
Salah NASR
CEM Laboratory ENIS Sfax
National Engineering School of Sousse
University of Sousse
Sousse, Tunisia
Email: nasrsalah.doc@gmail.com

Kais BOUALLEGUE
Department of Electrical Engineering
Higher Institute of Applied Sciences and Technology of Sousse
Sousse, Tunisia
Email: kais_bouallegue@yahoo.fr

Hassen MEKKI
CEM Laboratory ENIS Sfax
National Engineering School of Sousse
University of Sousse
Sousse, Tunisia
Email: mekki.hassen@gmail.com

Abstract—In this paper, based on non-linear chaotic systems, the motion control of an autonomous mobile robot is studied. Control the behavior of the mobile robot is obtained by adding the non-linear chaotic equations to the kinematic equations of the mobile robot. This chaotic equations is inspired from Lorenz attractor, Chua attractor double-scroll, therefore with generation of multi-scroll Chua attractor, which are well known equations having a chaotic behavior. Simulations results show the effectiveness of the proposed control based on non-linear chaotic systems applied to a wheeled mobile robot.

keywords: Autonomous mobile robot, non-linear chaotic systems, multi-scroll Chua attractor, Lorenz attractor, motion control.

I. INTRODUCTION

Mobile robotics, after decades of research and vital developments, remains an interesting research area following its increasing demands, and the relevance of its economic and technological impacts. Mobile robot has become a topic of great interest thanks to the continued growth of its applications in different activities. The devices of the fire fighting and floor cleaning were developed by exploiting autonomous mobile robots as useful tools in civil and industrial life [1]. In addition, several military activities that put the integrity of the man in danger, such as monitoring and exploration of terrains for explosives or dangerous materials and intrusion patrols at military installations, have driven to the development of intelligent robotic systems [2]. Especially, robotic systems in their military missions should have a very important feature as the perception and target identification and the positioning of the robot on the ground. However, the most interesting feature for those successful military missions, is path planning. Various control methods have been studied, including adaptive neural control [3] and fuzzy control design using genetic algorithm [4] to name a few. Among these control methods, many researchers have focused on the sliding mode control [5-7], simple neural network-based controllers were proposed for real-time fine control of mobile robots in [8,9].

Therefore, the unpredictability of the trajectory is also a crucial factor for the success of mission for such autonomous mobile robot. To meet this challenge Sekiguchi and Nakamura have suggested a strategy in 2001 to solve the problem of path planning based on chaotic systems [10].

Several other researches that interested in the chaotic trajectories of the mobile robot have been carried with other equations[11,12]. The main goal in the use of chaotic signals for autonomous mobile robot is to increase and benefit coverage areas resulting from its path of movements. Vast coverage areas are desirable for many applications of robots such as those dedicated for scanning unknown workspace, for cleaning or patrolling[13].

In our work, we focus on the specific problem of terrain exploration with research or vigilance goals. In such missions additional features like quick scan of the entire work area are highly appropriate. Chaotic behavior, typical of a class of non-linear dynamical systems can guarantee an unpredictable robot motion that scans the entire workspace. In this work, the chaotic behavior of Lorenz attractor and Chua attractor are imparted to the mobile robots motion control. For the sake of clarity, we present the Lorenz chaotic system, the double-scroll chaotic system of Chua. Then, after modifying the mathematical equations, we generate a new behavior of multi-scroll dynamic system behaving as a multi-scroll chaotic attractor. These chaotic systems will be explored to control the mobile robot.

The rest of this paper is organized as follows: The kinematic model of the robot is introduced in the next section. Then, the proposed chaotic systems are given in section III, which presents the generation of multi-scroll chaotic attractor. In Section IV, our control method of mobile robot and simulation results are given. Our concluding remarks are contained in the final section.

II. MODEL DESCRIPTION

An electrically driven non-holonomic mobile robot can be modeled via kinematic and dynamic equations. A non-holonomic mobile robot consists of two active wheels and a passive supporting wheel. The two driving wheels are independently driven by two DC motors to realize the robot motion and orientation.

Assume that the mobile robot is made up of a rigid frame equipped with non-deformable wheels as described in [14-16], considering the robot configurations wheeled according to its position $(x,y)$ and the direction $\theta$ in a two-dimensional...
environment. The space of the robot configurations is then constituted by all the triples of values \((x, y, \theta) \in \mathbb{R} \times \mathbb{R} \times [0, 2\pi]\), as shown in Fig. 1.

The kinematic model of the robot can be described as a differential system comprising of two control parameters \(v\) and \(\omega\) which represent respectively the values of linear and angular speeds as follows:

\[
\begin{pmatrix}
\dot{x}(t) \\
\dot{y}(t) \\
\dot{\theta}(t)
\end{pmatrix} =
\begin{pmatrix}
\cos \theta(t) & 0 & 0 \\
\sin \theta(t) & 0 & 0 \\
0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
v(t) \\
\omega(t)
\end{pmatrix}
\tag{1}
\]

The implementation of these dynamic system is achieved in Fig. 2.

