

## The Inverted Pendulum in Nonlinear Control Theory: A review

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**Abstract**— *Inverted Pendulum is a classic example of an inherently unstable system. Inverted pendulum control is one of the fundamental problems in the field of nonlinear control theory. For at least fifty five years, the inverted pendulum has been the most popular benchmark, among others, in nonlinear control theory.*

*This paper presents a review of various controllers & methods for Inverted Pendulum for the last decade. A variety of techniques with different concepts are mentioned for the control in nonlinear theory.*

*The purpose of this paper is to present a survey of the inverted pendulum system from a control engineer's point of view. The scope of the survey includes Possible future trends that we can envision based on the review.*

**Keywords:** *Inverted pendulum; Fuzzy; H $\infty$ ; LQR; PID;*

### 1-Introduction

Inverted Pendulum is a classic example of an inherently unstable system.

Inverted pendulum is a system that is stable under specific conditions.

In simple words, an inverted pendulum can be explained with the analogy of balancing a long stick on one finger. Inverted pendulum systems (IPS) represent a significant class of nonlinear under actuated mechanical systems, well-suited for verification and practice of ideas emerging in control theory and robotics [1]. Much work has been done on inverted pendulum since last 50 years[4]. The first solution to stabilize open-loop unstable systems was described in 1960 by Roberge [7], and then by Schaefer and Cannon in 1966[8]. More than 100 references in the open literature, dating back to 1960, are compiled to provide a survey of emerging ideas and challenging problems in nonlinear control theory accomplished and verified using this robotic system[3]. For at least fifty years, the inverted pendulum has been the most popular benchmark, among others, in nonlinear control theory. [3].

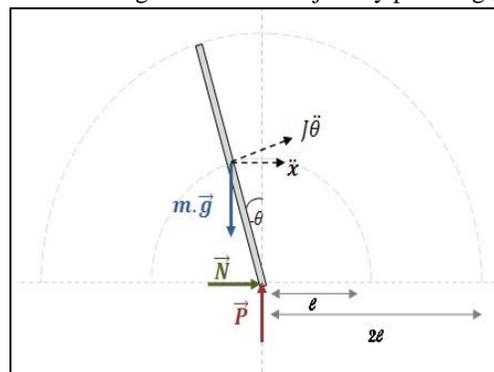
Inverted pendulum is a standard problem in Control system and is implemented in areas of precision control and robotics. The concept is applied for high precision robotic arms, launching of a rocket, control of a Vertical Take-Off and Land (VTOL) aircraft, a satellite, aircraft stabilization in the turbulent air-flow and stabilization of a cabin in a ship[4].

Another very important place where this concept is used is the field of education. Here new students can get a very good idea of this classic problem[4].

In spite of its simple structure, the inverted pendulum is considered, among the most common robotic benchmarks as Acrobot, Pendubot, the Furuta Pendulum, the Reaction Wheel Pendulum, the bicycle, the VTOL aircraft, the Beam-and-Ball system and TORA, the most fundamental benchmark[3].

A typical unstable non-linear Inverted Pendulum system is often used as a benchmark to study various control techniques in control engineering[2].

Most researches have mainly focused on the balance, while few have tried the driving control and trajectory planning [4].

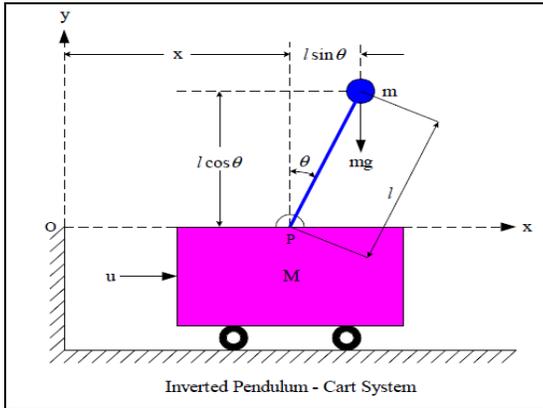


The family of inverted pendulum systems can be classified into single inverted pendulum systems, series-type double inverted pendulum systems, parallel-type double inverted pendulum systems, and so on[6].

The most familiar types of the inverted pendulum are the rotational single-arm pendulum, the cart inverted pendulum and the double inverted pendulum [3].

The less common versions are the rotational two-link pendulum, the parallel type dual inverted pendulum, the triple inverted pendulum, the quadruple inverted pendulum and the 3D or spherical pendulum[3].

A double pendulum is a well-known example of chaotic behavior[11].



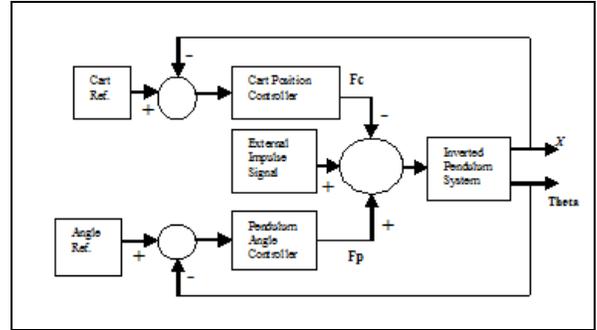
### 2-Why is the focus on inverted pendulum?

