International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608

State and Parameters estimation of nonlinear systems described by Wiener mathematical models

Houda SALHI

University of Sfax, National Engineering School of Sfax (ENIS), Laboratory of Sciences and Techniques of Automatic control & computer engineering (Lab-SAT), B.P. 1173, 3038 Sfax
salhi.houda85@yahoo.fr

Samira KAMOUN

University of Sfax, National Engineering School of Sfax (ENIS), Laboratory of Sciences and Techniques of Automatic control & computer engineering (Lab-SAT), B.P. 1173, 3038 Sfax

Abstract— **This paper deals with the state and parametric estimation of nonlinear systems described by Wiener mathematical models. We propose an approach, where we combine a state estimation algorithm based on the linear Kalman Filter (for state estimation) and a parametric estimation algorithm based on the Recursive Least Squares techniques and the adjustable model (for parametric estimation). The proposed algorithm can be extended to other block-oriented models, such as Hammerstein mathematical models. A simulation example is treated to test the effectiveness of the proposed algorithm.**

Keywords— *Nonlinear systems; State estimation; Parametric estimation; Wiener mathematical models; Kalman Filter; Recursive estimation algorithm.*

I. INTRODUCTION

In order to describe adequately a nonlinear system over the entire range of operating conditions, a nonlinear block-oriented model is often used and the identified system is generally subdivided into linear dynamic system and nonlinear static system [1]. The well-known models are the Hammerstein mathematical models and Wiener mathematical models, which correspond to processes with linear dynamic, but a nonlinear gain [2]. These mathematical models reveal the capability of describing a wide class of different systems and apart from industrial examples, such as distillation and pH neutralization process [3,4].

The Hammerstein and Wiener mathematical models are useful in representing the nonlinearities of a system without introducing the complications associated with general nonlinear operator [5].

This paper deals with recursive state and parametric estimation of a nonlinear system described by Wiener mathematical models. We will show how we can use the linear Kalman Filter KF, with some changes, to estimate the state variable of this type of models. Then, we will use the least squares method for parametric estimation. After, we will combine these two approaches in order to estimate jointly the state variable and the parameters of these considered mathematical models.

In order to describe adequately a nonlinear system over the entire range of operating conditions, a nonlinear block-oriented model is often used and the identified system is generally subdivided into linear dynamic system and nonlinear static system [1]. The well-known models are the Hammerstein mathematical models and Wiener mathematical models, which correspond to processes with linear dynamic, but a nonlinear gain [2]. These mathematical models reveal the capability of describing a wide class of different systems and apart from industrial examples, such as distillation and pH neutralization process [3,4].

The Hammerstein and Wiener mathematical models are useful in representing the nonlinearities of a system without introducing the complications associated with general nonlinear operator [5].

This paper deals with recursive state and parametric estimation of a nonlinear system described by Wiener mathematical models. We will show how we can use the linear Kalman Filter KF, with some changes, to estimate the state variable of this type of models. Then, we will use the least squares method for parametric estimation. After, we will combine these two approaches in order to estimate jointly the state variable and the parameters of these considered mathematical models.

II. WIENER MATHEMATICAL MODEL

Let us assume that the considered system can be described by the following Single-Input Single-Output (SISO) discrete-time Wiener mathematical model [6, 7]:

International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608

$$x(k+1) = Ax(k) + Bu(k) + v(k) \qquad (1)$$
$$z(k) = C^T x(k) + w(k)$$
$$y(k) = f[z(k)] + e(k)$$

where $u(k) \in R$, $x(k) \in R^n$ and $y(k) \in R$ are the input, the unmeasurable state and the output, respectively, $e(k)$ is a white noise zero mean, $f[.]$ is a nonlinear function, which can be described by a matrix form, such as:

$$f[z(k)] = \alpha z(k) + \theta^T \psi(z(k), k) \qquad (2)$$

where $\theta \in R^{p\times 1}$ is the parameters vectors and $\psi(k) \in R^{p\times 1}$ is the nonlinear observation vectors, which depend to $z(k)$.

The matrix A and the vectors B and C are definite respectively by:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \qquad (3)$$

$$B^T = [b_1 \quad \cdots \quad b_n] \qquad (4)$$

$$C^T = [c_1 \quad \cdots \quad c_n] \qquad (5)$$

Let the orders n and p be known. The parameters in the matrix $A \in R^{n\times n}$, in the vectors $B \in R^{n\times 1}$ and $C \in R^{n\times 1}$ are supposed to be unknown, the parameters in the vector $\theta \in R^{p\times 1}$ are supposed also unknown.

The purpose of this paper is to present a recursive algorithm to estimate the state and the parameters of the considered system (1), which is described by a Wiener mathematical model, from given input-output data $\{u(k), y(k)\}$.

