# Modeling and Indirect Force Control of Linear Switched Reluctance Motor

# ZAAFRANE Wajdi<sup>1</sup>, KEDIRI Jalel<sup>1</sup> and REHAOULIA Habib<sup>1</sup>

<sup>1</sup> Laboratory Signal, *Signal, Image and Energy Management* (SIME), University of Tunis, Tunisia. High school of Sciences and Technology of Tunis (ESSTT), 5 av. Taha Hussein BP 56 – 1008 Tunis.

# wajdi.zaafrane@gmail.com

Abstract—The present paper deals with the modeling and control of a single sided Linear Planner Switched Reluctance Motor (LPSRM) with active translator and passive stator. Force ripple are the majors disadvantage of linear switched reluctance motor (LSRM), thing that can decrease his performance reduce. This paper investigates a new modeling method based on dqo transformation in linear domain for control design. The actuator under consideration is composed by three phases magnetically coupled thing that give the advantage of low cost fabrication without the need of permanent magnet. In order to control the motor open and closed loop control are detailed and tested, this paper reports a study of indirect force control using a feedback linearization with PI controller and force distribution function (FDF).

Keywords—Linear switched reluctance motor; linear actuator; closed loop control; Force control; Force distribution function; PID controller; feedback linearization,

# I. INTRODUCTION

The linear switched reluctance motor/actuator (LSRM/LSRA) is an interesting alternative in many industrial applications especially in high precision application. [3] [6]. This motor is characterized by its simple structure and low construction cost. The major advantage of this actuator is the direct motion so canceling mechanical subsystems or rotary to linear motion converters thing that decrease friction and maintain problems and increase performance.

In recent years, control of this actuator is attracting much attention to linear or rotary switched reluctance motors which have the same configuration. Due to the development of control theory and computer hardware many controls strategies are applied to LSRM in open or closed loop control [1],[2],[16].

Today, thanks to advances in power electronics and in computer science applications, LSRMs are used in closed loop control especially in robotics and in biomedical applications [8]. Position an speed control are detailed in [8][9] and various control theory are presented like PID controller in [8], sliding mode control in[5], fuzzy logic control in [9] but force control is not described extensively in literature however controlling force is very important because that is related to qualities of the positioning.

The major disadvantage of LSRM is the large force ripple, in fact solving this problem represent an important objective so in recent years, several control strategies has been proposed in the literatures in linear or rotary domain [12],[13],[14],[15],[ by using a force distribution function FDF.

This paper intends to describe a novel method to LSRM modeling based on dqo transformation which is usually used in rotary domain for a force control design.

The force control loop is the key of high qualities of positioning, so in this work just force control is presented which can be used with other loops like position or speed.

The objective of this paper is double. The first part consists of modeling the LSRM neglecting magnetic saturation based on dqo transformation. The second part concerns the closed loop speed control of the actuator by continuous excitation of all phases. The new force controller is based on torque controller of classical synchronous reluctant machines.

Converting modeling and control from rotary to linear domain conduct to an optimum and easy strategy of control by using feedback linearization and PI controller to control the force and minimize ripple.

# II. LINEAR SWITCHED RELUCTANCE MOTOR CONFIGURATION AND MODELING

The proposed actuator is a planner longitudinal linear switched reluctance motor with active mover composed by three-phases and passive stator. This configuration consists of 6 translator pole wich was similar to 6/4 RSRM (6 stator and 4 rotor poles).

LSRM is composed by two elements, the first one is a toothed magnetic material fixed to a support and called stator. The second is a sliding part on rails called the mover (translator). This latter is formed by different modules regularly distributed lodging the winding. This topology is a single sided LSRM with active translator and passive stator, figure (1).

If teeth of an active module are aligned with teeth of the mover, the other stator modules must be unaligned in order to create a translation force. International Conference on Control, Engineering & Information Technology (CEIT'14) Proceedings - Copyright IPCO-2014, 345-351 ISSN 2356-5608



Fig. 1: Actuator configuration

The LSRM has a highly nonlinear characteristic due to its nonlinear flux behaviour [10]. In order to simplify equations, the modelling is performed without taking into account magnetic saturation, phases are considered identical and end effect is neglected [10].

