International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014 , pp.54-59
ISSN 2356-5608

Generalized H_2 sliding mode control for a class of (TS) fuzzy descriptor systems with time-varying delay and nonlinear perturbations

Mourad Kchaou and Ahmed Toumi
Laboratory of Sciences and Techniques of Automatic control & computer engineering (Lab-STA),
Email:mouradkchaou@gmail.com

***Abstract*—In this paper the problem of sliding mode control (SMC) for a class of uncertain (TS) fuzzy descriptor systems with time-varying delay is studied. An integral-type sliding function is proposed and a delay-dependent criterion is developed in terms of linear matrix inequality (LMI), which ensures the sliding mode dynamics to be robustly admissible with generalized H_2 disturbance rejection level. Moreover, a SMC law is established to satisfy the reaching condition of the specified sliding surface for all admissible uncertainties and time-varying delay. The developed results are tested on a representative example to illustrate the theoretical developments.**

I. Introduction

The class of descriptor (singular) systems has recently received great interest from mathematical and control theorists to properly describe the behaviour of some practical systems such as large-scale systems, economic systems, power systems and other areas [4]. Time delay phenomena constitute an intrinsic characteristic of several practical systems. Some of them can be modelled by the class of descriptor systems with delays. It should be pointed out that the robust stability problem for descriptor systems is much more complicated than that for state-space systems because it involves not only stability and robustness, but also regularity and impulse immunity for continuous descriptor systems or causality for discrete-time descriptor systems simultaneously [2], [16], [17], [18]. Recently, the (TS) fuzzy model has been extended to deal with descriptor nonlinear systems with time delay and many scholars have paid much attention to deal with fuzzy descriptor systems and various problems of analysis and synthesis have been treated [1], [7], [17].

As the dual of the robust control problem, the generalized H_2 ($L_2 - L_\infty$) control for dynamic systems has been extensively investigated. As H_∞ , generalized H_2 has been well recognized to be most appropriate for systems with noise input, whose stochastic information is not precisely known. The objective of this problem is to design a controller such that the resulting closed-loop system is stable and ensures that the peak value of the controlled output is often required to be within a certain range [1], [9], [14]

It is well known that the sliding-mode control (SMC) is an effective method to achieve robustness and invariance to matched uncertainties and disturbances on the sliding surface[3], [5], [10], [12]. The SMC strategy has been successfully applied to many kinds of systems due to its inherent attractiveness, for example, easy realization, fast response, good transient response and insensitivity to plant parameter variation or external disturbance. The SMC strategy has been successfully applied to many kinds of systems, such as, uncertain time-delay systems, stochastic systems, and Markovian jump systems [6], [11], [13], [15]. However, to the authors' knowledge, there is little related results reported on SMC of (TS) singular systems [8]. By using LMI technique, the present paper extends the sliding mode control to (TS) fuzzy descriptor systems which may contain the un-modelled dynamics, varying parameters and disturbance.

The remaining parts of this paper are organized as follows. Section 2 formulates the system description and presents some preliminaries.The integral sliding mode controller design is presented in section 3. Illustrative example is given in section 4. Finally a conclusion is provided in section 5.

Notations. The notation $X > 0$ (respectively, $X \geq 0$) means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). L_2 is the space of integral vector over $[0,\infty)$. The L_2 norm over $[0,\infty)$ is defined as $||g||_2^2 = \int_0^\infty g^T(t)g(t)dt$. The symbol $(*)$ stands for matrix block induced by symmetry, $\text{sym}(X)$ stands for $X + X^T$.

II. System description and preliminaries

The (TS) fuzzy dynamic model is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of nonlinear systems. A continuous fuzzy descriptor model with delay and parameter uncertainties can be described by :

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\theta)\Big\{A_i(t)x(t) + A_{hi}(t)x(t-h(t)) + B_i(u(t) \\ \qquad + f_i x(t)) + B_{wi}(t)w(t)\Big\} \\ z(t) = \sum_{i=1}^{r} \mu_i(\theta)C_i x(t) \\ x(t) = \varphi(t),\ t \in [-h_M, 0]. \end{cases} \tag{1}$$

where $\mu_i(\theta) = \frac{\prod_{j=1}^{s} F_j^i(\theta_j)}{\sum_{i=1}^{r}\prod_{j=1}^{s} F_j^i(\theta_j)}$, $i = 1,2,\cdots,r$, are the normalized membership functions, $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^w$ is the external disturbance input, $f_i(t,x(t))$ represents the system non-linearity and any model uncertainties in the system including external disturbances, $z(t) \in \mathbb{R}^s$ is the controlled output; F_j^i $(j = 1 \dots s)$ are fuzzy sets, $\theta = [\theta_1, \dots, \theta_s]$ is the premise variable vector. The delay $h(t)$ is time-varying and satisfies