III. KALMAN FILTER AND RLS ESTIMATION

In this section, we present recursive method for state and parametric estimation of a Wiener mathematical model [8]. First, we formulate the state estimation problem assuming that the parameters in the considered model (1) are known. Second, we treat the inverse case, i.e., we suppose that the state vector is measurable, but the parameters are unknown. Finally, we combined these two problems and we suppose that the state vector and the parameters are unknown.

A. Kalman Filter

The well-known state estimation filter is the Kalman filter, which has been widely used in the literature. In the case of general nonlinear system, this filter cannot be used. For that, several approximate solutions have been proposed, such that, the extended Kalman filter, which utilizes a linearized model along the estimated state trajectory; and the unscented Kalman filter, which utilizes a nonlinear transformation to approximate the probability density function of the state at each time instant [9].

In our case, we consider the nonlinear systems which are described by blocks oriented models. It is known that the study of a dynamic nonlinear system in term of connected systems, simplify the formulation of the identification diagram and control system. Since, one can apply the developed results for the linear systems, with some techniques of practical implementation [10].

In this section, we will present a state estimation algorithm based on the linear kalman filter, in order to estimate the state vector of the considered system, which is described by a wiener mathematical model (1).

First, we define $\hat{x}(k)$ the *a priori* estimate of the state $x(k)$, formulated at discrete-time k, given the data at discrete-time $k-1$. We define also $\hat{x}^0(k)$ the *a posteriori* estimate of the state $x(k)$, developed at discrete-time k, based on the data at the same time k.

We can define the *a priori* estimation error $\xi(k)$ and the *a posteriori* estimation error $\xi_0(k)$, by the following expressions, respectively:

$$\xi(k) = x(k) - \hat{x}(k) \qquad (6)$$

and

$$\xi_0(k) = x(k) - \hat{x}_0(k) \qquad (7)$$

International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608

The variance-covariance matrix $P(k)$ of the *a priori* estimation error $\xi(k)$ can be defined by:

$$P(k)=E\left[\xi(k)\xi^{T}(k)\right] \quad (8)$$

The variance-covariance matrix $P_0(k)$ of the *a posteriori* estimation error $\xi_0(k)$ is defined as follow:

$$P_0(k)=E\left[\xi_0(k)\xi_0^{T}(k)\right] \quad (9)$$

Based on the equation of the Kalman filter, we can express the *a posteriori* estimate $\hat{x}^0(k)$ of the considered model (6) as follow:

$$\hat{x}_0(k)=\hat{x}(k)+K(k)\left(z(k)-C^T\hat{x}(k)\right) \quad (10)$$

where $K(k)$ is an adaptation gain vector, which can be calculated by the following expression:

$$K(k)=\frac{P(k)C^T}{\sigma^2+CP(k)C^T} \quad (11)$$

where σ^2 is the variance of the noise $w(k)$ affecting the output of the linear part. The problem is that the output of the linear part $z(k)$ is inaccessible to measurement. For that, it must be estimated using measured data of the considered system.

Consider the following equations:

$$\begin{aligned} z(k+1)&=C^T x(k+1)+w(k) \\ y(k)&=\alpha z(k)+\theta^T\psi\left(z(k),k\right)+e(k) \end{aligned} \quad (12)$$

This can be written in the following form:

$$\begin{aligned} z(k+1)&=C^T Ax(k)+C^T Bu(k)+w(k) \\ y(k)&=\alpha C^T x(k)+\theta^T\psi\left(z(k),k\right)+e(k) \end{aligned} \quad (13)$$

Define the *a priori* estimation error $\xi_z(k)$ and his variance-covariance matrix $R(k)$, by the following equations, respectively:

$$\xi_z(k)=z(k)-\hat{z}(k) \quad (14)$$

and

$$R(k)=E\left[\xi_z(k)\xi_z^{T}(k)\right] \quad (15)$$

We can express the estimate $\hat{z}(k+1)$ of the considered model (10) as follow:

$$\begin{aligned} \hat{z}(k+1)=&\,C^T\left(A\hat{x}^0(k)+C^T Bu(k)\right) \\ &+K_z(k)\left(y(k)-\alpha C^T\hat{x}^0(k)-\theta^T\tilde{\psi}(k)\right) \end{aligned} \quad (16)$$

where $K_z(k)$ is an adaptation gain vector, which is defined by the following expression:

$$K_z(k)=\frac{C^T AP(k)\alpha C}{\alpha C^T P(k)\alpha C+\sigma_2{}^2+\theta^T G(k)\theta} \quad (17)$$

where $\sigma_2{}^2$ is the variance of the noise $e(k)$ affecting the output system and $G(k)$ is the variance-covariance matrix, defined by:

$$G(k)=E\left[\left(\psi(k)-\tilde{\psi}(k)\right)\left(\psi(k)-\tilde{\psi}(k)\right)^T\right] \quad (18)$$

where $\tilde{\psi}(k)$ is the approximated vector of $\psi(k)$.