In reference [5],[8] modeling of the motor is based on electrical and mechanical equations (1) and (2) when its clear the dependence variable thing that can complicate the control design

$$v_{j} = Ri_{j} + L_{0}\frac{di_{j}}{dt} + L_{1}\cos(\frac{2\pi x}{\lambda} - (j-1)\frac{2\pi}{3})\frac{di_{j}}{dt} + \frac{2\pi}{\lambda}L_{1}\sin(\frac{2\pi x}{\lambda} - (j-1)\frac{2\pi}{3})\frac{dx}{dt}i_{j}$$
(1)

$$\frac{dv}{dt} = -\frac{\pi L_1}{m\lambda} \begin{vmatrix} i^2 \sin(\frac{2\pi}{\lambda}x) \\ +i^2 \sin(\frac{2\pi}{\lambda}x - \frac{\pi}{3}) \\ +i^2 \sin(\frac{2\pi}{\lambda}x - \pi) \end{vmatrix} - \frac{\xi}{m}v - \frac{Fc}{m} (2)$$
$$-\frac{F0}{m}sign(v)$$

j = 1, 2, 3 For the phases A, B, C

Where V and *i* designate voltage and current of phases A,B and C, x the displacement,  $\lambda$  the tooth pitch,  $L_1$  and  $L_2$  the minimum and the maximum inductance, v the speed, Fc the load force, m and  $\xi$  the mass and friction.

A finite element analysis (2D-FEA) of the motor is done and the results obtained in figure (2) of inductance as function of displacement for different current values shows that the inductance shape is characterized by one minimum value when the mover and stator are unaligned whatever the value of the current.



Fig. 2: Inductance as function of position for different current values.

$$L_0 = \frac{L_{\text{max}} + L_{\text{min}}}{2} = \frac{0.063 + 0.01}{2} = 0.0365H$$
 (3)

$$L_1 = \frac{L_{\text{max}} - L_{\text{min}}}{2} = \frac{0.063 - 0.01}{2} = 0.0265H \quad (4)$$

$$u = 18V \quad R = 9\Omega, m = 5Kg, \lambda = 10mm$$
$$\xi = 65Nm / s, F_0 = 0.2N$$

The LSRM is characterized by a non linear equation in his force and voltage equations. In order to simplify the model for control design dqo transformation is used to eliminate dependence variable. The novel model is based on *wye* winding connection and sinusoidal inductances expression



Fig. 3: dqo axis in LSRM

The phase voltages equations of the actuator are given by

$$v_{123} = Ri_{123} + \frac{d\Phi_{123}}{dt}$$

$$u_{123} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} i_{123} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \phi_{123} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix},$$
(5)

International Conference on Control, Engineering & Information Technology (CEIT'14) Proceedings - Copyright IPCO-2014, 345-351 ISSN 2356-5608

 $\phi_i = L(x)i_i$  is flux linkages in various phases

The actuator is characterized by magnetically coupled phases so the position depend inductance matrix L(x) is described by:

$$L(x) = \begin{pmatrix} L_s & -M_s & -M_s \\ -M_s & L_s & -M_s \\ -M_s & -M_s & L_s \end{pmatrix}$$
  
+
$$L_m \begin{pmatrix} \cos(\frac{2\pi}{\lambda}x) & \cos(\frac{2\pi}{\lambda}x + \frac{2\pi}{3}) & \cos(\frac{2\pi}{\lambda}x + \frac{4\pi}{3}) \\ \cos(\frac{2\pi}{\lambda}x + \frac{2\pi}{3}) & \cos(\frac{2\pi}{\lambda}x + \frac{4\pi}{3}) & \cos(\frac{2\pi}{\lambda}x) \\ \cos(\frac{2\pi}{\lambda}x + \frac{4\pi}{3}) & \cos(\frac{2\pi}{\lambda}x) & \cos(\frac{2\pi}{\lambda}x + \frac{2\pi}{3}) \end{pmatrix}$$
  
(6)