$$0 \le h(t) \le h_M, \quad \dot{h}(t) \le h_d. \tag{2}$$

where h_M are constants representing the bounds of the delay, h_d is a positive constant. $\varphi(t)$ is a compatible vector-valued initial function in $[-h_M, 0]$ representing the initial condition of the system. The system disturbance, $w(t)$, is assumed to belong to $L_2[0,\infty)$. The matrix $E \in \mathbb{R}^{n\times n}$ may be descriptor and assume that $\text{rank}(E) = q \le n$. $A_i(t) = A_i + \Delta A_i(t)$, $A_{hi}(t) = A_{hi} + \Delta A_{hi}(t)$ and $B_{wi}(t) = B_{wi} + \Delta B_{wi}(t)$ are time-varying system matrices. A_i, A_{hi}, B_i, B_{wi} and C_i are constant matrices with appropriate dimensions. Note that the normalized weights $\mu_i(\theta)$ satisfy

$$\mu_i(\theta) \ge 0, \quad i = 1, 2, \cdots, r \quad \sum_{i=1}^{r} \mu_i(\theta) = 1. \tag{3}$$

Without loss of generality, we introduce the following assumption for technical convenience.

1) $\Delta A_i(t)$, $\Delta A_{hi}(t)$ and $\Delta B_{wi}(t)$ are the unmatched uncertainties satisfying
$$\begin{bmatrix}\Delta A_i(t) & \Delta A_{hi}(t) & \Delta B_{wi}(t)\end{bmatrix} = M_i F(t) \begin{bmatrix} N_i & N_{hi} & N_{wi}\end{bmatrix}, \tag{4}$$
where M_i, N_i and N_{di} are known real constant matrices and $F(t)$ is unknown time-varying matrix function satisfying $F^T(t)F(t) \le I$.
2) The matrices B_i, $i = 1, 2, \cdots, r$ are assumed to satisfy $B_1 = B_2, \cdots, B_r = B$.
3) The matched nonlinearities $f_i(x)$ satisfies the inequality
$$f_i(x) \le \eta_i(x) \tag{5}$$
where $\eta_i(x)$ are non-negative known vector-valued functions.
4) The exogenous signal, $w(t)$ is bounded.

First of all, we recall some definitions.
Consider an unforced linear descriptor system with delay described by

$$\begin{aligned} &E\dot{x}(t) = Ax(t) + A_h x(t - h(t)), \quad 0 \le h(t) \le h_M \\ &x(t) = \varphi(t), \; t \in [-h_M, 0]. \end{aligned} \tag{6}$$

Definition *1:* [4] System (6) is said to be admissible if it is regular ($det\Big(sE - A\Big) \neq 0$), impulse-free ($\deg\Big(\det\Big(sE - A\Big)\Big) = \text{rank}(E)$) and stable.

Definition *2:* The open-loop fuzzy descriptor system (6) is said asymptotically stable with generalized H_2 performance if the open-loop system is asymptotically stable and under the zero initial condition, the L_2-L_∞ norm of the open-loop transfer function $T_{zw}(s)$ from external disturbance $w(t)$ to controlled output $z(t)$ satisfies

$$||T_{zw}(s)||_{L_2-L_\infty} = \sup_{0\neq w(t)\in L_2} \frac{||z(t)||_\infty}{||w(t)||_2} < \gamma \tag{7}$$

where γ is a given positive scalar.

III. Integral sliding mode controller design

SMC design involves two basic steps. The first one is to design an appropriate switching surface such that the sliding mode dynamics restricted to the surface is admissible with generalized H_2 disturbance rejection level γ. In the second step an SMC law is synthesized to guarantee that the sliding mode is reached and the system states maintain in the sliding mode thereafter.

A. Integral sliding mode surface

The integral sliding-mode control completely eliminating the matched-type non-linearities and uncertainties of (1) while keeping $s = 0$.
In this work, the following integral sliding surface is considered:

$$\begin{aligned} s(x,t) = \mathcal{M}Ex(t) - \mathcal{M}\Big(Ex_0 + \int_0^t \sum_{i=1}^{r} \mu_i \Big\{ \Big(A_i + BK_i\Big)x(\theta) \\ + A_{hi}x(\theta - h(\theta))d\theta\Big) \end{aligned} \tag{8}$$

where $K_i \in \mathbb{R}^{m\times n}$ is real matrix to be designed and $\mathcal{M} \in \mathbb{R}^{m\times n}$ is designed to satisfy that $\mathcal{M}B$ is nonsingular. According to SMC theory, when the system trajectories reach onto the sliding surface, it follows that $s(x,t) = 0$ and $\dot{s}(x,t) = 0$. Therefore, from $\dot{s}(x,t) = 0$, the equivalent control law can be established as

$$\begin{aligned} u_s = (\mathcal{M}B)^{-1}\mathcal{M} \sum_{i=1}^{r} \mu_i \Big\{ \big(\Delta A_i(t) + BK_i\big)x(t) \\ + \Delta A_{hi}(t)x(t - h(t))) + B_{wi}(t)w(t) \Big\} \\ - \sum_{i=1}^{r} \mu_i f_i(x(t)) \end{aligned} \tag{9}$$