The resulting recursive state estimation algorithm of the wiener mathematical model, in the sense of Kalman filter, consists of the following stages:

1) Step 1 (estimation at discrete-time k):

a) Determining the a posteriori estimate $\hat{x}^0(k)$ of the state $x(k)$:

$$\begin{aligned} \hat{x}_0(k)&=\hat{x}(k)+K(k)\left(\hat{z}(k)-C^T\hat{x}(k)\right) \\ K(k)&=\frac{P(k)C^T}{\sigma^2+CP(k)C^T} \end{aligned} \quad (19)$$

b) Determining the variance-covariance matrix $P_0(k)$ of the a posteriori estimation error $\xi_0(k)$:

International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608

$$P^0(k) = P(k) - K(k)CP(k) \tag{20}$$

2) Step 2 (prediction):

a) Determining the a priori estimate $\hat{x}(k)$ of the state $x(k)$:

$$\hat{x}(k+1) = A\hat{x}^0(k) + Bu(k) \tag{21}$$

b) Determining the variance-covariance matrix $P(k+1)$ of the a priori estimation error $\xi(k)$:

$$P(k+1) = AP^0(k)A^T + Q \tag{22}$$

c) Determining the estimate $\hat{z}(k+1)$ of the output linear part:

$$\begin{aligned}\hat{z}(k+1) &= C^T\hat{x}(k+1) \\ &\quad + K_z(k)\left(y(k) - \alpha C^T\hat{x}^0(k) - \theta^T\tilde{\psi}(k)\right) \\ K_z(k) &= \frac{C^T AP\alpha C}{\alpha C^T P\alpha C + \sigma_2{}^2 + \theta^T G(k)\theta}\end{aligned} \tag{23}$$

d) Determining the estimate vector $\tilde{\psi}(k+1)$ of the vector $\psi(k+1)$ at the discrete-time $k+1$:

$$\tilde{\psi}(k+1) = h\left(\hat{z}(k+1)\right) \tag{24}$$

B. Recursive parametric estimation algorithm

Several recursive algorithms have been proposed to solve the parametric estimation problem, which are based on the prediction error method and such as Least squares technique [11].

The Least Squares technique is the simplest technique, which can be extended for an online implementation, namely, Recursive Least Squares (RLS) [12].

In this section, we will propose a recursive parametric estimation algorithm for parameters estimation, based on the adjustable model and the Least Squares technique.

Referring to the Wiener mathematical model (1), we propose the following adjustable mathematical model:

$$\begin{aligned}x_p(k+1) &= \hat{A}(k+1)x(k) + \hat{B}(k+1)u(k) \\ z_p(k) &= \hat{C}^T(k)x(k) \\ y_p(k) &= \hat{\phi}^T(k)\upsilon(k)\end{aligned} \tag{25}$$

where $\hat{A}(k+1)$ and $\hat{B}(k+1)$ represent the estimated parameters of the matrix A and the vector B , respectively, at the discrete-time *k+1*, $\hat{C}(k)$ and $\hat{\phi}(k)$ is the estimated vectors of C and ϕ, respectively, where $\phi^T = \begin{bmatrix}\alpha & \theta^T\end{bmatrix}$ is the parameters vector and $\upsilon^T(k) = \begin{bmatrix}z(k) & \psi^T(k)\end{bmatrix}$ is the observation vector.

The recursive parametric estimation algorithm allowing to estimate the various parameters in the matrix A , in the vectors *B*, *C*, and ϕ , is given by:

$$\begin{aligned}
&\hat{A}(k+1) = \hat{A}(k) + \xi_x(k)G_x\delta_x(k)x^T(k-1) \\
&\hat{B}(k+1) = \hat{B}(k) + \xi_x(k)G_x\delta_x(k)u^T(k-1) \\
&\hat{C}(k+1) = \hat{C}(k) + \xi_z(k)G_z\delta_z(k)x(k) \\
&\hat{\phi}(k+1) = \hat{\phi}(k) + \xi_y(k)G_y\delta_y(k)\upsilon(k) \\
&\delta_x(k) = x(k) - \hat{A}(k)x(k-1) - \hat{B}(k)u(k-1) \\
&\delta_z(k) = z(k) - \hat{C}^T(k)x(k) \\
&\delta_y(k) = y(k) - \hat{\phi}^T(k)\upsilon(k) \\
&\xi_x(k) = \frac{l_x}{\lambda_{G_x}\rho^2(k-1)} \\
&\rho^2(k-1) = x^T(k-1)x(k-1) + u^2(k-1) \\
&\xi_z(k) = \frac{lz}{\lambda_{G_z}\left(x^T(k)x(k)\right)} \\
&\xi_y(k) = \frac{ly}{\lambda_{G_y}\left(\upsilon^T(k)\upsilon(k)\right)}
\end{aligned} \tag{26}$$

where G_x , G_z and G_y are definite positive symmetrical matrices, λ_{G_x} , λ_{G_z} and λ_{G_y} are, respectively the maximum eigenvalue of the matrix G_x , the maximum eigenvalue of the matrix G_z and the maximum eigenvalue of the matrix G_y . l_x, l_z and l_y are three parametric gains, that must be chosen in a proper way to ensure the stability of the parametric estimation scheme. The convergence analysis of the proposed parametric estimation algorithm was made in [13].

