

combined drying process. The modeling tool used is the pseudo bond graph approach [3].

In the second part, we study the application of the Sliding Mode Control to the dryer; this method is designed for a class of non linear dynamic systems to tackle the problems with model uncertainties, parameter fluctuations and external disturbances. Sliding Mode Control (SMC) is a type of robust control design that plays an important role in the class of variable structure control systems (VSCS) [4].

II. DESCRIPTION OF THE SYSTEM

The system to describe is the solar tunnel dryer with an auxiliary heating system (Fig. 1). This system has been conceived to dry agricultural products that are placed on a tray located in a flow of hot air, and then moisture starts moving from the surface of the product to the chamber. The moist air exits the chamber through the chimney because of the vapour pressure difference between the chamber and the outside in the natural mode of operation.

The studied device admits following assumptions:

- ❖ Air density is constant.
- ❖ Air velocity distribution in the dryer is uniform.
- ❖ Air and water vapor are considered as perfect gases.
- ❖ The shrinkage of the product during the drying is neglected.
- ❖ The system studied is considered with lumped parameters, that has an evenly distributed temperature.

III. SYSTEM ORIENTED MODELLING IN BOND GRAPH SENS

A suitable unifying concept which can be used for this purpose is energy. Pseudo bond graphs are a graphical method which emphasizes the energetic interactions in a system by coupling components with energy bonds. The mathematical model of the system is then derived from the graph using techniques analogous to those of network analysis [5]. The technological level of modelling can be represented by word

pseudo bond graph model as shown in Fig. 2. Also different heats transfers took in consideration are well described in Fig. 2. Then the developed pseudo bond graph model is shown in Fig. 3.

A. Pseudo Bond Graph Elements And Mathematical Equations

1) effort sources

Se1 is an effort source used in this pseudo bond graph model and it represents the average temperature of the hot air T_{ha} (drying air) obtained from the solar and auxiliary module by:

$$T_{ha} = \frac{T_{aux} + T_s}{2} \quad (1)$$

Se2 is also an effort source and it represents the external air temperature T_{ex} .

2) C-fields

Cpr models the accumulation of energy at the surface of the product, the product temperature is given by:

$$T_{pr} = \frac{1}{C_{pr}} \int \dot{Q}_{pr} dt \quad (2)$$

Where \dot{Q}_{pr} is the thermal heat flow accumulated on the surface of the product and C_{pr} is thermal capacity of products.

Cch represents the accumulation of energy inside the chamber, the air temperature in the chamber is given by:

$$T_{ch} = \frac{1}{C_{ch}} \int \dot{Q}_{ch} dt \quad (3)$$

Where \dot{Q}_{ch} is the thermal heat flow accumulated in the chamber and C_{ch} is thermal capacity of the moist air.

Cwa models the accumulation of energy in the inner wall of the chamber, the temperature of the chamber wall is given by:

$$T_{wa} = \frac{1}{C_{wa}} \int \dot{Q}_{wa} dt \quad (4)$$

Where \dot{Q}_{wa} is the thermal heat flow accumulated in the chamber walls and C_{wa} is thermal capacity of the chamber wall.

3) R-fields

Rc1 models the convection heat transfer phenomena between the hot air and the agricultural product, the convective heat flow is given by:

$$\dot{Q}_{c1} = \frac{1}{R_{c1}} (T_{ha} - T_{pr}) A_{pr} = h_{c1} (T_{ha} - T_{pr}) A_{pr} \quad (5)$$

A_{pr} is the area of product and h_{c1} is the convective heat transfer coefficient between the hot air and the agricultural product.

$$\text{Where } h_{c1} = h_c = Nu \frac{\lambda_a}{D} \quad (6)$$

In which Nu is the number of Nusselt established on the basis of Reynolds number (Re) [6], that gives an idea about the flow regime, λ_a is the thermal conductivity of air and D is the characteristic diameter of the layer of the product.

Rc2 models the convection heat transfer phenomena between the agricultural product and the moist air in the chamber, the convective heat flow is given by:

$$\dot{Q}_{c2} = \frac{1}{R_{c2}} (T_{pr} - T_{ch}) A_{pr} = h_{c2} (T_{pr} - T_{ch}) A_{pr} \quad (7)$$

Where $h_{c2} = h_c$ is the convective heat-transfer coefficient between the agricultural product and the moist air.

