

A Fuzzy Stochastic Dominance Approach to the Measurement of Regional Disparities: Application to The Tunisian Case

Amal Jmaïi

Faculty of Economic Sciences and
Management

University of Tunis El Manar

Tunisia, BP num 94-ROMMANA 1068

Email: jemaïamal@yahoo.fr

Besma Belhadj

Faculty of Economic Sciences and
Management

University of Carthage

Nabeul, Tunisia

Email: besma.kaabi@isg.rnu.tn

Abstract—This paper propose a fuzzy version of dynamic stochastic approach to analyze *between-region* inequalities. We propose a class of fuzzy index and include them in a stochastic dominance framework to present a dominance criterion able to class regional distribution. This study contribute to the literatures of inequalities as it take into consideration the intrinsec nature of poverty.

I. INTRODUCTION

Reducing poverty is a major objective in many countries, especially in developing ones. Several essays have been made in terms of studies and researches to understand the phenomenon. The reason for choosing such a subject is justified by the scale and urgency of the problems related to individual wellbeing. However, despite the considerable improvements in poverty reduction policies over the world, some elements of individuals deprivation are not sufficiently explored in developing countries, which may handicap effective policy against poverty.

The debate on regional poverty disparities in Tunisia has been dealt by several researchers they maintained (for a long period) different public policies. However, until now the gap between region stil exist.

There is a consensus on the need of poverty and inequalities reduction strategies, designed to facilate empowerment of poor and marginalized people. As poverty is a social complex phenomenon, in recent decades it has been the subject of many theoretical debates, often complementary. The interest of different approaches resides in the strict identification of poverty, a necessary condition to implement efficient policies against this scourge.

The goal of this study is to develop a "fuzzy stochastic" dominance approach to compare the extent of poverty across different region of Tunisia. Two mainly reasons justify our interest in analysing regional disparity. First, in Tunisia, poverty is a central issue after the revolution. Second, given the unequal distribution of poverty in the regional level, ensuring adequate access to education, health and basic needs with equity is an important policy objective.

II. METHODOLOGY

A. Fuzzy set approach

Fuzzy logic based on membership values that can range (inclusively) between 0 and 1. In addition, the degree of truth of a statement can also range between 0 and 1 and is not restricted to the two values (false (0), true (1)) as in classic logic ([1]). In contrast to fuzzy set theory, the crisp logic affirm that variables have only two possible value of membership (0 or 1). In fact, fuzzy logic is considered as a consequence of the development of fuzzy sets theory proposed by Zadeh[2]. Let A be the set of "non-poor" individuals and B be the set of "poor" individuals. Assume that we have a continuous variable "Y" that describes objectively the poverty status of individuals. Then, using the crisp set theory sets as follows:

$$A = \{i/y_i > s\} \quad (1)$$

and

$$B = \{i/y_i \leq s\} \quad (2)$$

where y_i represent the income of the i^{th} individual and s is the poverty line below (above) which an individual is considered as poor (non-poor). This definition requires the following property: if $i \in A(B)$ than $i \notin B(A)$.

In fuzzy framework, we consider a continuous membership function $\mu_P(y) \in [0, 1]$ wich represents as an extension of valuation. Therefore, individuals can be considered simultaneously as poor ($\mu_P(y)$) or non-poor ($1 - \mu_P(y)$) where being poor is weighted by a membership function such that individuals status can vary between on a continuum of degrees of poverty.

a) Definition 1: Let θ be the set of membership functions $\mu_P(y)$ which are non-increasing, continuous and dierentiable.

To define the fuzzy set of poor, membership function must be drawn in θ . This implies that individuals with

higher "y" are more likely to be considered as non-poor. The classic denition of the set of poor individuals given in equation (2) is a particular case for which $\mu_P(y) = 1$ if $y \leq s$ and $\mu_P(y) = 0$ if $y > s$. Building on the literature, we consider two thresholds: s^- below which we assume that the membership function $\mu_P(y) = 1$, and a maximum threshold s^+ over which we assume that the membership function $\mu_P(y) = 0$. The interval $[s^-, s^+]$ represents the fuzzy frontier between the sets of poor and non-poor individuals.

b) **Definition 2:** Let $b \hat{\theta}$ be the subset of θ for which:

$$\begin{aligned} \mu_P(y) &= 1 & \text{if } y \leq s^- \\ \mu_P(y) &\in [0, 1] & \text{if } y^- < y < y^+ \\ \mu_P(y) &= 0 & \text{if } y \geq s^+ \end{aligned}$$

To build this membership function one has to find the correspondence between the value of the poverty index used and the adjectives "poor" and "non-poor" status. Taken into consideration the intrinsec nature of poverty, this task involves normative judgement on what is to be considered as poor.