Theorem 1. Consider a nonlinear dynamic system described by Wiener mathematical model, which is composed of a

International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608

dynamic linear part, modulated by a state-space equation, and a static nonlinear part. The estimation of the parameters in the matrix A and in the vectors B, C and ϕ of this mathematical model can be made using the recursive estimation algorithm (26). The choice of parameters l_x , l_z and l_y , must satisfy the following conditions:

$$0 < l_x < 2 \tag{27}$$

$$0 < l_z < 2 \tag{28}$$

and

$$0 < l_y < 2 \tag{29}$$

to ensure the stability of the proposed scheme.

C. Units Kalman Filter combined with recursive parametric estimation algorithm

In this section, we present a recursive algorithm that combines the state estimation method with the recursive parametric estimation algorithm, in order to estimate the state and the parameters of the mathematical model (1).

The steps of the proposed algorithm are as follow:

Initialization: An initial estimate is given for various parameters of the considered model, i.e, $\hat{A}(0)$, $\hat{B}(0)$, $\hat{C}(0)$, $\hat{\phi}(0)$, $\hat{x}(0)$, $\hat{x}(1)$, $\hat{x}(2)$, $\hat{x}^0(0)$, $\hat{z}(0)$, $\hat{z}(1)$ $\hat{\psi}(0)$, $P(0)$, $G(0)$ and $P(0)$. The good choice of initial values can make the recursive algorithm more stable and can give convergence to the global minimum.

Parametric estimation: Given $\hat{x}(k-1)$, $\hat{x}(k-2)$, and $\hat{\upsilon}(k-1)$, we can use the following algorithm:

$$\begin{aligned}
&\hat{A}(k) = \hat{A}(k-1) + \xi_x(k) G_x \delta_x(k-1) \hat{x}^{\mathrm{T}}(k-2) \\
&\hat{B}(k) = \hat{B}(k-1) + \xi_x(k) G_x \delta_x(k-1) u^T(k-2) \\
&\hat{C}(k) = \hat{C}(k-1) + \xi_z(k) G_z \delta_z(k-1) \hat{x}(k-1) \\
&\hat{\phi}(k) = \hat{\phi}(k-1) + \xi_y(k) G_y \delta_y(k-1) \hat{\upsilon}(k-1) \\
&\delta_x(k-1) = \hat{x}(k-1) - \hat{A}(k-1)\hat{x}(k-2) - \hat{B}(k-1)u(k-2) \\
&\delta_z(k-1) = \hat{z}(k-1) - \hat{C}^T(k-1)\hat{x}(k) \\
&\delta_y(k-1) = y(k-1) - \hat{\phi}^T(k-1)\hat{\upsilon}(k-1) \\
&\xi_x(k) = \frac{l_x}{\lambda_{G_x} \rho^2(k-2)} \\
&\rho^2(k-2) = \hat{x}^{\mathrm{T}}(k-2)\hat{x}(k-2) + u^2(k-2) \\
&\xi_z(k) = \frac{lz}{\lambda_{G_z}\left(\hat{x}^T(k-1)\hat{x}(k-1)\right)} \\
&\xi_y(k) = \frac{ly}{\lambda_{G_y}\left(\hat{\upsilon}^T(k-1)\hat{\upsilon}(k-1)\right)}
\end{aligned}$$

State estimation: this step consists of:

a) calculate the gain filter

$$K(k) = \frac{P(k)\hat{C}(k)^T}{\sigma^2 + \hat{C}(k)P(k)\hat{C}(k)^T} \tag{31}$$

b) estimate the a posteriori estimate $\hat{x}^0(k)$ *of the state* $x(k)$*:*

$$\hat{x}_0(k) = \hat{x}(k) + K(k)\left(\hat{z}(k) - \hat{C}(k)^T \hat{x}(k)\right) \tag{32}$$

c) determine the variance-covariance matrix $P_0(k)$*:*

$$P^0(k) = P(k) - K(k)\hat{C}(k)P(k) \tag{33}$$

d) determine the a priori estimate $\hat{x}(k)$*:*

$$\hat{x}(k+1) = \hat{A}(k)\hat{x}^0(k) + \hat{B}(k)u(k) \tag{34}$$

e) determine the variance-covariance matrix $P(k+1)$*:*

$$P(k+1) = \hat{A}(k)P^0(k)\hat{A}(k)^T + Q \tag{35}$$

International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608

f) calculate the gain $K_z(k)$:

$$K_z(k)=\frac{\hat{C}^T(k)\hat{A}(k)P\hat{\alpha}(k)\hat{C}(k)}{\hat{\alpha}(k)\hat{C}^T(k)P\hat{\alpha}(k)\hat{C}(k)+\sigma_2{}^2+\hat{\theta}^T(k)G(k)\hat{\theta}(k)} \tag{36}$$

g) determine the estimate $\hat{z}(k+1)$:

$$\begin{aligned}\hat{z}(k+1)&=\hat{C}^T(k)\hat{x}(k+1)\\&+K_2(k)\left(y(k)-\hat{\alpha}(k)\hat{C}^T(k)\hat{x}^0(k)-\hat{\theta}^T(k)\tilde{\psi}(k)\right)\end{aligned} \tag{37}$$

h) determine the estimate vector $\hat{\psi}(k+1)$:

$$\tilde{\psi}(k+1)=h\left(\hat{z}(k+1)\right) \tag{38}$$

In this section, we propose to use the parameters gains l_x, l_z and l_y as a variables in discrete-time k, in order to give more robustness of the state and parametric estimation algorithm. The variation range of each gain should be chosen adequately, in order to verify the convergences conditions (27), (28) and (29) of the parametric estimation algorithm.

Calculation of the three parametric gains can be made as follow [11]:

$$l_i(k)=l_{0i}\left(1-j_i(k)\right) \qquad i=x,z,y \tag{39}$$

where l_{0i}, a positive parameter, must be selected as: $0<l_{0i}<2$ and the parameter $j_i(k)$ can be calculated using the following recursive equation:

$$j_i(k)=j_{0i}j_i(k-1)+j^{0i}(1-j_{0i}) \tag{40}$$

We can easily show that:

$$\lim_{k\to\infty} j_i(k)=j^{0i} \tag{41}$$

and

$$\lim_{k\to\infty} l_i(k)=l_0\left(1-j^{0i}\right) \tag{42}$$

we represent, Fig.1, an example of evolution curve of the parameters gains l_x, l_z and l_y, which are described by (39).

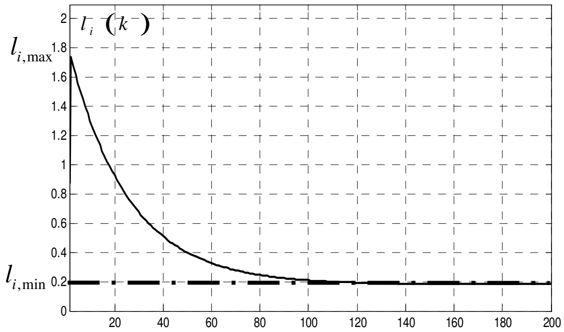

Fig. 1. The evolution curve of the parametric gains l_x, l_z and l_y.

It can be noted that the proposed recursive algorithm of state and parametric estimation can be divided into three blocks: a parametric estimation block, a state parametric block and a prediction block.

There is a coupling between the three blocks, in the sense that the practical implementation of one block requires a transfer of information and data with the other blocks.

IV. ILLUSTRATIVE EXAMPLE

In this section, we consider an example to illustrate the proposed algorithm for recursive state and parameters estimation.

Consider a nonlinear system, described by the following Wiener mathematical model:

$$\begin{aligned}x(k+1)&=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}x(k)+\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}u(k)+v(k)\\z(k)&=\begin{bmatrix}c_1&c_2&c_3\end{bmatrix}x(k)+w(k)\\y(k)&=f\left[z(k)\right]+e(k)\end{aligned} \tag{43}$$

where $x^T(k)=\left[x_1(k)\quad x_2(k)\quad x_3(k)\right]$, $u(k)$, $z(k)$ and $y(k)$ represent, respectively, the state vector, the system input, the output of the linear part and the system output at the discrete-time k, $v^T(k)=\left[v_1(k)\quad v_2(k)\quad v_3(k)\right]$ is the noise vector that acts the state system $x(k)$, $w(k)$ and $e(k)$ denote the noises affecting, respectively, the output of the linear part $z(k)$ and the measured value of the output system $y(k)$ and

$f: R \to R$ is a nonlinear function, (see Figure 2), with hysteresis-relay nonlinearity such that:

$$f\left[z(k)\right] = \begin{cases} \beta & z(k) > -\tau \\ -\beta & z(k) < \tau \end{cases} \tag{44}$$