 $L_{s}$ : The average self inductance

 $L_m$ : The variation inductance due to air gap variation

 $M_{\rm s}$ : The average mutual inductance

Employing mathematical transformation the novel LSRM model can be written in dqo reference frame as follow,

$$u_{dqo} = Su_{123} \ i_{dqo} = Si_{123} \ \phi_{dqo} = S \ \phi_{123} \quad (7)$$

$$S(x) = \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\frac{\pi}{\lambda}x) & \cos(\frac{\pi}{\lambda}x + \frac{2\pi}{3}) & \cos(\frac{\pi}{\lambda}x - \frac{2\pi}{3}) \\ -\sin(\frac{\pi}{\lambda}x) & -\sin(\frac{\pi}{\lambda}x + \frac{2\pi}{3}) & -\sin(\frac{\pi}{\lambda}x - \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$
(8)

The model of the LSRM can be written by substituting (7) into (5) as follows:

$$U_{d} = Ri_{d} + L_{d} \frac{di_{d}}{dt} - \frac{\pi}{\lambda} L_{q}i_{q} \frac{dx}{dt}$$
(9)

$$U_{q} = Ri_{q} + L_{q} \frac{di_{q}}{dt} + \frac{\pi}{\lambda} L_{d}i_{d} \frac{dx}{dt}$$
(10)

$$U_{q} = Ri_{q} + L_{q} \frac{di_{q}}{dt} + \frac{\pi}{\lambda} L_{d}i_{d} \frac{dx}{dt}$$
(11)

$$U_o = Ri_o \tag{12}$$

$$F_{(i_d,i_q)} = \frac{\pi}{\lambda} (L_d - L_q) i_d i_q$$
(13)

To test the developed models and to verify the effectiveness of various applied controls, MATLAB/ SIMULINK was used as a simulation tool.

# A. Open loop control

The open-loop control have the merit of simplicity and consequent low cost, its consists to supplying the motor phases in a fixed order according to the direction, so there is no return and no regulation possible and there is no guarantee that the actuator has responded to the command.





Fig 5: Phases Currents



Figure (5) shows the simulation results of the LSRM in open loop control when the energizing sequence of the different phases is B,C then A because we assumed that initially the aligned teeth correspond to A phase teeth.

The phase current, when just one phase is supplied for 1 second in every step are shows in figure (15.a)

The mover displacement shown in figure (15.b) for one step proves that the motion is characterized by great over-shoot and strong oscillations. The oscillations are also observed in force in figure (15.c)

The force and displacement ripple are the major disadvantage of the LSRM open loop control so including this actuator into high precision application need a closed loop control by using controller and motion sensor.

#### III. CLOSED LOOP CONTROL

The purpose of this section is an elaboration of LSRM control strategy that can reduce the force ripple so facilitate his integration into a high precision application.

# A. Current command generation

When only one phase of the LSRM is excited at a time, the force generated by the actuator is characterized by a high force ripple. The solution is using function that can distribute the force on multiple phases. The idea is that the three phase of the motor contribute to produce the desired force. The multiphase excitation helps to reduce the peak of phase's currents also with minimum power dissipation. This part is an optimization of the three currents value that can provide the desired force and which minimizes the copper losses.

This nonlinear constrained optimization problem can be solved by introducing the Lagrangian which is presented as follow:

$$L(x,\lambda) = f(x) + K^T h(x)$$
(14)

Where K contains the Lagrange multipliers.

The first-order necessary conditions in this problem are:  $\nabla_x L(x, \lambda) = 0$  (15)

$$\nabla_{\lambda} L(x, \lambda) = 0 \tag{16}$$

Minimizing f(x) subject to h(x) = 0

As discussed in [4] the optimization problem is defined and solved just once in a position independent way.

$$f(i_d, i_q) = \frac{1}{2}(i_d^2 + i_q^2)$$
(17)

$$h(i_d, i_q) = \alpha i_d i_q - F^d \tag{18}$$

Where  $F^{d}$  is the desired force and  $\alpha = \frac{\pi}{\lambda}(L_{d} - L_{q})$ 

$$L = \frac{1}{2} (i_d^2 - i_q^2) + K (\alpha i_d i_q - F^d)$$
(19)