Rc3 models the convection heat transfer phenomena between the moist air and the inner wall of the chamber, the convective heat flow is given by:

$$\dot{Q}_{c3} = \frac{1}{R_{c3}} (T_{ch} - T_{wa}) A_{wa} = h_{c3} (T_{ch} - T_{wa}) A_{wa} \quad (8)$$

Where A_{wa} is the area of the wall and $h_{c3} = h_c$ is the convective heat transfer coefficient between the moist air and the inner wall, using another expression of Nusselt number [6].

Rd models the conduction heat transfer phenomena between the chamber walls and the external environment through the insulation, the conduction heat flow is given by:

$$\dot{Q}_d = \frac{1}{R_d} (T_{wa} - T_{ex}) A_{wa} = \frac{1}{h_d} (T_{wa} - T_{ex}) A_{wa} \quad (9)$$

Where h_d (W/m²°C) is the conductive heat-transfer coefficient across the insulation and estimated by:

$$h_d = \frac{\lambda_i}{d_i} \quad (10)$$

Where λ_i (W/m°°C) is the thermal conductivity of the insulation and d_i (m) is the average mean thickness of the insulation.

Revap models the evaporation heat transfer phenomena from the agricultural product and the moist air in the chamber, the evaporation heat flow is given by:

$$\dot{Q}_{evap} = \frac{1}{R_{evap}} (T_{pr} - T_{ch}) A_{pr} = h_{evap} (T_{pr} - T_{ch}) A_{pr} \quad (11)$$

$$h_{evap} = 0.016 h_c \frac{[P(T_{pr}) - \gamma_{ch} P(T_{ch})]}{(T_{pr} - T_{ch})} \quad (12)$$

h_{evap} is the evaporative heat transfer coefficient and γ is the relative decimal humidity and P the saturated vapour pressure in (N/m²) given by Jain and Tiwari [7].

Revac models the phenomenon of discharge of humid air to the outside through the chimney for a natural type of flow [7], the corresponding heat flow is given by:

$$\dot{Q}_{evac} = c_d A_e \sqrt{2g \Delta H \Delta P} \quad (13)$$

Where:

$$\Delta P = [P(T_{ch}) - \gamma_{ex} P(T_{ex})] \quad (14)$$

$$\Delta H = \frac{\Delta P}{\rho_a g} \quad (15)$$

are the difference in partial pressure (N/m²) and the difference in pressure head (m) respectively.

c_d : coefficient of diffusivity

A_e : area of the exit section of the chimney

g : gravitational acceleration, (m.s⁻²)

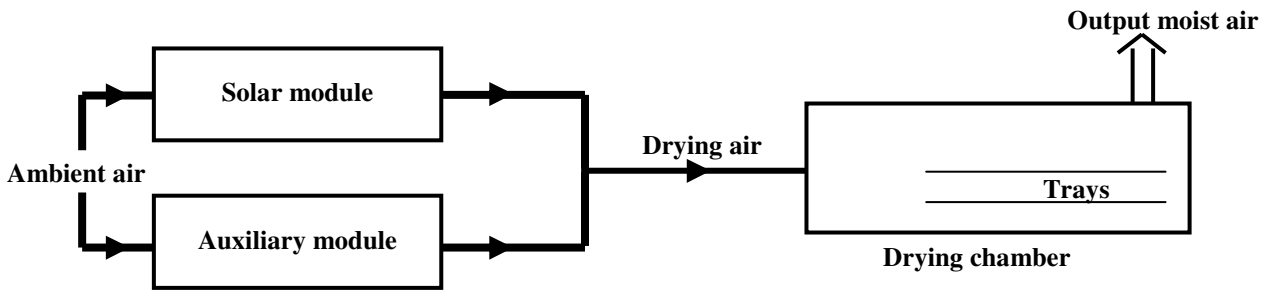


Fig. 1. Flow diagram of drying process in a tunnel dryer.