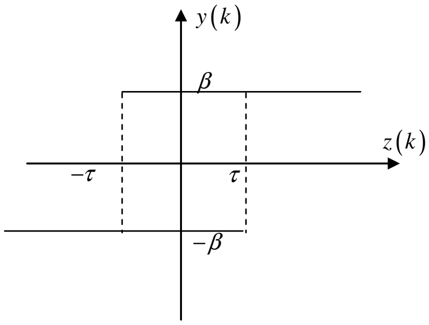

Fig. 2. Hysteresis-relay nonlinearity

This function can be expressed as:

$$f\left[z(k)\right] = \frac{\beta}{2\tau}\left[(z+\tau)\,\mathrm{sgn}(z+\tau) - (z-\tau)\,\mathrm{sgn}(z-\tau)\right] \tag{45}$$

where

$$\mathrm{sgn}[*] = \begin{cases} 1 & * \geq 0 \\ -1 & * < 0 \end{cases} \tag{46}$$

Then, (38) can be written as:

$$f\left[z(k)\right] = \phi^T \upsilon(k) \tag{47}$$

with

$$\phi^T = \begin{bmatrix} 1 & -1 & \dfrac{\beta}{2\tau} & \dfrac{\beta}{2} & -\dfrac{\beta}{2\tau} & \dfrac{\beta}{2} \end{bmatrix} \tag{48}$$

and

$$\begin{aligned} \vartheta^T(k) = [&z(k) \quad -z(k) \quad z(k)\,\mathrm{sgn}(z(k)+\tau) \\ &\mathrm{sgn}(z(k)+\tau) \quad z(k)\,\mathrm{sgn}(z(k)-\tau) \quad \mathrm{sgn}(z(k)-\tau)] \end{aligned} \tag{49}$$

The parameters a_{ij}, b_i and c_i of the linear part are supposed unknown, for $i, j = 1, \ldots, 3$. The parameters in vector ϕ are supposed also unknown. The state variables $x_1(k)$, $x_2(k)$ and $x_3(k)$ are assumed immeasurable for all values of the discrete-time k. The input signal $u(k)$ and the output signal $y(k)$ are measurable. These measured values are assumed independent of the noise component $v_1(k)$, $v_2(k)$ and $v_3(k)$ and of the noise $e(k)$.

In order to estimate the state variables and the parameters of the mathematical model (36), we will use the proposed state and parametric estimation algorithm given by (27) to (35).

The relative data to this example of numerical simulation, for the practical implementation of the proposed algorithm, are given as hereafter:

a) the input $u(k)$ is taken as a random sequence with zero mean and constant variance.

b) the components $v_1(k)$, $v_2(k)$ and $v_3(k)$ of the noise vector $v(k)$ consist of independent random variables with zero mean. The variance-covariance matrix of the noise vector is given by: $Q = 0.02 I_{3\times3}$.

c) the noise $w(k)$ and $e(k)$ are consists of an independent random sequences with zero mean and variances $\sigma_w^2 = 0.05$ and $\sigma_e^2 = 0.015$.

d) the number of measurements M being chosen, such as: $M = 1, \ldots, 200$.

e) the adaptation gain and the initial conditions of the various quantities involved in the proposed state and parameters estimation algorithm are selected with an adequate way.

The quality estimation of the parameters intervening in the Wiener model (36) can be made by considering the parametric distance $d(k)$ given by:

$$d(k) = \left[\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\frac{a_{ij} - \hat{a}_{ij}(k)}{a_{ij}}\right)^2 + \sum_{i=1}^{3}\left(\frac{b_i - \hat{b}_i(k)}{b_i}\right)^2 + \sum_{i=1}^{3}\left(\frac{c_i - \hat{c}_i(k)}{c_i}\right)^2 + \sum_{p=1}^{6}\left(\frac{\phi_p - \hat{\phi}_p(k)}{\phi_p}\right)^2\right]^{0.5} \tag{50}$$

International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608

The evolution curves of the three state variables $x_1(k)$, $x_2(k)$ and $x_3(k)$ with the estimate state variable $\hat{x}_1(k)$, $\hat{x}_2(k)$ and $\hat{x}_3(k)$ and the system output $y(k)$ are given Fig.3. The evolutions curves of the three state estimation error $\delta_1(k)$, $\delta_2(k)$ and $\delta_3(k)$ and the evolution curve of the output system $\delta_y(k)$ are illustrated Fig.4.

Fig.5 shows the evolutions curves of the variance of the state estimation error $\sigma_1(k)$, $\sigma_2(k)$ and $\sigma_3(k)$ and the evolution curve of the parametric distance $d(k)$. The real parameters values and the average of its estimated are given by Table 1.