Substituting (9) into (1), we obtain the following sliding mode dynamics:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\theta) \Big\{ \overline{A}_i(t)x(t) + \overline{A}_{hi}(t)x(t - h(t)) + \overline{B}_{wi}(t)w(t) \Big\} \\ z(t) = \sum_{i=1}^{r} \mu_i(\theta) C_i x(t) \end{cases} \tag{10}$$

where $\overline{\mathcal{M}} = I - B(\mathcal{M}B)^{-1}\mathcal{M}$ and

$$\begin{aligned} &\overline{A}_i(t) = \overline{A}_i + \Delta\overline{A}_i(t), \qquad \overline{A}_i = A_i + BK_i, \quad \overline{A}_{hi}(t) = A_{hi} + \Delta\overline{A}_{hi}(t), \\ &\overline{B}_{wi}(t) = \overline{B}_{wi} + \Delta\overline{B}_{wi}(t), \quad \overline{B}_{wi} = \overline{\mathcal{M}}B_{wi}, \qquad \overline{M}_i = \overline{\mathcal{M}}M_i, \end{aligned} \tag{11}$$

$$\begin{bmatrix}\Delta\overline{A}_i(t) & \Delta\overline{A}_{hi}(t) & \Delta\overline{B}_{wi}(t)\end{bmatrix} = \overline{M}_i F(t) \begin{bmatrix} N_i & N_{hi} & N_{wi}\end{bmatrix}. \tag{12}$$

B. Sliding Mode Dynamics generalized H_2 analysis

In this subsection, we develop a delay-dependent sufficient condition that ensure for sliding mode dynamics (10) to be robustly admissible with generalized H_2 performance.

Theorem *3.1:* Let γ, h_M and h_d given positive scalars. The fuzzy descriptor system (10) is regular, impulse free and asymptotically stable with generalized H_2 norm bound γ, if a non-singular matrix P exists, some matrices $Q_1 > 0$, $Q_2 > 0$, $S > 0$, of appropriate dimensions and positive scalars ε_i such that the following set of LMIs holds:

$$E^T P = P^T E \geq 0 \tag{13}$$

$$\begin{bmatrix} \Phi_i & \mathbf{B}_{wi} & \sqrt{h_M}\mathbf{A}_i S & P^T \mathbf{M}_i & \varepsilon_i \mathbf{N}_i \\ * & -\gamma I & \sqrt{h_M}\,\overline{B}_{wi}^T S & 0 & \varepsilon_i N_{wi} \\ * & * & -S & 0 & 0 \\ * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} -E^T P & C_i^T \\ * & -\gamma I \end{bmatrix} < 0, \tag{15}$$

where

$$\Phi_i = \begin{bmatrix} \Phi_{11i} & P^T A_{hi} + \frac{1}{h_M} E^T SE & 0 \\ * & -(1-h_d)Q_1 - \frac{2}{h_M} E^T SE & \frac{1}{h_M} E^T SE \\ * & * & -Q_2 - \frac{1}{h_M} E^T SE \end{bmatrix}$$

$$\Phi_{11i} = Q_1 + Q_2 + \mathrm{sym}(P^T \overline{A}_i) - \frac{1}{h_M} E^T SE$$

$$\mathbf{A}_i = \begin{bmatrix} \overline{A}_i & A_{hi} & 0 \end{bmatrix}^T, \ \mathbf{B}_{wi} = \begin{bmatrix} \overline{B}_{wi}^T P & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{N}_i = \begin{bmatrix} N_i & N_{hi} & 0 \end{bmatrix}^T, \ \mathbf{M}_i = \begin{bmatrix} \overline{M}_i^T P & 0 & 0 \end{bmatrix}^T$$

Proof: The proof of this theorem is divided into two parts. The first one is concerned with the regularity and the impulse-free characterizations, and the second one treats the stability property of system (10). Since $\mathrm{rank}(E) = q \leq n$, there always exist two non singular matrices M and $N \in \mathbb{R}^{n\times n}$ such that

$$\overline{E} = MEN = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix} \tag{16}$$

Set

$$\mathbb{A}_i = M\overline{A}_i N = \begin{bmatrix} A_{i11} & \mathbb{A}_{i12} \\ \mathbb{A}_{i21} & \mathbb{A}_{i22} \end{bmatrix}, \quad \mathbb{A}_{hi} = MA_{hi}N \begin{bmatrix} \mathbb{A}_{hi11} & \mathbb{A}_{hi12} \\ \mathbb{A}_{hi21} & \mathbb{A}_{hi22} \end{bmatrix},$$
$$\mathbb{P} = M^{-1}PN = \begin{bmatrix} \mathbb{P}_{11} & \mathbb{P}_{12} \\ \mathbb{P}_{21} & \mathbb{P}_{22} \end{bmatrix}. \tag{17}$$