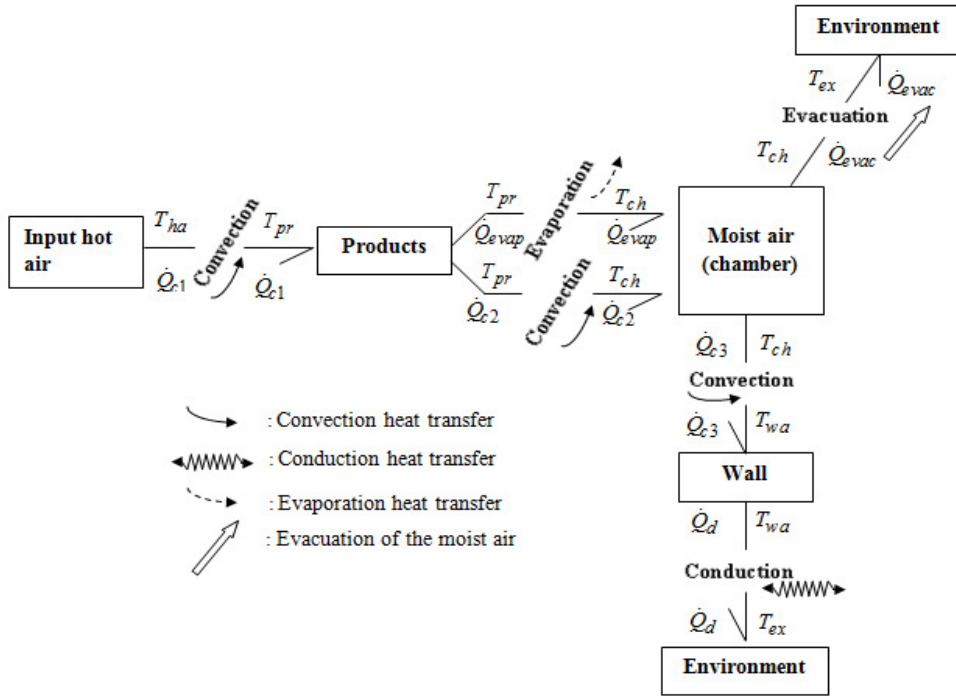


Fig. 2. Word pseudo bond graph model of the tunnel dryer.

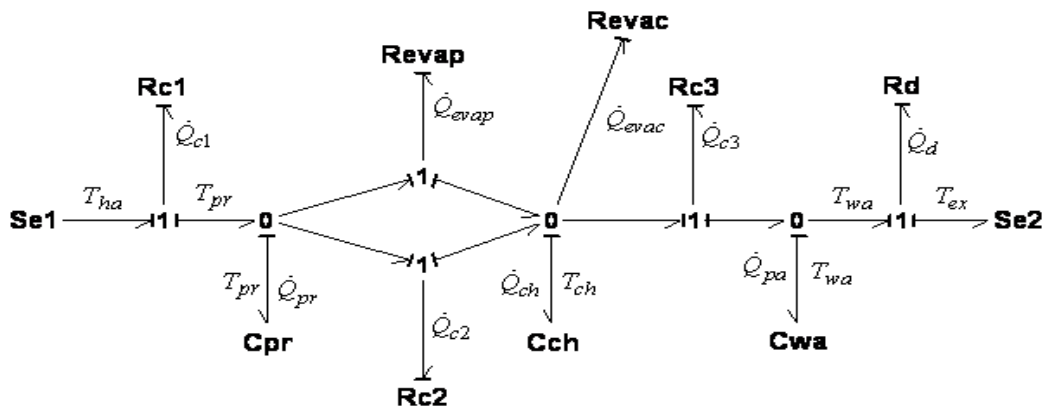


Fig. 3. Pseudo bond graph model of the tunnel dryer.

4) (0.1)-junctions

1-junctions correspond to the equality of flows

$$\left(\sum_i e_i = 0\right) \quad (16)$$

0-junctions correspond to the equality of effort and represent the energy flow balances.

$$\left(\sum_i f_i = 0\right) \quad (17)$$

Using the above equations determined by pseudo bond graph elements, the detailed equations for the energy flow balance become:

- energy balance equation of the product

$$C_{pr} \frac{dT_{pr}}{dt} = h_{c1}(T_{ha} - T_{pr})A_{pr} - (h_{c2} + h_{evap})(T_{pr} - T_{ch})A_{pr} \quad (18)$$

- energy balance equation of the moist air in the drying chamber

$$C_{ch} \frac{dT_{ch}}{dt} = (h_{c2} + h_{evap})(T_{pr} - T_{ch})A_{pr} - h_{c3}(T_{ch} - T_{wa})A_{wa} - c_d A_e \sqrt{2g \Delta H \Delta P} \quad (19)$$

- energy balance equation of the wall of the drying chamber

$$C_{wa} \frac{dT_{wa}}{dt} = h_{c3}(T_{ch} - T_{wa})A_{wa} - h_d(T_{wa} - T_{ex})A_{wa} \quad (20)$$