Hence the necessary conditions are:

$$i_d + K \alpha i_q = 0 \tag{20}$$

$$i_a + K \alpha i_d = 0 \tag{21}$$

$$\alpha i_d i_q - F^d = 0 \tag{22}$$

Solving equation (20), (21) and (22)

$$i_d = \sqrt{\frac{|F^d|}{\alpha}}$$
 And  $i_q = \sqrt{\frac{|F^d|}{\alpha}} sign(F^d)$ 

The current controls of the three phases are:

International Conference on Control, Engineering & Information Technology (CEIT'14) Proceedings - Copyright IPCO-2014, 345-351 ISSN 2356-5608

$$i_1^{d} = \sqrt{\frac{\left|F^{d}\right|}{\alpha}} \left(\cos(\frac{\pi x}{\lambda}) - \sin(\frac{\pi x}{\lambda}) * sign(F^{d})\right)$$
(23)

$$i_{2}^{d} = \sqrt{\frac{\left|F^{d}\right|}{\alpha}} \left( \cos(\frac{\pi x}{\lambda} + \frac{2\pi}{3}) - \sin(\frac{\pi x}{\lambda} + \frac{2\pi}{3}) * sign(F^{d}) \right) (24)$$

$$i_{3}^{d} = \sqrt{\frac{\left|F^{d}\right|}{\alpha}} \left( \cos(\frac{\pi x}{\lambda} + \frac{4\pi}{3}) - \sin(\frac{\pi x}{\lambda} + \frac{4\pi}{3}) * sgn(F^{d}) \right) (25)$$

$$\begin{bmatrix} i_{1}^{d} \\ i_{2}^{d} \\ i_{3}^{d} \end{bmatrix} = \sqrt{\frac{\left|F^{d}\right|}{\alpha}} \left( \cos(\frac{\pi}{\lambda}x) - \sin(\frac{\pi}{\lambda}x + \frac{2\pi}{3}) \\ \cos(\frac{\pi}{\lambda}x + \frac{2\pi}{3}) - \sin(\frac{\pi}{\lambda}x + \frac{2\pi}{3}) \\ \cos(\frac{\pi}{\lambda}x + \frac{4\pi}{3}) - \sin(\frac{\pi}{\lambda}x + \frac{4\pi}{3}) \\ \end{bmatrix} \begin{bmatrix} 1 \\ sgn(F^{d}) \end{bmatrix}$$

$$(26)$$

## B. Current controller

In this part we present the current controller using a proportional plus integral controller based on input output feedback linearization [6]. The voltage equation of a phase of the LSRM neglecting the mutual inductance is given by:

$$V_{j} = Ri_{j} + \frac{dL_{j}}{dt}i_{j} + \frac{di_{j}}{dt}L_{j}$$
<sup>(27)</sup>

$$\frac{di_j}{dt} = -\frac{R}{L_j}i_j - \frac{\frac{dL_j}{dx}}{L_j}\frac{dX}{dt}i_j + \frac{V_j}{L_j}$$
(28)

We assumed that:  $a = \frac{1}{L_A}$ ,  $b = \frac{R}{L_A}$ ,  $c = \frac{dL_A}{dx}$ 

$$\dot{i}_A = -bi_A - c x i_A + aV_A \tag{29}$$

The system (27) can be liberalized by defining the control

input 
$$V_j$$
 as,  $V_j = \frac{c x i_A}{a} + U_j$ 

Where  $U_j$  is the new control input  $i_j = -bi_j - aU_j$ 

The new control input  $U_i$  is given by a PI controller as,

$$U_{j} = K_{p} \left[ (i_{j}^{*} - i_{j}) + K_{i} \int (i_{j}^{*} - i_{j}) dt \right]$$

Where  $K_p$ ,  $K_i$  are the gain of PI controller



Fig. 8: Control block diagram of LSRM

The control block diagram shown in figure 6 illustrate the simplicity of the proposed control strategy where current command generation is determined by equation (25) and current control stroller is controlled by PI controller based on feedback linearization.





Figure 10: Force produced by the LSRM



Figure 11: Different Phases voltage

The proposed is tested using matlab/simulik. With force load equalize 5N the purpose is to produce a force around this value with minimum ripple.

