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Discrete-Time Robust Saturated Sliding Mode Control Applied to a Quarter-Vehicle MR Suspension System

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Abstract*— The saturation problem is the one of the most common handicaps for applying linear control to real applications, especially the actuator saturation. This paper proposes a new design approach of discrete time sliding mode control of linear systems in presence of saturation constraint. The saturation is reported on inputs vector. The design of the sliding surface is formulated as a pole assignment of linear saturated systems in a specific region through convex optimization. The solution to this problem is therefore numerically tractable via linear matrix inequalities (LMI) optimization, which leads to the development of a discrete and non-linear control law. Finally, the validity and the applicability of this approach are illustrated by a multivariable numerical example of a quarter of vehicle system to give simulation results.

***Key words**: Sliding Mode; Discrete-Time; Saturated Systems; Robustness; LMI.*

I. INTRODUCTION

Variable structure control (VSC) is a robust control strategy characterized by a sliding mode. It is obtained by designing a nonlinear control which drives the state trajectory from an arbitrary initial point into a pre-specified subspace, called the sliding surface, and thereafter the motion of the state trajectory remains within this subspace, such that stability of the system is assured.

Most industrial processes operate in the areas delineated by many physical and technological constraints (saturation, limit switches...). The implementation of the control law design without considering these limitations can have dire consequences for the system.

The recent development of the concept of sliding mode control (SMC) of saturated systems has led to linear continuous systems for constructing a robust controller which satisfies the constraints imposed on the control [1], [2]. However, in the recent years with the rapid development of computer technologies and DSP chips, it is imperative to realize a digital system controller by computer. Therefore, it is more significant to extend the design method of sliding mode control in continuous systems into the discrete-time control system. A primary reason is that most control strategies nowadays are implemented in discrete-time.

Research in discrete-time sliding mode control has been intensified in recent years; and many interesting techniques are available [3], [4], [5]. However, there is a lack of design approaches that consider the saturated systems. Nevertheless with the fast evolution of industrial technology, especially in the actuators, it is necessary to envisage methods of resolution for this problem.

Used in early days, let, quote of these methods, the anti-windup design [6], [7] and many other methods which introduce conditions on systems containing saturation functions [8], [9].

This paper is organized as follows: in first section, we give the form of the saturation structure reported on the control vector. Then, in the second section, we present a design procedure of robust saturated discrete-time-sliding mode control. This controller development procedure contains the classical steps of sliding mode design. The first one is to build an optimal sliding surface using the technique of pole placement in LMI region, and the second one is to choose a control law to enforce the system behavior to reach and stay in the desired sliding surface. Finally, we apply the proposed approach to a numerical example.

II. SYSTEM WITH SATURATION CONSTRAINT

Let us consider that the structure of the saturation constraint is described by figure 1:

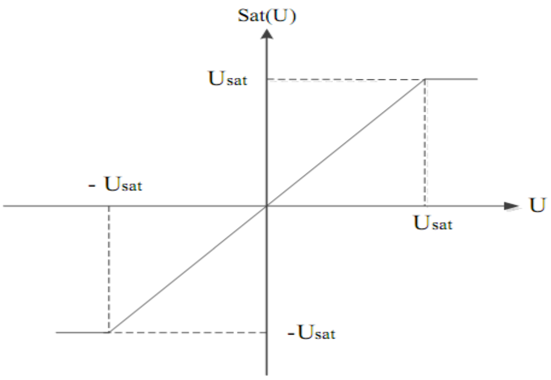

Figure1: The Structure of the saturation constraint

Assumption: The control vector is subjected to constant limitations in amplitude. It's defined by:

$$u \in \Re^m = \left\{ u \in \Re^m / -U_{sat} \le u(k) \le U_{sat} \ ; U_{sat} > 0 \right\} \quad (1)$$

For $0 < \psi < 1$ such as $sat\left(u\left(k\right)\right) = \Psi u(k)$, the term of saturation $sat\left(u\left(k\right)\right)$ and Ψ are given by, [10].

$$sat\left(u\left(k\right)\right) = \begin{cases} U_{sat} & if \quad U > U_{sat} \\ u(k) & if \quad -U_{sat} < U < U_{sat} \\ -U_{sat} & if \quad U < U_{sat} \end{cases} \quad (2)$$