Using the fact that $\mathbb{P}$ is non-singular, it is easy to see from (13) and (17) that $\mathbb{P}_{11} > 0$, $\mathbb{P}_{12} = 0$ and $\mathbb{P}_{22}$ is also non-singular. From (14), it is easy to see that the inequality

$$\mathrm{sym}(P^T \overline{A}_i) - \frac{1}{h_M} E^T SE < 0 \tag{18}$$

holds. Pre- and post-multiplying (18) by N^T and N, respectively, we obtain

$$\begin{bmatrix} \star & \star \\ \star & \mathrm{sym}\,(\mathbb{P}_{22}^T \mathbb{A}_{i22}) \end{bmatrix} < 0 \tag{19}$$

where $\star$ will not be used in the following development. Hence, we can deduce that $\overline{\mathbb{A}}_{22}$ is non-singular. Therefore, descriptor time-delay system (10) is regular and impulse free for any time-delay $h(t)$ satisfying (2).

Now, let us choose the following Lyapunov-Krasovskii functional as

$$\begin{aligned} V(x_t) &= V_1(t) + V_2(t) + V_3(t) \\ V_1(t) &= x^T(t) E^T P x(t) \\ V_2(t) &= \int_{t-h(t)}^{t} x^T(s) Q_1 x(s) ds + \int_{t-h_M}^{t} x^T(s) Q_2 x(s) ds \\ V_3(t) &= \int_{-h_M}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) E^T SE \dot{x}(s) ds d\theta \end{aligned} \tag{20}$$

The derivative along the trajectories of (10) satisfies that

$$\begin{aligned} \dot{V}_1(t) &= 2x^T(t) P^T E \dot{x}(t) = 2x^T(t) P^T \Big(\sum_{i=1}^{r} \mu_i(\theta) \Big\{ \overline{A}_i x(t) \\ &\quad + A_{hi} x(t-h(t)) \Big) \\ \dot{V}_2(t) &= x^T(t) Q_1 x(t) - (1-\dot{h}(t)) x^T(t-h(t)) Q_1 x(t-h(t)) \\ &\quad + x^T(t) Q_2 x(t) - x^T(t-h_M) Q_2 x(t-h_M) \\ &\leq x^T(t)(Q_1+Q_2)x(t) - (1-h_d) x^T(t-h(t)) Q_1 x(t-h(t)) \\ &\quad - x^T(t-h_M) Q_2 x(t-h_M) \\ \dot{V}_3(t) &= h_M \dot{x}^T(t) E^T SE \dot{x}(t) - \int_{t-h_M}^{t} \dot{x}^T(s) E^T SE \dot{x}(s) ds \\ &= h_M \dot{x}^T(t) E^T SE \dot{x}(t) - \int_{t-h_M}^{t-h(t)} \dot{x}^T(s) E^T SE \dot{x}(s) ds \\ &\quad - \int_{t-h(t)}^{t} \dot{x}^T(s) E^T SE \dot{x}(s) ds \end{aligned} \tag{21}$$

According to Jensen Lemma we have

$$\dot{V}_3(t) \leq -\frac{1}{h_M} \gamma_1 E^T SE \gamma_1 - \frac{1}{h_M} \gamma_2 E^T SE \gamma_2 \tag{22}$$

where $\gamma_1 = x(t-h(t)) - x(t-h_M)$ and $\gamma_2 = x(t) - x(t-h(t))$. Define $\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & x^T(t-h_M) \end{bmatrix}^T$. Then, we have

$$\dot{V}(x_t) \leq \sum_{i=1}^{r} h_i \xi^T(t) \Big(\Phi_i + h_M \mathcal{A}_i S \mathcal{A}_i^T \Big) \xi(t) \tag{23}$$

Hence, $\dot{V}(x_t) \leq -\alpha \|\xi(t)\|^2$ which implies that nominal singular system (10), with $w(t) = 0$ is asymptotically stable.