III. CHAOTIC ATTRACTOR WITH MULTI-SCROLLS

The concept of deterministic chaos has been greatly influencing not only science but also engineering, technology, and even arts along with substantial progress in our understanding of deterministic chaos since 1970’s.

In order to have a chaotic trajectory of the mobile robot, this is achieved by the use of a controller that guarantees chaotic motion. The chaotic models used to generate the robot path are presented as the Lorenz attractor and the circuit equations of Chua.

A. LORENZ ATTRACTOR

In this subsection, we recall Lorenz attractor. The Lorenz system has become one of paradigms in the research of chaos [19]. Lorenz system is utilized for the investigation. The dynamical equation of Lorenz attractor is given by:

\[
\begin{align*}
\dot{X}_1 &= -10X_1 + 10X_2 \\
\dot{X}_2 &= 28X_1 - X_2 - X_1X_3 \\
\dot{X}_3 &= -\frac{8}{3}X_3 + X_1X_2
\end{align*}
\tag{2}
\]

The implementation of these dynamic system is achieved in Fig. 2.

B. CHUA ATTRACTOR

Chua attractor, which was introduced by Leon Ong Chua in 1983, are simplest electric circuits operating in the mode of chaotic oscillations. Different dynamic systems have been inspired from Chua circuit such as:

\[
\begin{align*}
\dot{X}_1 &= \alpha(Y_1 - f(X_1)) \\
\dot{Y}_1 &= X_1 - Y_1 + Z_1 \\
\dot{Z}_1 &= -\beta Y_1
\end{align*}
\tag{3}
\]

where \(f(X_1) = bX_1 + \frac{1}{2}(a - b)(|X_1 + 1| - |X_1 - 1|)\), \(\alpha = 9, \beta = \frac{100}{7}, a = -\frac{8}{7}, b = -\frac{5}{7}\)

The implementation of these dynamic system is achieved in Fig. 3.

C. GENERATION OF CHUA ATTRACTOR WITH MULTI-SCROLL

In the work presented by Suresh Rasappan and Sundarapandian Vaidyanathan [17], the authors present the chaotic Chua
system to generate a n-scroll attractor. By choosing the value of parameter c=1,2,3 and 5 have been obtained 2-scroll, 3-scroll, 4-scroll and 6-scroll attractors respectively. Then the maximum of scrolls can be obtained is n=6. In[18], the authors presented a family of hyperchaotic multi-scroll attractor in $\mathbb{R}^n$, $n \geq 4$, based on unstable dissipative systems. This class of systems is constructed by a switching control law changing the equilibrium point of an unstable dissipative system.

In our approach, in order to obtain more complex scrolls, we use a simple method based on a modification of the system of equations presented in [17]. This leads to a new dynamic system is described by Eqs.4, Eqs. 10 and Eqs. 6, then we generate a multi-roll chaotic system with a variable number of scrolls and may be too superior to 6. Fig. 4 shows the implementation of the multi-scroll Chua attractor.

The new dynamical equation of multi-scroll of Chua Circuit is given by:

\[
\begin{align*}
\dot{X}_1 &= \alpha(Y_1 - f(X_1)) \\
\dot{Y}_1 &= ((X_1 - Y_1 + Z_1) - g(Y_1)) \\
\dot{Z}_1 &= -\beta Y_1
\end{align*}
\]

(4)

Where $f(X_1)$ is given by:

\[
\begin{align*}
f(X_1) &= \begin{cases} 
\frac{b}{2a}(X_1 + 2ac) & \text{if } X_1 \geq 2ac \\
-b \sin\left(\frac{\pi X_1}{2ac}\right) + d & \text{if } -2ac \leq X_1 \leq 2ac \\
\frac{b}{2a}(X_1 + 2ac) & \text{if } X_1 < -2ac 
\end{cases}
\]

(5)

and $g(Y_1)$ is given by:

\[
\begin{align*}
g(Y_1) &= \begin{cases} 
\frac{b}{2a}(Y_1 - 2bc) & \text{if } Y_1 \geq 2bc \\
-b \sin\left(\frac{\pi Y_1}{2bc}\right) + d & \text{if } -2bc \leq Y_1 \leq 2bc \\
\frac{b}{2a}(Y_1 + 2bc) & \text{if } Y_1 < -2bc 
\end{cases}
\]

(6)

Where $a, b, c, d$ are positive real constants.

Fig. 4 shows the generation of multi-scroll Chua system implemented with a value of parameter $c = 3$, using the two equations: Eqs.4 and Eqs.10.

IV. CONTROL OF MOBILE ROBOT USING CHAOTIC ATTRACTOR

Motion planning or path planning of mobile robots explores an approximate non-collision path consistent with a certain performance objective. This subject has attracted much attention in recent years in robotics. Without mapping, path planning is a difficult task for mobile robots. Chaotic trajectory can be a solution to this predicament. In this section, the following proposed control system will be applied to control the movement of the robot. We adopt the chaos approach for controlling the trajectories of robot. Due to topological transitivity the chaotic mobile robot searches the entire workspace and the sensitivity to initial conditions makes the robot exceedingly unpredictable.