The parameters of the nonlinear part can be deduced from the following expression:

$$\hat{\beta}(k) = \hat{\phi}(3,k) + \hat{\phi}(5,k) \tag{51}$$

and

$$\hat{\tau}(k) = \frac{1}{2}\left[\frac{\hat{\beta}(k)}{2\hat{\phi}(3,k)} - \frac{\hat{\beta}(k)}{2\hat{\phi}(5,k)}\right] \tag{52}$$

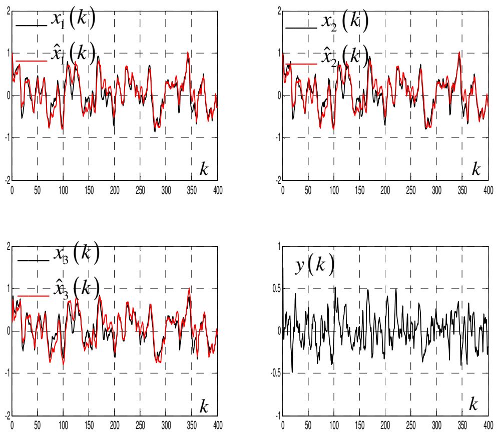

Fig. 3. Evolution curves of the state variables $x_1(k)$, $x_2(k)$ and $x_3(k)$ with the estimate state variable $\hat{x}_1(k)$, $\hat{x}_2(k)$ and $\hat{x}_3(k)$ and the output system $y(k)$.

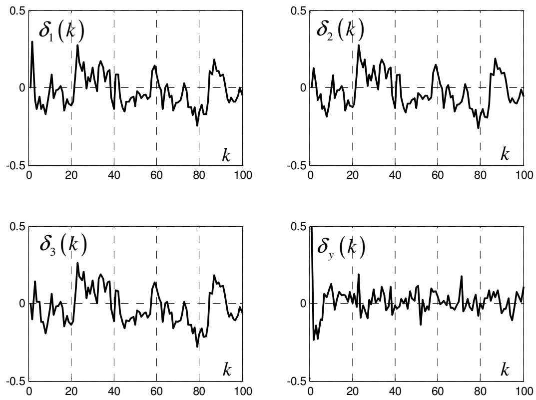

Fig. 4. Evolutions curves of the three states estimation errors $\delta_1(k)$, $\delta_2(k)$ and $\delta_3(k)$ and the evolution curve of the output system $\delta_y(k)$.

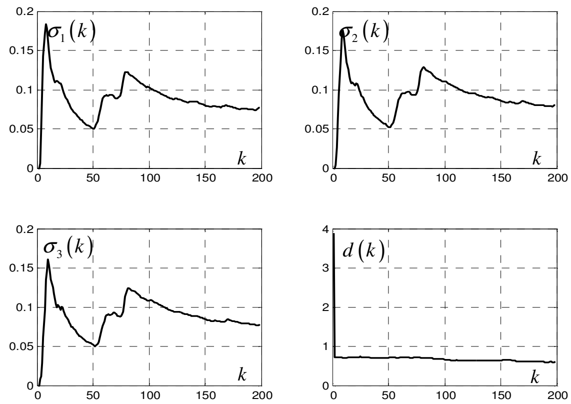

Fig. 5. Evolutions curves of the variances of the state estimation errors $\sigma_1(k)$, $\sigma_2(k)$, $\sigma_3(k)$ and of the parametric distance $d(k)$.

Figure3 shows the state estimation performance using the proposed state estimation method based on linear Kalman filter. The averages of the state estimation error of the three state variables $x_1(k)$, $x_2(k)$ and $x_3(k)$ are, respectively, $\overline{m}_{\delta_1(k)} = 0.1241$, $\overline{m}_{\delta_2(k)} = 0.1125$ and $\overline{m}_{\delta_3(k)} = 0.1139$.

Moreover, the variances $\sigma_1(k)$, $\sigma_2(k)$, $\sigma_3(k)$ of the state estimation errors and the parametric distance in figure 5, decreases asymptotically towards a low minimum.

From Tables 1 and 2, we can draw the following conclusions:

International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608

- the parameters estimates given by the proposed parameters and state estimation algorithm converge to their true values as the parameters gains l_x, l_z and l_y are variables in discrete-time k.
- the state estimation errors depend on the quality of the parametric estimation, i.e., it become smaller when the estimates parameters converge rapidly to their real parameters.

This shows that the proposed algorithm of the parametric and state estimation is effective.