Let us now prove that system (10) has the generalized H_2 performance. For this purpose, consider the following performance index:

$$J_0 = V(x(t)) - \gamma \int_0^t w^T(s) w(s) ds \tag{24}$$

where $V(x(t))$ is defined as in (20). For any non-zero $w(s) \in L_2$, $t > 0$ and zero initial state condition $\varphi(t) = 0$, $t \in [-h_M, 0]$, we have

$$\begin{aligned} J_0 &= V(x(t)) - V(0) - \gamma \int_0^t w^T(s) w(s) ds \\ &= \int_0^t \dot{V}(x(s)) - \gamma w^T(s) w(s) ds \end{aligned} \tag{25}$$

Define $\zeta(t) = \begin{bmatrix}\xi^T(t) & w^T(t)\end{bmatrix}^T$. Following the same procedure as used above, we get

$$\dot{V}(x(t)) - \gamma w^T(t)w(t) \leq \sum_{i=1}^{r} h_i(\theta)\zeta^T(t)\Psi_i\zeta(t) \tag{26}$$

From (14), it follows that $\Psi_i < 0$, which implies that $J_0 < 0$. Therefore, we can obtain the following inequality

$$x^T(t)E^T Px(t) \leq V(x(t)) < \gamma \int_0^t w^T(s)w(s)ds \tag{27}$$

On the other hand, from (15) it yields $\gamma^{-1}z^T(t)z(t) - EP < 0$ which, in turn, leads to

$$\begin{aligned} z^T(t)z(t) &\leq \gamma x^T(t)E^T Px(t) \leq \gamma V(x(t)) \\ &< \gamma^2 \int_0^t w^T(s)w(s)ds \leq \gamma^2 \int_0^\infty w^T(s)w(s)ds \end{aligned} \tag{28}$$

Taking the maximum value of $||z(t)||_\infty^2$, we have $||z(t)||_\infty^2 < \gamma^2||w(t)||_2^2$ for any $0 \neq w(t) \in L_2$ which means that system (10) is delay-dependent asymptotically stable with generalized H_2 norm bound γ. This completes the proof. ■

C. Sliding Mode Dynamics generalized H_2 synthesis

Given all the system matrices in (1). Based on the previous results, we focus on this section to determine the gain K_i in the switching surface function of (8) such that the sliding mode dynamics (10) is robustly admissible with generalized H_2 performance.

Theorem *3.2:* Let h_M, h_d and γ given positive scalars. Then, the sliding mode dynamics (10) is robustly admissible with H_2 performance γ, for any delay $h(t)$, satisfying (2), and any tuning parameters σ, if there exists a non-singular matrix X, symmetric positive-definite matrices $\tilde{Q}_1$, $\tilde{Q}_1$, $\tilde{S}$ and some positive scalars ε_i, $i = 1, \cdots, r$ such that the following LMIs hold:

$$EX = X^T E^T \geq 0 \tag{29}$$

$$\Upsilon_i = \begin{bmatrix} \tilde{\Phi}_i & \tilde{\mathbf{B}}_{wi} & \sqrt{h_M}\tilde{\mathbf{A}}_i & \tilde{\mathbf{N}}_i & \varepsilon_i\tilde{\mathbf{M}}_i \\ * & -\gamma I & \sqrt{h_M}\, B_{wi}^T & N_{wi}^T & 0 \\ * & * & \sigma^2\tilde{S} - \sigma\,\mathrm{sym}(X) & 0 & \varepsilon_i\sqrt{h_M}\ \overline{M}_i \\ * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0 \tag{30}$$

$$\Gamma_i = \begin{bmatrix} -X^T E^T & \tilde{C}_i^T \\ * & -\gamma I \end{bmatrix} < 0, \qquad i, j = 1, \cdots, r \tag{31}$$

where

$$\Phi_i = \begin{bmatrix} \Phi_{1i} & \overline{A}_{hi}X + E\tilde{S}E^T & 0 \\ * & -(1-h_d)\tilde{Q}_1 - 2E\tilde{S}E^T & E\tilde{S}E^T \\ * & * & -\tilde{Q}_2 - E\tilde{S}E^T \end{bmatrix}$$

$$\Phi_{1i} = \mathrm{sym}(A_iX + BF_i) + \tilde{Q}_1 + \tilde{Q}_2 - \frac{1}{h_M}E\tilde{S}E^T,\ \tilde{C}_i = C_iX \tag{32}$$

$$\tilde{\mathbf{A}}_i = \begin{bmatrix} A_iX + BF_i & A_{hi}X & 0 \end{bmatrix}^T,\quad \tilde{\mathbf{B}}_{wi} = \begin{bmatrix} \overline{B}_{wi}^T & 0 & 0 \end{bmatrix}^T$$
$$\tilde{\mathbf{M}}_i = \begin{bmatrix} \overline{M}_i^T & 0 & 0 \end{bmatrix}^T, \qquad \tilde{\mathbf{N}}_i = \begin{bmatrix} N_iX & N_{hi}X & 0 \end{bmatrix}^T. \tag{33}$$