We propose a membership function taken into account definition 1 and 2 as follow:

$$\mu^p(y) = \begin{cases} 1 & \text{if } y \leq s^- \\ \left(\frac{s^+ - y}{s^+ - s^-}\right)^\alpha & \text{if } s^- < y < s^+ \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where α defines the degree of concavity of the individual poverty function and is connected to the aversion parameter of poverty presented by [3]. When $\alpha = 0$, we fall back in the standard definition of the crisp membership function taking a value of 1 for all those whose income is under s^+ . Otherwise, the membership function varies between 0 and 1.

B. Fuzzy logic to Stochastic Dominance approach

Stochastic dominance is relatively a new method which make possible obtaining robust results in a change in poverty index or poverty line ([4]). One of the properties of stochastic dominance is to detect situations where a change in poverty index could reverse the order established between two groups. However, in the crisp logic this approach may provide a particular judgement based on the choice of a specific membership function. in this paper we develop a new approach of stochastic dominance criterion based on fuzzy logic.

Let $y \in [y_{min}, y_{max}]$ be the poverty status and h the situation of good wellbeing such that $h \in [0, 1]$, where $h = 1$ indicates that the individual has always a good wellbeing and $h = 0$ indicates that he faced, at least once, a poverty situation. Also, let $f(y, h)$ represent the joint density of individuals characterized by the vector (y, h) then,

$$F(y, h) = \sum_{s=0}^h \int_{y_{min}}^y f(t, a) dt \quad (4)$$

TABLE I
L ESTIMATES BY URBAN-RURAL DECOMPOSITION

	Urban	Rural
$\alpha = 0.5$	0.0296	0.0450
$\alpha = 1$	0.0211	0.0316
$\alpha = 2$	0.0197	0.0249

indicates the joint cumulative distribution function

$$L_i(\mu_P) = \int_{y_{min}}^{y_{max}} \mu_P(y) f_i(y, 0) dt \quad (5)$$

a) **Proposition 1:** Let i and j be two region, for all membership functions $\mu_P(y) \in \hat{\theta}$ $L_i \geq L_j$ if and only if

$$\int_{y_{min}}^y [f_i(t, 0) - f_j(t, 0)] dt \geq 0 \forall y \in [s^-, s^+] \quad (6)$$

III. EMPIRICAL ILLUSTRATION

A. Data

The present empirical analysis has been conducted using the data set of the Tunisian Household Expenditure Survey from 2010, the most recent available survey. We used as attribute, the region of residence: Great Tunisia, North east, Middle west, Sud east, North west, Middle east and Sud west and a urban-rural decomposition. We choose this decomposition since we are interested by the urban rural disparities over time. In addition, we selected the educational level of household heads: Illiterate, primary level, secondary level and higher level, and finally the sex of household head.

B. Results

a) *Fuzzy stochastic dominance by Urban-Rural decomposition:* We can observe clearly that the prevalence of poverty in urban area dominates rural area at every point of the distribution. Otherwise, we register in urban environment the low poverty compared to rural areas.

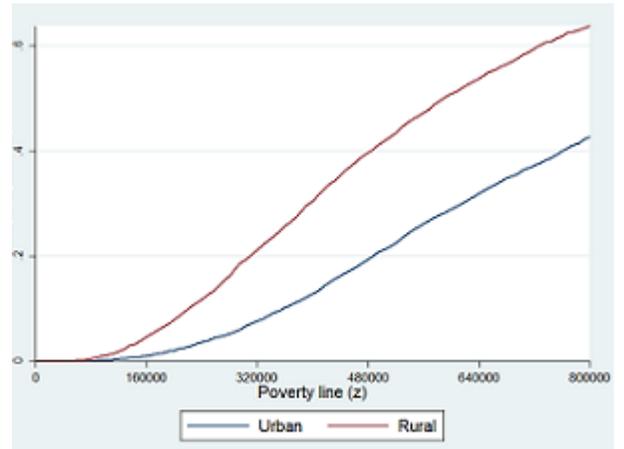


Fig. 1. Dominance test according to urban-rural decomposition

TABLE II
L ESTIMATES BY REGION

	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 2$
Great Tunis	0.0178	0.0126	0.0103
North east	0.0197	0.0120	0.0092
Middle east	0.0402	0.0322	0.0266
Sud east	0.0492	0.0431	0.0320
North west	0.0502	0.0443	0.0319
Middle west	0.0614	0.0564	0.0448
Sud west	0.0522	0.0483	0.0329

TABLE III
L ESTIMATES BY EDUCATIONAL LEVEL

	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 2$
Illiterate	0.0175	0.0106	0.094
primary level	0.0228	0.0195	0.0105
secondary level	0.0347	0.0263	0.0201
higher level	0.0480	0.0360	0.0271

b) *Fuzzy stochastic dominance by Region:* In addition, middle west dominates all other regions following by the north west. Moreover, the west and east south and the west and east north intersect each other and gives some ambiguity in our analysis.

In total, dominance stochastic of first order reveals that the extent of monetary poverty is on average higher in the interior cities than in other and especially in rural and west area.