Table . real parameters values, estimated parameters values and the average of the states estimation errors

	$l_i(k)$	a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}
Real parameters		1.800	-1.0400	0.1920	1.000	0.000	0.000
Estimate parameters	***Constants***	1.5063	-1.0061	0.1767	0.8534	-0.0094	-0.0097
Estimate parameters	***variables***	1.7994	-1.0516	0.1867	0.9980	-0.0055	-0.0010

	$l_i(k)$	a_{33}	b_1	b_2	b_3	ϕ_1	ϕ_2
Real parameters		0.000	1.000	0.00	0.00	1	-1
Estimate parameters	***Constants***	-0.0011	0.8973	-0.0019	-0.0002	0.8794	-0.8238
Estimate parameters	***variables***	0.000	0.9987	0.0010	0.0003	0.9140	-0.9715

	$l_i(k)$	ϕ_3	ϕ_4	ϕ_5	ϕ_6	c_1	c_2
Real parameters		1	0.5	-1	0.5	1	-0.2240
Estimate parameters	***Constants***	0.9783	0.2383	-0.8553	0.2784	0.8573	-0.0266
Estimate parameters	***variables***	0.9738	0.3812	-0.9866	0.4600	1.0034	-0.2238

	$l_i(k)$	c_3	$\bar{\delta}_1(k)$	$\bar{\delta}_2(k)$	$\bar{\delta}_3(k)$
Real parameters		-0.6470			
Estimate parameters	***Constants***	-0.5550	0.2346	0.2128	0.1940
Estimate parameters	***variables***	-0.6493	-0.0572	-0.0496	-0.0439

V. CONCLUSION

This paper was presented a state and parametric estimation algorithm for nonlinear systems described by Wiener mathematical models. The latter are composed by a dynamic linear part, described by a space-state equation, and a static nonlinear part. The developed work was divided into four parts. The first part was devoted to the development of a state estimation algorithm making it possible to estimate the state of the considered nonlinear model, assuming that their parameters are known. The formulation of this algorithm is based on the linear Kalman Filter. The second part was reserved to the development of a parametric estimation algorithm, in order to estimate the parameters of the Wiener model, assuming that the state vector is measurable. The formulation of this algorithm is made using the least square technique and the adjustable model. In the third part, the state vector and the parameters of the considered system are supposed unknown. For that, we present a recursive state and parametric estimation algorithm, which combines the state estimation algorithm with the recursive parametric estimation algorithm.

In the last part, an example of numerical simulation was treated, in order to test the performances and the effectiveness of the developed recursive state and parametric estimation algorithm. The results of obtained numerical simulations are satisfactory.

ACKNOWLEDGMENT

We thank the ministry of higher education and scientific research of Tunisia for funding this work.

REFERENCES

[1] S.Biagiola, o. Agamennoni "Identification and control of wiener process: a direct synthesis approach" 2 nd Mercosur congress on Chemical Engineering. Argentina.

[2] T.A. Glaria Lopez, D.Sbarbaro. "Observer design for nonlinear processes with Wiener structure". IEEE Conference on Decision and control and European Control Conference. Orlando, FL, USA, December 12-15, 2011.

[3] S.I. Biagiola, J.L. Figueroa. "Wiener and Hammerstein uncertain models identification". Elsevier, Mathematics and computers in simulation 79,pp. 3296-3313, 2009.

[4] Y. Tang, L.Giao, X. Guan. "Identification of Wiener model using Step signals and particle swarm optimization". Expert systems with application, Elsevier Vol.37, pp. 3398-3404, 2010.

[5] S. Norquay, A. Palazoglu. "Model predictive control based on Wiener models" Chemical Engineering Science, Vol. 53, No.1, pp. 75-84, May 1998.

[6] R. Orjuela. "Contribution à l'estimation d'état et au diagnostic des systèmes représentés par des multimodèles". Thesis in automatic and signal traitement, Nancy, Novembre 2008.

[7] A. Hagenblad. "Aspects of the identification of Wiener models". Studies in science and thechnology thesis No. 793, 1999.

[8] W. Yu, D. Wilson, B. Young. "Control performance assessment for block-oriented nonlinear systems". 8th IEEE international conference on control and automation. China, June 2010.

[9] K. Xiong, C.L. Wei, L. Liu. "Robust Kalman filtering for discrete-time nonlinear systems with parameter uncertainties". Aerospace Science and Technology Vol.18, pp. 15-24, 2012.

[10] R. Lopez, J. Yuz. "Recursive parameter and state estimation for a mining industry process". 20th mediterranean conference on control & automation. Spain, Jully 2012.

[11] S.Kamoun. "Contribution of identification and adapative control of complexes systems ". Thesis in automatic and computers engeneering, ENIS, 2003.

[12] D. Wang, Y. chu, F. Ding. "Auxiliary model based recursive generalized least squares parameter estimation for Hammersetin OEAR systems. Mathematical and computer Modelling, Vol. 52, pp. 309-317, 2010.

[13] D. Wang, Y. Chu, F. Ding. "Auxiliary model-based RELS and MI-ELS algorithm for Hammersetin OEAR systems". Computer and Mathematics with Application, Vol. 59, pp. 3092-3098, 2010.

International Conference on Control, Engineering & Information Technology (CEIT'14)
Proceedings - Copyright IPCO-2014, pp.73-82
ISSN 2356-5608