The stabilising controller gains are given by $K_i = F_iX^{-1}$

Proof: Under the conditions of Theorem 3.2, a feasible solution satisfies the condition $-\sigma\,\mathrm{sym}(X) + \sigma^2\tilde{S} < 0$ which implies that X is nonsingular.
On another hand, we note for any $\sigma > 0$ that

$$0 \leq (X - \sigma\tilde{S})^T\tilde{S}^{-1}(X - \sigma\tilde{S}) = X^T\tilde{S}^{-1}X - \sigma\,\mathrm{sym}(X) + \sigma^2\tilde{S} \tag{34}$$

which implies that

$$-X^T\tilde{S}^{-1}X \leq -\sigma\,\mathrm{sym}(X) + \sigma^2\tilde{S} \tag{35}$$

Let $P = X^{-1}$, $Y_i = K_iX$, $\tilde{Q}_l = X^T Q_l X$ $(i = 1, 2)$, and $\tilde{S} = X^T SX$. Considering (35) and checking a congruence transformation to (29), (30) and (31) by P, $diag\{P, P, I, I, I, I\}$ and $diag\{P, I\}$, respectively, the inequalities (13), (14) and (15) hold. ■

D. SMC law synthesis

Now, we are in position to synthesize a SMC law, by which the trajectories of the uncertain fuzzy singular time-delay systems (1) can be driven onto the pre-specified switching surface $s(t) = 0$ in a finite time and then are maintained there for all subsequent time.

Theorem *3.3:* Consider the uncertain singular time-delay system (1). Suppose that the switching surface function is given by (8), then the trajectories of system (1) can be driven onto the switching surface $s(t) = 0$ in a finite time by the following SMC law:

$$u(t) = \sum_{i=1}^{r} \mu_i(\theta)\big(K_ix(t) - \alpha_i\frac{s(t)}{\|s(t)\|}\big) \tag{36}$$

where

$$\begin{aligned}\alpha_i = \lambda + \eta_i(x) + \|\big(\mathcal{M}B\big)^{-1}\mathcal{M}M_i\|\Big\{\|N_ix(t)\| + \|N_{hi}x(t - h(t))\| \\ + \|N_{wi}w(t)\|\Big\} + \|\big(\mathcal{M}B\big)^{-1}\mathcal{M}B_{wi}\|\|w(t)\|\end{aligned} \tag{37}$$

Proof: Choose $\mathcal{M}$ under the condition of $\mathcal{M}B$ is nonsingular. Consider the following Lyapunov function:

$$V_s(t) = \frac{1}{2}s^T(t)\big(\mathcal{M}B\big)^{-1}s(t) \tag{38}$$

According to (8), we have

$$\begin{aligned}\dot{s}(t) = \mathcal{M}\sum_{i=1}^{r}\mu_i\Big\{\big(\Delta A_i(t) - BK_i\big)x(t) + \Delta A_{hi}(t)x(t - h(t))\big) \\ + B_{wi}(t)w(t) + B\Big(u(t) + f_i(x(t))\Big)\Big\}\end{aligned} \tag{39}$$

Thus, taking the derivative of $V_s(t)$ and considering the above equation, we have

$$\begin{aligned}\dot{V}_s(t) =& s^T(t)\big(\mathcal{M}B\big)^{-1}\dot{s}(t)\\ =& s^T(t)\big(\mathcal{M}B\big)^{-1}\mathcal{M}\sum_{i=1}^{r}\mu_i\Big\{\Delta A_i(t)x(t)\\ &+\Delta A_{hi}(t)x(t-h(t)))+B_{wi}(t)w(t)\Big\}\\ &+s^T(t)\Big(u(t)+\sum_{i=1}^{r}\mu_i\big(f_i(x(t))-K_ix(t)\big)\Big)\\ \leq& \|s(t)\|\sum_{i=1}^{r}\mu_i\Bigg\{\|\big(\mathcal{M}B\big)^{-1}\mathcal{M}M_i\|\\ &\Big(\|N_ix(t)\|+\|N_{hi}x(t-h(t))\|+\|N_{wi}w(t)\|\Big)\\ &+\|\big(\mathcal{M}B\big)^{-1}\mathcal{M}B_{wi}\|\|w(t)\|+\eta_i(x)\Bigg\}\\ &+s^T(t)\Big(u(t)-\sum_{i=1}^{r}\mu_iK_ix(t)\Big)\end{aligned} \tag{40}$$

Substituting (36) into (40), we have

$$\dot{V}_s(t) = -\lambda\|s(t)\| < 0, \quad \forall\|s(t)\|\neq 0 \tag{41}$$

Then the system trajectories converges to the predefined sliding surface and is restricted to the surface for all subsequent time, thereby completing the proof. ■

