Analysis of the Chaotic Discrete Model of Boost Converter

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Abstract—The hybrid dynamical systems (SDH) are complex systems that involve both continuous-time and discrete-time dynamics. This hybrid aspect presents a variety of nonlinear phenomena such as bifurcations and chaos, which is the case of switching power circuits. As the discrete time model is more accurate and simpler, in this paper, our aim is to analyze the chaotic behavior of the current-programmed boost converter as a case of study from an exact discrete time model. The MATLAB simulation results are then compared to the ones using the continuous time model and validated from the responses of the system simulated in PSIM. By considering the reference current as a parameter bifurcation, we present the strange attractor and the bifurcation diagram. We also calculate its Lyapunov exponents to determine if the DC-to-DC boost converter behaves in a chaotic manner.

Keywords— dynamical hybrid system, DC-DC boost converter, current programmed, exact discrete time model, bifurcation, Lyapunov exponents.

I. INTRODUCTION

Lately, research on hybrid systems has known a strong interest, motivated primarily by the industrial applications. Indeed, industrial process control requires the control of the interactions in the time between discrete events generated by the computer and the continuous dynamics of the physical phenomenon. On the other hand, the hybrid nature of the system may come from the system itself, which is for example the case of the switching power converter circuits.

DC-DC converters (Buck, Boost, Buck-Boost...) are a typical example of power electronic circuits and are characterized by cyclic switching of circuit topologies [8]. Despite their simple schemes, the switching aspect of such systems makes it complex and may have a variety of nonlinear behaviors including bifurcations, quasi-periodicity and chaos[2],[5],[9]. In academic community and industry, many research teams work on analysis and control of these systems in order to provide efficient control laws that are stable within the operating range, in response to certain specifications and taking into account the problem of being industrially applicable.

The hybrid aspect of power circuits makes it difficult to analyze and predict its behavior. Thus, as it is known, exact discrete-time model has been successfully used to study the nonlinear dynamics of switching power converters [3], [12].

A simple example of DC-DC power converter is the DC-to-DC boost converter. Considering its hybrid aspect, this chapter aims at examining and studying the dynamic behavior of the current-mode controlled boost converter. We show also with Matlab simulations that variation of the current reference can lead to interesting route to chaos. The simulation results are approved by calculating Lyapunov exponents.

The paper is organized as follows. Section 2 provides an overview of bifurcation, chaotic theory and Lyapunov exponents. The operating mode of boost converter is defined in section 3 using PSIM simulation. We also present the state space model and the exact-discrete time model of the circuit in section 3. PSIM and MATLAB simulation results of discrete and continuous time models are included in section 4. We show also the analyze of the chaotic behavior of the converter by presenting the bifurcation diagram and calculating the Lyapunov exponents. Section 5 presents the conclusion of this work and shows briefly our future researches.

II. BASIC PRINCIPLES OF BIFURCATION, CHAOTIC THEORY AND LYAPUNOV EXPONENTS

The study of nonlinear dynamics of DC-DC converters started in 1984 by Brockett’s and Wood’s research[4]. Since then, chaos and nonlinear phenomena in power electronic circuits have stolen the spotlight and have attracted the attention of different research groups[4].

“Chaos theory is the qualitative study of unstable aperiodic behavior in deterministic nonlinear dynamical systems.” It studies the behavior of dynamical systems that are hypersensitive to initial condition [7].

Generation of a chaotic system is not immediate. Indeed, the system does not change of nonexistent state to a chaotic state without going through transitions. Consider the nth-order discrete dynamical system parameterized by a single parameter\(\mu\):

\[ x_{n+1} = f(x_n, \mu) \]  \hfill (1)

We assume that \(f\) is a smooth function.

By changing the system parameter, the limit sets of the system also change [1], [3]. Typically, a small change in \(\mu\) produces small quantitative changes in a limit set. For instance, perturbing \(\mu\) could change the position of a limit set slightly, and if the limit set is not an equilibrium point, its shape or size could also change. There is also the possibility that a small change in \(\mu\) can cause a limit set to undergo a qualitative change[11], [13]. Such a qualitative change is called a bifurcation and the value of \(\mu\) at which a bifurcation occurs is called a bifurcation value [6].
Doubling-period bifurcation is decidedly the most known one. It corresponds to the instability of a stable period-k solution of a map and the generation of a stable period-2k solution. A sequence of period doubling bifurcations where the system proceeds from period-one to period-two, then from period-two to period-four, etc... leads to chaos[8]. Indeed, the bifurcation values accumulate at a particular parameter value for which the system becomes chaotic [11]. The most commonly used tool for analysis of the system’s dynamics when the system has chaotic behavior is the bifurcation diagram.