Figure (7) shows the different current phases versus time and proves the efficiency of the proposed method where it's clear that the force is disturbed on multiple phases. As shown in figure (8) force ripples also decrease because phases motor contribute to produce the desired force, therefore it's clear that the generated force flow the load force (5N).

The obtained results are characterized by a smoothed force behavior without strong over-shoots.

Figure (15) shows the voltage supply of the three phase which that depends on the motion and the force developed by the motor.

The proposed command strategy in this work allows a flexibly force control of LSRM with minimizing force ripple thing that proves the efficiency of the proposed control strategy.

This control strategy can be used with other cascade control like velocity and position control into a height precision application.

## V. CONCLUSIONS

In this paper a new method of Linear Switched Reluctance Motor modeling is presented. Based on dqo transformation, the proposed method allowed us to simplify the model and eliminate dependence variable for control design. The novel indirect force controls using a feedback linearization with PI controller generate smooth force and produce minimal copper losses which maximize motor efficiency for any desired load force

## References

- T. R. Fredriksen, "Application of the Closed-Loop Stepping Motor," *IEEE Trans. on Automatic Control*, vol. AC-13, pp. 464– 474, Oct. 1968.
- [2] B. C. Kuo, "Closed-Loop and Speed Control of Step Motors," *The 3rdAnnual Symposium Incremental Motion Control Systems and Devices*, Urbana-Champaign, IL, May6–8, 1974.
- [3] Ghislain REMY, " Commande optimisée d'un actionneur linéaire synchrone pour un axe de positionnement rapide" Thèse de Docteur, Ecole doctorale n° 432 : Sciences des Métiers de l'Ingénieur,2007.
- [4] Dvid G.Taylor and Raga Ahmed. "Force Control of a linear Variable Reluctance Motor with Magnetically Coupled Phases". Proceedings of the Thirty-Fourth Southeastern Symposium on System Theory, 2002.
- [5] FAVRE.E, BRUNNER.D, PRAGET.C. "Principes et applications des moteurs linéaires", Revue d'automatisme, N°9. Mars 2000.
- [6] Han-Kyung Bae, Byeong-Seok Lee, Praveen Vijayraghavan and R. Krishnan. " Linear Switched Reluctance Motor: Converter and Control", IEEE transaction on industry application, vol 36,No.5 September/October 2000.
- [7] MAHMOUD Imed, "Design and modelling of a linear switched reluctance actuator for biomedical applications", International Journal of the Physical Sciences Vol. 6(22), pp. 5171-5180, 2 October, 2011.
- [8] ZAAFRANE Wajdi, MAHMOUD Imed, FATHALAH Mourad, and REHAOULIA Habib "control of linear switvched reluctance motor for biomedical application". ICESSA .2013.
- [9] Ben saad Kamel. Mhir dursun and Fatmagul koc " Linear switched reluctance motor control with PIC18F452 microcontrôleur". Turkich journal of electrical engineering and computer science Turk J El ec Eng & Com(2013) 21: 1107 – 1119.
- [10] Ahmed K ."A Fourier Series Generalized Geometry-Based Analytical Model of Switched Reluctance Machines". IEEE transactions on industry application. 43(3).2007
- [11] ACARNLEY P. "Stepping motors a guide to theory and practice", The Institution of Electrical Engineers, Edition IEE, London 2002.
- [12] P. Pillay, Y. Liu, W. Cai, and T. Sebastian, "Multiphase operation of Switched Reluctance Motor Drives," in Rec. of IEEE Industry Applications Society Annual Meeting, pp. 310-317, 1997.
- [13] Husain and M. Ehsani, "Torque Ripple minimization in Switched Reluctance Motor Drives by PWM Current Control," in IEEE Transactions on Power Electronics, vol. 11, no.1, pp. 83-88,1996.
- [14] H.K. Bae, B.S. Lee, P. Vijayranghavan, and R. Krishnan, "A Linear Switched Reluctance Motor: converter and control", in IEEE Trans. on Industry Application, vol.36, pp. 1351 – 1359, Sept./Oct. 2000
- [15] D.S. Schramm, B.W. Williams, and T.C. Green, "Torque ripple reduction of switched reluctance motors by phase current optimal profiling," in Proc. IEEE Annual Power Electronics Specialists Conference, 1992, pp. 857–860.