B. Drying Rate Equation

The theory of drying is described by Lewis based on the analogous of Newton's law of cooling in heat transfer and is often used to mass transfer in thin layer drying and is as follows:

$(-\frac{dM}{dt})$ is one of the most important parameters used in process drying. The following drying rate equation was obtained by Lopez et al [8]:

$$\left(-\frac{dM}{dt}\right) = k(M - Me) \quad (21)$$

Where k is the drying constant and it is related to the temperature of the moist air by:

$$k = 0.00719 \exp\left(-\frac{130.64}{T_{ch}}\right) \quad (22)$$

M is the instantaneous moisture content and Me is the equilibrium moisture content of the vegetable or the wet agricultural product, it was calculated by determining experimentally the equilibrium moisture isotherms at 25(°C), 40(°C), 60(°C) and 90(°C). GAB model was selected to predict Me because it was the model that better fit to experimental data. The following expression was obtained [8]:

$$Me = \frac{W_m C K a_w}{(1 - K a_w) [1 + (C - 1) K a_w]} \quad (23)$$

Where W_m , C and K are parameters related with air temperature by the following expressions:

$$W_m = 0.0014254 \exp\left(\frac{1193.2}{T_k}\right) \quad (24)$$

$$C = 0.5923841 \exp\left(\frac{1072.5}{T_k}\right) \quad (25)$$

$$K = 1.00779919 \exp\left(-\frac{43.146}{T_k}\right) \quad (26)$$

Where T_k is air absolute temperature (K) and a_w is the water activity.

IV. SLIDING MODE CONTROL

After modeling the system by bond graph approach in the first part of this study, the non linear dynamical model of the dryer is deduced from the energy balance equations (18, 19, and 20) which can be represented by the following state space model as:

$$\begin{cases} \frac{dT_{pr}}{dt} = \frac{1}{C_{pr}} \left[h_{c1}(T_{ha} - T_{pr})A_{pr} - (h_{c2} + h_{evap})(T_{pr} - T_{ch})A_{pr} \right] \\ \frac{dT_{ch}}{dt} = \frac{1}{C_{ch}} \left[(h_{c2} + h_{evap})(T_{pr} - T_{ch})A_{pr} - h_{c3}(T_{ch} - T_{wa})A_{wa} - c_d A_e \sqrt{2g \Delta H \Delta P} \right] \\ \frac{dT_{wa}}{dt} = \frac{1}{C_{wa}} \left[h_{c3}(T_{ch} - T_{wa})A_{wa} - h_d(T_{wa} - T_{ex})A_{wa} \right] \\ \frac{dM}{dt} = -k(M - Me) \end{cases} \quad (27)$$

Where:

$X(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T = [T_{pr}, T_{ch}, T_{wa}, M]^T$ is the state vector and $U(t) = T_{ha}$ is the control input vector.

The development of SMC for temperature tracking control of the dryer system consists of two steps. The first step is to select an equilibrium manifold or sliding surface, $S(t)$, to prescribe the desired dynamic characteristic of the controlled system. The second one is to design a control law to drive the system to the sliding mode $S(t) = 0$ and to maintain it there forever [9].

When we are in the "sliding mode", the trajectory remains on the switching surface. This can be expressed by:

$$S(t) = 0 \text{ and } \dot{S}(t) = 0 \quad (28)$$

A sufficient condition to guarantee that the trajectory of the error will translate from reaching phase to sliding phase is to select the control strategy, also known as the reaching condition [10]: ($S \dot{S} < 0$)

The trajectories are enforced to lie on the sliding surfaces. Let the tracking error in a closed-loop control system be $e(t)$, then a sliding proportional + integral (PI) surface in the space of error can be defined as:

$$S(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau \quad (29)$$

Where the coefficients $k_p, k_i \in \mathfrak{R}^+$ and $e(t)$ is the tracking error and defined as:

$$e(t) = x(t) - x_d(t) \quad (30)$$

The objective is to regulate the product temperature in the dryer. So, the equation (30) becomes:

$$e(t) = x_1(t) - x_{1d}(t) \quad (31)$$

Which $x_{1d}(t)$ is the desired trajectory and $x_1(t)$ the controlled state variable.

The control law of the sliding mode control is proposed as:

$$u(t) = u_s(t) + u_{eq}(t) \quad (32)$$

Where, u_{eq} and u_s are the signals for equivalent control and switching control, respectively.