The chaotic regime of the converters has been detected and analyzed through Lyapunov exponents [10]. As it is well known, Lyapunov exponents measure the exponential rates of divergence or convergence of nearby orbits of an attractor in the state space. A positive Lyapunov exponent found indicates that the system is chaotic. In fact, in chaotic regions the exponent is positive because it indicates the rate at which one loses the ability to predict the system response. Considering the discrete dynamical system:

$$x_{n+1} = f(x_n)$$  \hspace{1cm} (2)

whose the solution is defined in terms of the initial condition $x_0$:

$$x_n(x_0) = f(x_{n-1})f(x_{n-2}) \ldots f(x_1)f(x_0)$$  \hspace{1cm} (3)

Lets the variational system associated:

$$\delta x_{n+1} = Df(x_n)\delta x_n$$  \hspace{1cm} (4)

where $Df(x_n)$ is the Jacobian of $f(x_n)$.

Thereby, the Lyapunov exponents are calculated as follows:

$$\lambda(e^n,x_0) = \lim_{n \to +\infty} \frac{\log \|Df_{x_0}e_1 \land \ldots \land Df_{x_0}e_n\|}{\|e_1 \land \ldots \land e_n\|}$$  \hspace{1cm} (5)

Where $e^n$ is a subspace of dimension n of the tangent space $E_{x_0}$, and $\{ e_i \}, i = 1, 2, \ldots , n$ the base of $e^n$, $\land$ represents the exterior product and $\| \ldots \|$ represents the norm.

III. CURRENT-MODE CONTROLLED BOOST CONVERTER IN OPEN LOOP

A. System description

PSIM is a program designed for power electronics simulation.

By default, components are ideal, which allow us to focus on understanding the functioning of the circuit.

A boost converter is a DC-DC power converter with an output voltage greater than its input voltage. It consists of two parts: a converter and a switching controller, as it is shown in Fig. 1.

The circuit includes the following elements: an inductor, a capacitor, a diode, a controlled electronic switch and a load resistance connected in parallel with the capacitor.

The working principle of the boost converter can be explained as follows. The switching controller drives the general circuit. It compares the inductor current to the reference current $I_{ref}$ and generates the on/off controlling signal for the switch by a set-reset Flip-Flop. Indeed, during the switch on period (the switch is closed), the diode is blocked and the inductor current increases until attaining the reference current $I_{ref}$ which leads to make the set-reset flip-flop turning the switch off until reaching the next pulse, in which the switch is closed again.

Assuming that we operate in continuous conduction mode (CCM), the boost converter switches between two states: switch on and switch off. The general equation that governs the operation of the boost converter is:

$$\dot{x}(t) = A(q)x(t) + B(q)$$ with $q \in \{1,2\}$.

For $q = 1$ and $q = 2$, we obtain:

$$S_{on} : \quad \dot{x} = A_1x + B_1$$  \hspace{1cm} (7)

$$S_{off} : \quad \dot{x} = A_2x + B_2$$  \hspace{1cm} (8)

Where $x = \begin{bmatrix} V_c \\ \frac{tL}{R} \end{bmatrix}$ is the state vector, and the matrices $A_1, A_2, B_1, B_2$ are given by:

$$A_1 = \begin{bmatrix} -\frac{1}{RC} & 0 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ E \end{bmatrix}$$
\[ A_2 = \begin{bmatrix} -1 & 1 \\ \frac{1}{RC} & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{E}{L} \end{bmatrix} \]

B. Continuous time modeling for a boost power converter

The continuous time modeling of boost converter is presenting using Simulink-Stateflow as it is shown in Fig. 2.

![Simulink-Stateflow simulation model of boost converter with current control mode.](image)

The Stateflow block Simulink is a graphical tool for design and development used in the modeling of state machines. Using this tool, we were able to simulate the behavior of the continuous model of the Boost converter.

C. Exact discrete-time modeling for a boost power converter

The aim of this part is to develop an iterative application that gives the ability to switch of a continuous complex dynamic system to a simplified discrete dynamic. This modeling will be able to study the behavior of the system via MATLAB simulation.

In order to model the dynamic of a two-state switching converter operated in the continuous conduction mode, it should be from its continuous model described as:

\[
\begin{aligned}
\frac{dx}{dt} &= A_1 x + B_1 e(t) \text{pouri} nT \leq t < nT + dT \\
\frac{dx}{dt} &= A_2 x + B_2 e(t) \text{pouri} nT + dT \leq t < (n + 1)T
\end{aligned}
\]

where \( x \) is the state vector, \( e \) is the input voltage and \( d \) is the duty ratio of the switch.

The exact discrete-time model can then be expressed as:

\[
\begin{aligned}
x((n + 1)T) &= e^{A_2(1-d)T}[e^{A_1dT}x(nT) + dB_1e] \\
&+ (e^{A_2(1-d)T} - I)A_2^{-1}B_2e
\end{aligned}
\]

\[ d = (I_{ref} - I_L(nT)) \frac{L}{re} \]  

where \( I_{ref} \) is the reference current, \( I_L \) is the inductor current, \( L \) is the independence, and \( e \) is the input voltage.