With

$$\Psi = \begin{cases} \dfrac{U_{sat}}{u(k)} & if \quad U > U_{sat} \\ 1 & if \quad -U_{sat} < U < U_{sat} \\ -\dfrac{U_{sat}}{u(k)} & if \quad U < U_{sat} \end{cases} \quad (3)$$

The saturated system can be written as:

$$x(k+1) = \Phi x(k) + \Gamma \Psi u(k) \quad (4)$$

$\Phi \in \Re^{n \times n}, \Gamma \in \Re^{n \times m}, x \in \Re^n$ and $u \in \Re^m$

Assumption: The pair (Φ, Γ) is controllable, Γ has full rank m, and $n > m$.

III. SLIDING MODE CONTROL (SMC)

1. Invariance of the sliding mode:

Let us consider a discrete linear uncertain system described by:

$$x(k+1) = \left(\Phi + \Delta\Phi\right) x(k) + \left(\Gamma + \Delta\Gamma\right) u(k) \quad (5)$$

$\Delta\Phi$ and $\Delta\Gamma$ are the uncertainty of matching conditions type written as

$$\Delta\Phi = \Gamma\Delta\tilde{\Phi}, \ \Delta\Gamma = \Gamma\Delta\tilde{\Gamma} \quad (6)$$

The sliding mode condition can be represented by the following equation:

$$s(k) = s(k+1) = \ldots = s(k+i) \quad \forall k \ge k_g \quad (7)$$

k_g represent the instant when the sliding mode is reached.
With i=1, 2 , …
Also, we have:

$$Gx\left(k+1\right) = Gx\left(k\right) = 0 \quad (8)$$

G: matrix which defines the sliding surface.

It can be written as:

$$G\left(\Phi + \Gamma\Delta\tilde{\Phi}\right) x(k) + G\left(\Gamma + \Gamma\Delta\tilde{\Gamma}\right) u(k) = Gx\left(k\right) \quad (9)$$

If $\left(I + \Delta\tilde{\Gamma}\right)^{-1} \left(G\Gamma\right)^{-1}$ exists, them

$$u_{eq} = -\left(I + \Delta\tilde{\Gamma}\right)^{-1} \left(G\Gamma\right)^{-1} \left(\Phi + \Gamma\Delta\tilde{\Phi} - I\right) Gx_k \quad (10)$$

As a result

$$x\left(k+1\right) = \left(\Phi + \Gamma\Delta\tilde{\Phi}\right) x(k) - \Gamma\left(G\Gamma\right)^{-1} G\left(\Phi + \Gamma\Delta\tilde{\Phi} - I\right) x\left(k\right) \quad (11)$$

We finally get:

$$x\left(k+1\right) = \left(\Phi - \Gamma\left(G\Gamma\right)^{-1} G\left(\Phi - I\right)\right) x\left(k\right) \quad (12)$$

The dynamics x (k+1) (Eq 12), describes the motion on the sliding surface which is independent of $\Delta\Phi$ and $\Delta\Gamma$ and depends only on the choice of the matrix G.

The dynamics behavior of the system is totally invariant with respect to a subset of uncertainties called matched uncertainties, and the dynamics are completely defined by the choice of the sliding surface. However, this class of uncertainties has no effect on the ideal dynamics, as it acts only within channels implicit in the control input.

2. Design of the sliding surface:

In this section, we will prove the existence of the sliding mode. Indeed the canonical form used by for VSC design can be extended to saturated systems to select the gain matrix G.

Assumption: There exists an (nxn) orthogonal transformation matrix T such that $y(k+1) = Tx(k+1)$ and $T\Gamma = \begin{bmatrix} 0 \\ \Gamma_2 \end{bmatrix}$ where Γ has full rank m and Γ_2 is (m x m) and non-singular.