The following is a possible choice of the structure of a sliding mode controller [4]:

$$u(t) = -k_s |S|^\alpha \text{sgn}(S) + u_{eq}(t) \quad (33)$$

Where $0 < \alpha < 1$ and $k_s > 0$ (control gain).

The controller described by Equation (33) allows to increase the switching speed, when the system state moves away from the sliding surface, it reduces it when the state approaches the surface. In this case, the phenomenon of chattering decreases with increasing the switching speed.

The objective is to regulate the product temperature in the dryer, the equivalent control is calculated as follow:

The derivative of the sliding surface defined by Eq (29) can be given as:

$$\dot{S}(t) = k_p \dot{e}(t) + k_i e(t) \quad (34)$$

A necessary condition for the output trajectory to remain on the sliding surface $S(t)$ is $\dot{S}(t) = 0$

This gives:

$$u_{eq}(t) = \frac{1}{h_{c1}} \left[(\dot{x}_{1d}(t) - k_i e(t)) \cdot \frac{C_{pr}}{A_{pr} k_p} + (h_{c2} + h_{evap}) \cdot (x_1(t) - x_2(t)) \right] + x_1(t) \quad (35)$$

V. SIMULATION RESULTS

The SMC has been applied to the solar-gas tunnel dryer and simulations are made with Matlab software. These simulation results are established for an average speed of drying air equal to $3m/s$ and ambient air temperature is $25^\circ C$. The calculations have been made by using the system parameters (Table 1).

Fig. 4 shows the outlet air temperature of the solar module measured in a day of the month of February (2012) for 5 hours, this temperature will be used as input to the tunnel dryer. In Fig. 5 we notice that we get the desired behaviour in SMC because the state variable follows the desired trajectory.

Figs. 6, 7 and 8 show the variation of the other state variable of the system, also they follow the desired trajectory.

Fig. 9 presents the evolution of the control law applied to dryer model and Fig. 10 shows that the sliding mode control still allows the outputs to converge to their reference values causing the components of the surface to slide toward zero, hence the SMC is a robust control. Also Fig. 11 illustrates the variation of air temperature provided by the auxiliary gas

burner; this temperature has the same shape as that of the state variables of the system. In addition it provides information on the mode of the gas burner.

TABLE I. VALUE OF PARAMETERS USED IN NUMERICAL SIMULATION

Parameters	Values	Parameters	Values
A_{pr}	0.5 (m ²)	M_{in}	6.14 kg/kg ⁻¹ (db)
A_{wa}	3 (m ²)	ρ_{ch}	1.16 (kg/m ³)
A_e	0.00785 (m ²)	ρ_{wa}	2700 (kg/m ³)
a_w	0.2	λ_a	0.0262 (W/m ² °C)
$C_{p,pr}$	4180 (J/kg°C)	λ_i	0.022 (W/m ² °C)
$C_{p,ch}$	1006 (J/kg°C)	ν	2.10^{-5} (m ² /s)
$C_{p,wa}$	860 (J/kg°C)	γ_{ch}	0.66 (dec)
c_d	0.6	γ_{ex}	0.35 (dec)
D	0.66 (m)	k_s	100
g	9.8 (ms ⁻²)	α	0.9
d_i	0.15 (m)	k_p	5
m_{pr}	10 (kg)	k_i	0.1

VI. CONCLUSION

In this paper, we used a unified approach of modeling based on graphic technique known as bond-graph. This technique is systematic and has a sufficient flexibility for thermal system and allows us to establish a mathematical model. The SMC method has been proposed and used for the control of the drying operation in the tunnel dryer. Reliable simulation results are presented to demonstrate the validity of the proposed control approach.

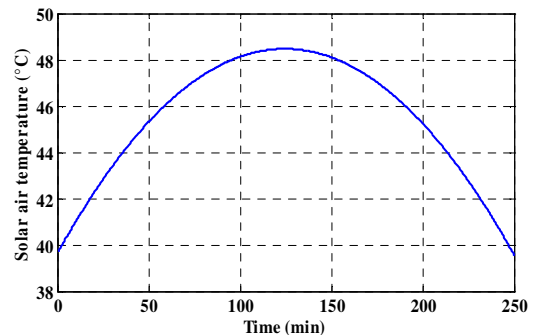


Fig. 4. Variation of the solar air temperature.