IV. NUMERICAL EXAMPLE

To illustrate the merit and effectiveness of our results, we consider the following nonlinear time delay system borrowed from [17]

$$\begin{aligned}&\Big(1+(a+\delta a)\cos(\theta(t))\Big)\ddot{\theta}(t) = -(b+\delta b)\ \dot{\theta}^3(t)\\ &+(c+\delta c)\theta(t)+(c_h+\delta c_h)\theta(t-h(t))\\ &+d(u(t)+f_i(t,x(t)))\end{aligned} \tag{42}$$

where the range of $\dot{\theta}(t)$ is assumed to satisfy $|\dot{\theta}(t)|<\phi$, $\phi=2$, $c_h=0.8$, $u(t)$ is the control input and $w(t)$ is the disturbance input. For simulation purposes, we set $a=1$, $b=e=1$, $c=1$ and $d=1$. As in [17], the time-delay system (42) can be expressed exactly by a (TS) fuzzy descriptor withy the following parameters :

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\ A_1=\begin{bmatrix}0&1&0\\0&0&1\\c&-b(\phi^2+2)&a-1\end{bmatrix},$$
$$A_2=\begin{bmatrix}0&1&0\\0&0&1\\c&0&-a-1-a\phi^2\end{bmatrix},\ A_3=\begin{bmatrix}0&1&0\\0&0&1\\c&0&a-1\end{bmatrix},$$
$$A_{hi}\begin{bmatrix}0&0&0\\0&0&0\\c_h&0&0\end{bmatrix},\ B=\begin{bmatrix}0\\0\\d\end{bmatrix},\ B_{wi}=\begin{bmatrix}0\\0\\e\end{bmatrix},$$
$$C_i=\begin{bmatrix}0.5&0&0\end{bmatrix},\quad i=1,2,3.$$
$$\mu_1=\frac{x_2^2(t)}{\phi^2+2},\ \mu_2=\frac{1+\cos(x_1(t))}{\phi^2+2},$$
$$\mu_3=\frac{\phi^2-x_2^2(t)+1-\cos(x_1(t))}{\phi^2+2}$$

When we assume that $\delta a(t)=\alpha\Delta(t)a$ and $\delta c_h(t)=\alpha\Delta(t)c_h$, the uncertain matrices can be described as (4) with

$$M_i=\begin{bmatrix}0\\0\\\alpha\end{bmatrix},\quad N_{1,3}=\begin{bmatrix}0&0&a\end{bmatrix},\quad N_2=\begin{bmatrix}0&0&-a(\phi^2+1)\end{bmatrix}$$

First, we consider the case where $\alpha=0.25$ and the time-varying delay is given as $h(t)=1.2+0.1sin(t)$, and a straightforward calculation gives $h_M=1.3$ and $h_d=0.1$.

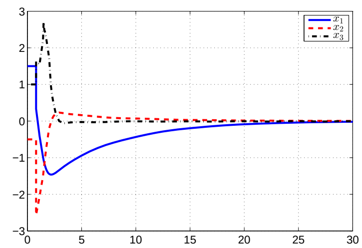
Our aim in this work is to design a SMC law $u(t)$ as given in (36) such that the closed-loop system is robustly stable with generalized H_2 performance.

Set $\mathcal{M}=\begin{bmatrix}0.3&0.2&1\end{bmatrix}$. Theorem (3.3) produces a feasible solution to the corresponding LMIs with the following controller gains

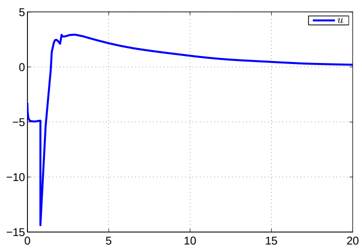
$$\begin{aligned}K_1&=\begin{bmatrix}-2.2383&3.78&-2.0092\end{bmatrix},\\ K_2&=\begin{bmatrix}-2.2206&-2.2258&4.1739\end{bmatrix},\\ K_3&=\begin{bmatrix}-2.2383&-2.22&-2.0092\end{bmatrix},\end{aligned} \tag{43}$$

Let $f_i(t,x(t))=0.3sin(x_1(t))x_1(t)$, $i=1,2,3$, and $w(t)=\frac{0.5}{1+t^2}$. By setting $\lambda=0.35$, the SMC law can be designed according to (36)-(37) and the simulation results are depicted in figures (1(a))-(1(c)) with initial condition $x(0)=\begin{bmatrix}1.5&-0.5&1\end{bmatrix}^T$. To prevent the control signals from chattering, we replace sign $\frac{s(t)}{\|s(t)\|}$ with $\frac{s(t)}{0.05+\|s(t)\|}$.

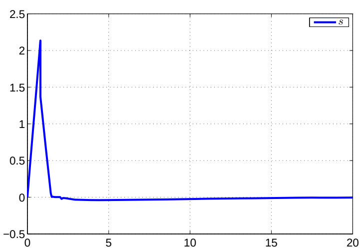
Figure (1(a)) plots the evolution of the system states and Figure (1(b)) depicts the control input vector. The response of $s(t)$ is given in Figure (1(c)). It is observed from Figure 1(a) that the state trajectories of the system all converge to the origin quickly. The system can be stabilized by the proposed method and the reaching motion satisfies the sliding reaching condition in spite of the time-varying delay, uncertainties and matched input. Figure (1(d)) shows the state trajectories of the closed-loop system without a sliding mode term. From this figure, we can see the effectiveness of the sliding mode term, which is used to compensate the effect of unknown input.