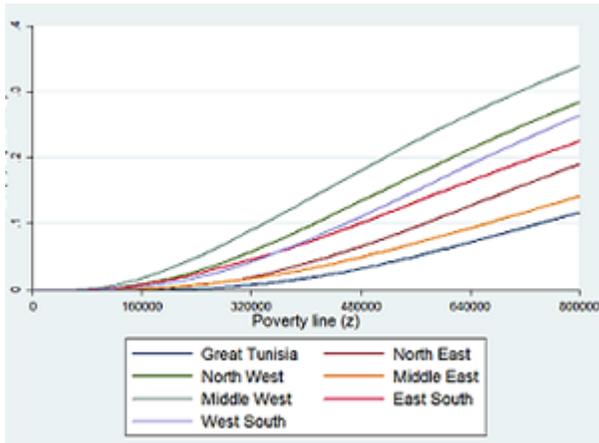


Fig. 2. Dominance test by Region

c) *Fuzzy stochastic dominance by educational level:* Figure 3 shows the inverse relationship between the level of education and the prevalence of poverty. The higher level of education is associated with a curve with a low prevalence of poverty, whatever the considered threshold levels, followed by secondary and primary education. We observe that illiterate is still dominated at all levels.

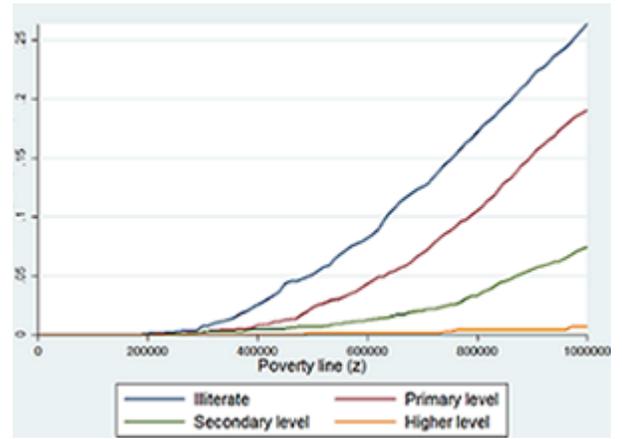


Fig. 3. Dominance test by educational level

TABLE IV
L ESTIMATES BY GENDER

	Females	Males
$\alpha = 0.5$	0.0420	0.0350
$\alpha = 1$	0.0329	0.0276
$\alpha = 2$	0.0256	0.0222

d) *Fuzzy stochastic dominance according to the sex of household heads:* Results of the proposed methodology, confirms the dominance of the curve of a men household head compared to women, whatever the level of poverty line.

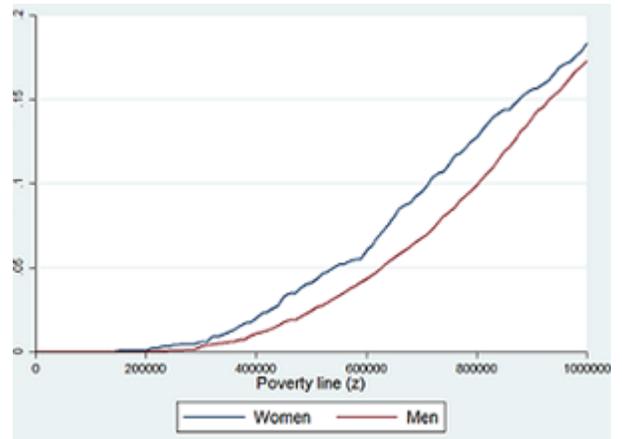


Fig. 4. Dominance test according to the sex of household heads

IV. CONCLUSION

Stochastic dominance results from between-region comparisons suggests that on the global level individuals who are in poor health are less likely to have unmet health care needs in Canada. In addition, it appears that characteristics such as gender and education based are responsible for the disparities between-region.

Fuzzy stochastic dominance results reveal the presence of

income driven disparities between regions, these disparities seem to be more accentuated in middle west. results show also that poverty is less prominent in north east and great Tunisia.

Regarding to the poverty level, Tunisia still succeeded in reducing poverty, but inequality keep up to be a challenge and poverty remains dominant in rural areas. In the light of these finding, it's further recommended that policymakers efforts should be addressed to creating employment for school leavers and unemployed individuals in marginalized regions.

REFERENCES

- [1] A. K. Mehta and A. Shah, "Chronic poverty in india: overview study," *Chronic Poverty Research Centre Working Paper*, no. 7, 2001.
- [2] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, pp. 338–353, 1965.
- [3] J. Foster, J. Greer, and E. Thorbecke, "A class of decomposable poverty measures," *Econometrica: Journal of the Econometric Society*, pp. 761–766, 1984.
- [4] J.-Y. Duclos, A. Araar, and C. Fortin, "Dad: A software for distributive analysis/analyse distributive," *MIMAP programme, International Development Research Centre, Government of Canada, and Université Laval* (www.mimap.ecn.ulaval.ca), 2003.

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