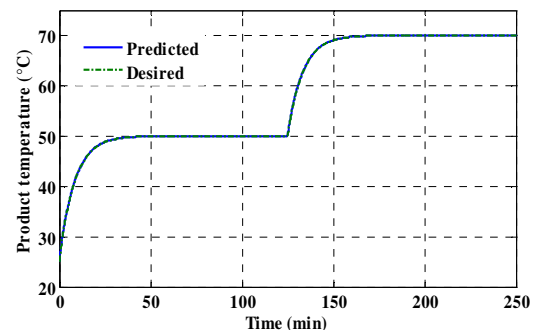


Fig. 5. Variation of the product temperature.

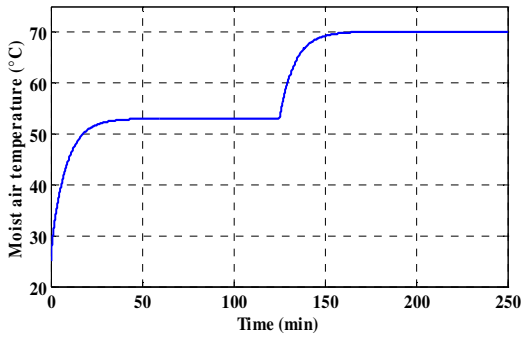


Fig. 6. Variation of the moist air temperature.

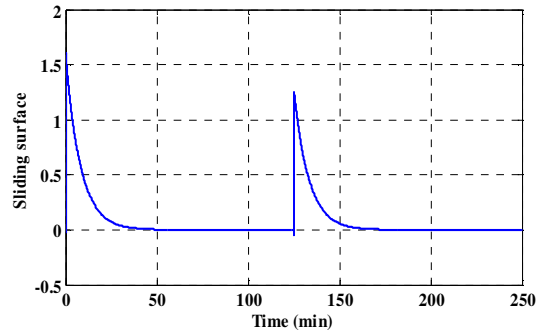


Fig. 10. Variation of the sliding surface.

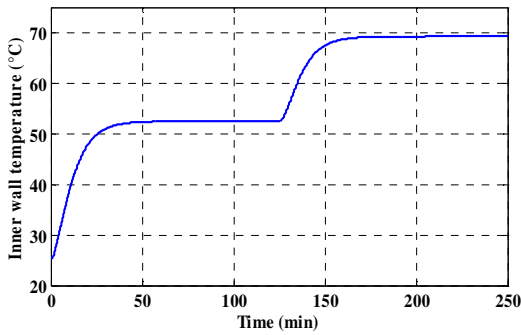


Fig. 7. Variation of the chamber wall temperature.

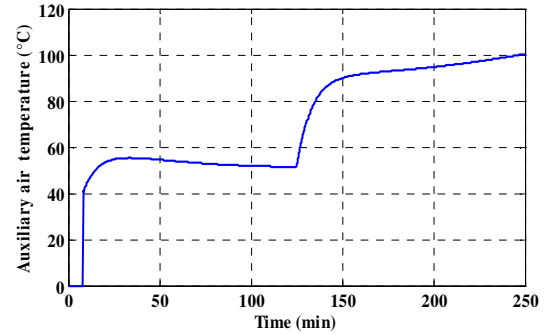


Fig. 11. Variation of the auxiliary air temperature.

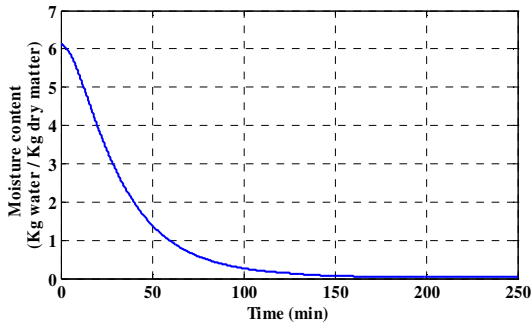


Fig. 8. Variation of the moisture content.

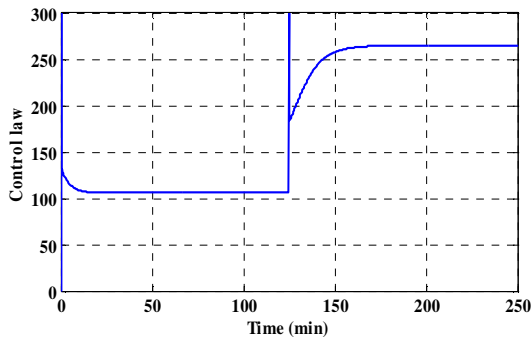


Fig. 9. Variation of the control law.

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