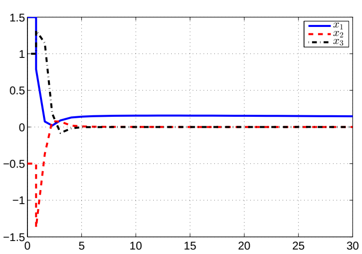
(a) States of the closed-loop system.



(b) Control input $u(t)$.



(c) Switching surface function $s(t)$.



(d) States without a sliding mode term.

V. Conclusion

Complete results have been developed for robust sliding mode control of a class of continuous (TS) fuzzy descriptor systems with time-varying delay and parameters uncertainties. Without resorting to the decomposition and equivalent transformation of the sliding mode dynamics, the question of robust admissibility with generalized H_2 performance is considered and a new delay-dependant LMI-based criterion is derived. Moreover, a SMC control law is designed such that the reaching condition is satisfied and the chattering can be reduced. The feasibility and the effectiveness of theoretical developments has been verified by a numerical example.

References

[1] M Kchaou, M Souissi, and A Toumi. Delay-dependent stability and robust $L_2 - L_\infty$ control for a class of fuzzy descriptor systems with time-varying delay. *International Journal of Robust Nonlinear Control*, doi: 10.1002/rnc.1832, 2011.

[2] E.K Boukas. Singular linear systems with delay : H_∞ stabilization. *Optimal Control Applications And Methods*, 28:259–274, 2007.

[3] J.L Chang. Dynamic output feedback integral sliding mode control design for uncertain systems. *Internal Journal and Robust Nonlinear Control*, 12:841–857, 2012.

[4] L Dai. *Singular Control Systems*, volume 118 of *Lecture Notes in Control and Information Sciences*. Springer, New York, 1989.

[5] Y Ding, H Zhu, and S Zhong. Exponential stabilization using sliding mode control for singular systems with time-varying delays and non-linear perturbations. *Commun Nonlinear Sci Numer Simulat*, 16:4099–4107, 2011.

[6] L GAO and Y WU. An lmi-based variable structure control for a class of uncertain singular markov switched systems. *Journal of Control Theory and Applications*, 4:415–419, 2007.

[7] H Gassara, A El Hajjaji, M Kchaou, and M Chaabane. Observer based $(Q,V,R) - \alpha$ -dissipative control for TS fuzzy descriptor systems with time delay. *Journal of the Franklin Institute*, 2013.

[8] C Han, G Zhang, L Wu, and Q Zeng. Sliding mode control of TS fuzzy descriptor systems with time-delay. *Journal of the Franklin Institute*, 2013.

[9] L Li, Y Jia, J Dub, and S Yuan. Robust $L_2 - L\infty$ control for uncertain singular systems with time-varying delay. *Progress in Natural Science*, 18:1015–1021, 2008.

[10] T.H.S Li, C.L Kuo, and N.R Guo. Design of an EP-based fuzzy sliding-mode control for a magnetic ball suspension system. *Chaos, Solitons and Fractals*, 33:1523–1531, 2007.

[11] A Seuret, C Edwards, and S.K Spurgeon. Static output feedback sliding mode control design via an arti?cial stabilizing delay. *IEEE Trans Automat Control*, 54(2):256–266, 2009.

[12] J Wu, Z Weng, Z Tian, and S Shi. Fault tolerant control for uncertain time-delay systems based on sliding mode control. *Kybernetika*, 2008:617–632, 44.

[13] L Wu and D.W.C. Ho. Sliding mode control of singular stochastic hybrid systems. *Automatica*, 46:779–783, 2010.

[14] L Wu and Z Wang. Robust $L_2 - L\infty$ control of uncertain differential linear repetitive processes. *Systems & Control Letters*, 57:425–435, 2008.

[15] L Wu and W.X Zheng. Passivity-based sliding mode control of uncertain singular time-delay systems. *Automatica*, 45:2120–2127, 2009.

[16] F Yang and Q.L Zhang. Delay-dependent H_∞ control for linear descriptor systems with delay in state. *Journal of Control Theory and Applications*, 3(1):76–84, 2005.

[17] H Zhang, Y Shenb, and G Feng. Delay-dependent stabilityand H_∞ control for a class of fuzzy descriptor systems with time-delay. *Fuzzy Sets and Systems*, 160:1689–1707, 2009.

[18] S Zhou and J Lam. Robust stabilization of delayed singular systems with linear fractional parametric uncertainties. *Circuits systems signal processing*, 22(6):579–588, 2003.