IV. SIMULATION RESULTS

Responses of exact-discrete time model and continuous time model were simulated in MATLAB for step changes in the reference current from 0.7 A to 1.5 A. The circuit parameters are set as follows: \( L = 1.5 \) mH, \( C = 5 \) \( \mu \)F, \( R = 40 \) \( \Omega \), \( T = 100 \) \( \mu \)s, \( V_{in} = 10 \) V.

A. PSIM simulation results

Fig. 3 shows the functioning of boost converter at periodic state \((I_{ref} = 0.7 \) A\) using PSIM.

![Simulated inductor current and capacitance voltage of boost converter using PSIM at periodic state \((I_{ref} = 0.7 \) A\).](image)

By increasing \( I_{ref} \) to 1.5A, the system goes into a chaotic state as it is shown in fig. 4:

![Simulated inductor current and capacitance voltage of boost converter using PSIM at chaotic state \((I_{ref} = 1.5 \) A\).](image)
B. Matlab simulation results: Continuous time model

Fig. 5: Simulated capacitance voltage and inductor current of continuous time model of boost converter in response to a step change in the reference current from 0.7 A to 1.5 A

(a) Period-1 \( (I_{\text{ref}} = 0.7\, \text{A}) \)
(b) Period-2 \( (I_{\text{ref}} = 1\, \text{A}) \)
(c) Period-4 \( (I_{\text{ref}} = 1.3\, \text{A}) \)
(d) Chaotic state \( (I_{\text{ref}} = 1.5\, \text{A}) \)

Concerning the continuous time model, we note in Figures 5 and 6 that for \( I_{\text{ref}} = 0.7\, \text{A} \), we are in a periodic regime. By increasing \( I_{\text{ref}} \) over 0.7 A, we will get period-2 for \( I_{\text{ref}} = 1\, \text{A} \), then period-4 for \( I_{\text{ref}} = 1.3\, \text{A} \), and thereafter the chaos starts from \( I_{\text{ref}} = 1.5\, \text{A} \).

C. Matlab simulation results: Exact-discrete time model

Fig. 7: Simulated capacitance voltage and inductor current of exact-discrete time model of boost converter in response to a step change in the reference current from 0.7 A to 1.5 A

(a) Period-1 \( (I_{\text{ref}} = 0.7\, \text{A}) \)
(b) Period-2 \( (I_{\text{ref}} = 1\, \text{A}) \)
(c) Period-4 \( (I_{\text{ref}} = 1.3\, \text{A}) \)
(d) Chaotic state \( (I_{\text{ref}} = 1.5\, \text{A}) \)

Fig. 8: Simulated phase portrait \( V_C-I_L \) of exact-discrete time model of boost converter in response to a step change in the reference current from 0.7 A to 1.5 A

(a) Period-1 \( (I_{\text{ref}} = 0.7\, \text{A}) \)
(b) Period-2 \( (I_{\text{ref}} = 1\, \text{A}) \)
(c) Period-4 \( (I_{\text{ref}} = 1.3\, \text{A}) \)
(d) Chaotic state \( (I_{\text{ref}} = 1.5\, \text{A}) \)
Figures 7 and 8 show period-doubling cascade from period 1 to chaos. The first bifurcation takes place at $I_{\text{ref}} = 1$ A where the period 1 bifurcates to period 2. The period 2 behavior again bifurcates to period 4 behavior at $I_{\text{ref}} = 1.3$ A.

D. Bifurcation structure

Figures 9 and 10 show the bifurcation diagram and its Lyapunov exponents.

![Bifurcation diagram](image)

Fig. 9: Bifurcation diagram

Fig. 10: Lyapunov Exponents

It is noted in figure 10-(d) that the Lyapunov exponents when $I_{\text{ref}} = 1.5$ A, are: $\lambda_1 = 0.291 > 0$, $\lambda_2 = -1.468$. This may confirm that the system has a chaotic behavior.

V. CONCLUSION

The exact discrete time model provides more accurate presentation of hybrid dynamical systems. Therefore, it has been used in this work to study the chaotic behavior of the current-mode control boost converter. Variation of the reference current as a bifurcation parameter shows that the circuit tracks a period doubling until chaos. This is illustrated in the bifurcation diagram. On the other side, calculating the Lyapunov exponents provides information about the circuit’s behavior. Indeed, the positive exponent found, proves the chaotic behavior of the boost converter.

In this work, the DC-to-DC boost converter was chosen as a case study. For future, our objective is to generalize this study for any kind of converter, and to apply control laws for the control of chaos in power converters.

REFERENCES