A. Using Lorenz attractor

By using the dynamic equation introduced in Eqs.2, we will find robot equation of motion as follows:

\[
\begin{align*}
\dot{X}_1 &= -10X_1 + 10X_2 \\
\dot{X}_2 &= 28X_1 - X_2 - X_1X_3 \\
\dot{X}_3 &= -\frac{8}{3}X_3 + X_1X_2 \\
\dot{x} &= v \cos(X_1) \\
\dot{y} &= v \sin(X_1)
\end{align*}
\]

(7)

Fig. 6 shows the implementation result of robot motion using the dynamic equation described in Eqs.7. In regard to the benefit coverage areas resulting from its path of movements, at initial conditions: $X_1(0) = 1, X_2(0) = 0, X_3(0) = 1, x(0) = 1, y(0) = 0, v = 3$. 

---

Fig. 5. Multi-scroll Chua attractor with $c = 5$.

Fig. 4. Multi-scroll Chua attractor with $c = 3$.

Fig. 6 shows the generation of multi-scroll Chua system implemented with a value of parameter $c = 3$. 

---

ISSN: 2356-5608
The feature of chaotic systems is that its chaotic orbits have to be dense. This means that, the trajectory of a dynamical system is dense, if it comes arbitrarily close to any point in the domain.

**B. Using Double-scroll Chua attractor**

Now, by using the dynamic equation introduced in Eqs.3, we will find robot equation of motion as follows:

\[
\begin{align*}
\dot{X}_1 &= \alpha (Y_1 - f(X_1)) \\
\dot{Y}_1 &= X_1 - Y_1 + Z_1 \\
\dot{Z}_1 &= -\beta Y_1 \\
\dot{x} &= v \cos(N \dot{X}_{1,n}) \\
\dot{y} &= v \sin(N \dot{X}_{1,n-1})
\end{align*}
\]

(8)

where \( f(X_1) = bX_1 + \frac{1}{2}(a - b)(|X_1 + 1| - |X_1 - 1|) \) and \( N=50r \).

we used the time delay of the first state on the model of Chua attractor, these states are used by combining with the model of robot. With \( \dot{X}_{1,n} \), describes the present state and \( \dot{X}_{1,n-1} \) described the previous state.

Fig. 6. Behavior of Robot controller with Lorenz attractor.

Fig. 7 shows very satisfactory results in regard to the fast scanning of the robots workspace with unpredictable way.

Fig. 7.

Fig. 8. Behavior of Robot controller with Double-scroll Chua attractor with 25000 iteration.

Fig. 8.

Fig. 9. Zooming Behavior of Robot controller with Double-scroll Chua attractor.

Fig. 9.

Fig.9 shows the zooming behavior of robot control with Double-scroll Chua attractor. It is clear that the scrolls presented in this behavior contains a large number of orbits.

**C. Using Multi-scroll Chua attractor**

The integrated system of the multi-scroll Chua circuit equation [17] as a controller of the mobile robot will be as follows:

\[
\begin{align*}
\dot{X}_1 &= \alpha (Y_1 - f(X_1)) \\
\dot{Y}_1 &= (X_1 - Y_1 + Z_1) \\
\dot{Z}_1 &= -\beta Y_1 \\
\dot{x} &= v \cos(N \dot{X}_{1,n}) \\
\dot{y} &= v \sin(N \dot{X}_{1,n-1})
\end{align*}
\]

(9)

Fig. 7. Behavior of Robot controller with Double-scroll Chua attractor with 15000 iteration.
Where \( f(X_1) \) is given by:

\[
f(X_1) = \begin{cases} 
\frac{a}{b} (X_1 + 2ac) & \text{if } X_1 \geq 2ac \\
-b \sin(\frac{a}{b} X_1 + d) & \text{if } -2ac \leq X_1 \leq 2ac \\
\frac{a}{b} (X_1 + 2ac) & \text{if } X_1 < -2ac
\end{cases}
\] (10)

The resultant trajectory of the mobile robot is controlled by Chua equations, at initial conditions: \( X_1(0) = 1, Y_1(0) = 0, Z_1(0) = 1 \), \( x(0) = 1 \), \( y(0) = 0 \), \( v = 3 \) and \( N=50\pi \).

As shown in Fig.10, when we used chaotic attractor with multi-scroll, the number of orbits in trajectory of the robot are decreased. There is relationship between the number of scrolls and the number of orbits.

This approach by chaotic attractor multi-scroll guarantee not only to accomplish the path planning of robot but also can optimize energy and reduce the time to finish its tasks.

V. CONCLUSION

In this work, we defined an approach based on non-linear dynamic systems that may be involved in the realization of a navigation trajectory for an autonomous mobile robot. It is based on a technique of control using the chaos, used to monitor the dynamics of Lorenz attractor, double-scroll Chua attractor and multi-scroll Chua attractor. This proposed control and implementation of chaotic behavior on a mobile robot, implies a mobile robot with a controller that guarantees its chaotic motion with the minimum of orbits. This will make the most economical robot in energy consumption and reduce the time to finish its tasks. Some numerical simulation results are provided to show the effectiveness of the method proposed in this work.

REFERENCES