The reasons for selecting the IP as the system used in the field of non-linear control theory are:-

- It is the most easily available system (in most academia) for laboratory usage.
- It is a nonlinear system, which can be treated to be linear, without much error, for quite a wide range of variation.
- It Provides a good practice for prospective control engineers.
- The Inverted Pendulum is essential in the evaluating and comparing of various control theories.
- It can be considered as the simplest robotic system, with only one rigid body and only one rotational joint.
- The inverted pendulum system is a good system to demonstrate the use of feedback control theory as the purpose is to stabilize an unstable open-loop system.

### 3-Control Techniques and methods.

Controller design is a key content of the Inverted Pendulum system. Controllers are used to stabilize the unstable system and make it robust to disturbances. Several techniques have been used for achieving the same, e.g. Sliding Mode Technique, Fuzzy Logic Controller, Partial Feedback Linearization, Fuzzy Servo Control Method, Real-Time Control.

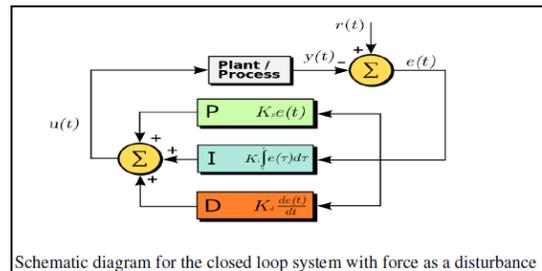


Block diagram of Inverted Pendulum-cart controller system

### 3-1 PID Controller

Proportional, integral derivative are the controllers whose output, a control variable (CV), is generally based on the error (e) between some user-defined set point (SP) and some measured process variable (PV) [2].

The Proportional-Integral-Derivative (PID) control gives the simplest and yet the most efficient solution to various real-world control problems [10].



Schematic diagram for the closed loop system with force as a disturbance

### 3-2 LQR (Linear Quadratic Regulator):

LQR is a method in modern control theory that uses state-space approach to analyze a system like inverted pendulum. The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where system dynamics are described by a set of linear differential equations and the cost is described by quadratic functions which are called LQ problem [6]. The goal of such problem is to find an optimal control that minimizes a quadratic cost functional associated with a linear system [2].

### 3-3 Fuzzy Logic Controller (FLC):

FLC provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information [7]. FLC's approach to control problems mimics how a person makes decisions. Fuzzy control describes the algorithm for process control as a fuzzy relation between information about the condition of

the process to be controlled,  $x$  and  $y$  and the input for the process. The control algorithm is given in the IF - THEN expression[2]. FLC is currently considered as the most popular control technique [3].

### 3-4 H- infinity controller( $H_{\infty}$ ):

In order to achieve robust performance or stabilization, the H-Infinity control method is used.

The  $H_{\infty}$  name derives from the fact that mathematically the problem may be set in the space  $H_{\infty}$ , which consists of all bounded functions that are analytic in the right-half complex plane.  $H_{\infty}$  method is also used to minimize the closed loop impact of a perturbation depending on the problem formulation the impact will be measured in terms of either stabilization or performance[2].

$H_{\infty}$  controller guarantees robustness, good performance in terms of sensitivity and provides high disturbance rejection, providing high stability for any operating conditions[9].

### 3-5 The Bang-Bang control :

Bang-Bang control strategy requires complex calculation and control variables are usually characterized by abruptly switching between two states[3].

### 3-6 The neural network {ANN:-}

Neural Network controllers have simple structures and usually avoid unnecessary and lengthy computations. For their simplicity, they are currently considered as the most popular control techniques. However, stability conditions are often not specified when such approaches are applied. The AI techniques, such as Artificial Neural Network (ANN) have brought a new area of expertise in motor drives[13].

Multi-layer Artificial Neural Networks are designed and trained to model the plant parameter variations. Improvements in the speed control performance are presented for smaller variations and larger variations in the motor parameters and the load conditions [14].

### 3-7 The energy-based methods:

Energy based methods, energy shaping techniques and the hybrid control approaches are the most rigorous control approaches applied to the inverted pendulum benchmark, for which Lyapunov tools are usually used and global stability conditions of the overall system are mathematically well proven [3].

### 3-8 The sliding mode control:

A sliding mode control approach is applied in such cases, stability conditions are well proven and robustness performances are often guaranteed. FOSMC has a good

response to achieve the desired characteristic compare to LQR. [12].

### 3-9 The Time optimal control:

Time optimal control techniques can give very satisfactory results by solving optimization problems[3].

### 3-10 The predictive control:

The predictive control techniques can give very satisfactory results by solving optimization problems[3].

### 4-Future trends and challenges:-

It can confirm that handling for nonlinear systems, delays, instable internal dynamics, uncertainty conditions, actuator saturation and chaos dynamics are the future trends and the most challenging problems to be solved in control theory [3].

For the inverted pendulum case, the problem can be approached by designing a single controller to realize both the upswing and the stabilization control. A complete solution to this crucial problem was recently proposed.

Friction is a very complicated phenomenon that occurs in all control systems.

When friction effects are not modelled, feedback controllers may fail. Indeed, frictions are usually associated with oscillatory behaviours or in limiting cycles. In this framework, very few results exist .

Application of Mobile Inverted Pendulum in robotics and personal transportation has boosted the amount of work done in this field in recent times. This area of research has a lot of scope and a bright future.[4]

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