The transformed state equation

$$y(k+1) = T\Phi T^T y(k) + T\Gamma\Psi u(k) \tag{13}$$

Such as $y_k{}^T = \begin{bmatrix} y_{1k}^T & y_{2k}^T \end{bmatrix}$,

with $y_{1k} \in \Re^{n-m}$ and $y_{2k} \in \Re^m$, can be rewritten as:

$$\begin{cases} y_1(k+1) = \Phi_{11}y_1(k) + \Phi_{12}y_2(k) \\ y_2(k+1) = \Phi_{21}y_1(k) + \Phi_{22}y_2(k) + \Gamma_2\Psi u(k) \end{cases} \tag{14}$$

Since the sliding condition is $Gx(k) = GT^T y(k) = 0$, with:

$$\mathrm{T\Phi T}^T = \begin{bmatrix} \Phi_{11}\Phi_{12} \\ \Phi_{21}\Phi_{22} \end{bmatrix}, \ \mathrm{T\Gamma} = \begin{bmatrix} 0 \\ \Gamma_2 \end{bmatrix} \text{ and } \mathrm{GT}^T = \begin{bmatrix} G_1 & G_2 \end{bmatrix} \tag{15}$$

We can obtain the new defining sliding condition:

$$G_1 \ y_1(k) + G_2 \ y_2(k) = 0 \tag{16}$$

By assumption $G\Gamma$ is non-singular then G_2 must be non-singular.

The sliding mode condition becomes:

$$y_2(k) = -G_2^{-1}G_1 \ y_1(k) = -Fy_1(k) \tag{17}$$

With $F = G_2^{-1}G_1$, F being an [m x (n-m)] matrix.

The reduced system is (n-m) order. y_2 becomes a state feedback control.

The sliding mode is then governed by:

$$\begin{cases} y_1(k+1) = \Phi_{11}y_1(k) + \Phi_{12}y_2(k) \\ y_2(k) = -Fy_1(k) \end{cases} \tag{18}$$

The closed loop system will have the dynamics:

$$y_1(k+1) = (\Phi_{11} - \Phi_{12}F)\, y_1(k) \tag{19}$$

This indicates that the design of a stable sliding mode requires the selection of a matrix F such that $(\Phi_{11} - \Phi_{12}F)$ has (n-m) eigenvalues in the unit circle.

If the stabilizing matrix F has been determined, G is given by:

$$G = \begin{bmatrix} F & \mathrm{I}_m \end{bmatrix} T \tag{20}$$

3. Determination of the gain of the reduced system:

To determine the matrix G defining the sliding surface and the gain F, the method of the LMI seems to us very effective. Indeed to improve the performances of the control law and the response of system, we select to place the poles in a defined area [11], called area LMI. For that, we propose to choose all the eigenvalues of the matrix $(\Phi_{11} - \Phi_{12}F)$ in an area defined by a disc of center q and ray r in the unit circle.

Poles must be placed in a circle with center on the positive real axis in order to obtain a reasonably damped response (damping ratio $\gamma < 1$), [12].

The system $(\Phi_{11} - \Phi_{12}F)$ is asymptotically stable such as:

The eigenvalues of system $(\Phi_{11} - \Phi_{12}F)$ are all in area Ω circle of center (q, 0) and of ray r of the complex plan so if $\exists\ Q > 0$:

$$f_\Omega = \begin{pmatrix} -rQ & qQ + Q\Phi_{11}{}^T + L^T\Phi_{12}{}^T \\ qQ + \Phi_{11}Q + \Phi_{12}L & -rQ \end{pmatrix} < 0 \tag{21}$$

With L: optimal gain given by the solution of LMIs.

Then the gain is given by $F = LQ$

IV. SATURATED CONTROL LAW DESIGN

To reach the sliding surface and ensures that trajectories are directed towards the switching surface from any point in the state space, we select a saturated feedback nonlinear control function $u = u_L + u_N$, where u_L and u_N are the linear and nonlinear control law parts.

The general form is the following

$$u(k) = Kx(k) + \beta sig(s(k)) \tag{22}$$

Where K and β are appropriate matrix.

To determine the sliding mode control with state feedback, we proceed in the following way [13]:

$$s(k+1) = (1 - qT)\, s(k) - \varepsilon T \ sig(s(k)) \tag{23}$$

With $\varepsilon > 0$, $q > 0$ and $1 - qT > 0$

Theorem 1. *If the reaching law in Eq (23) is respected, and at any* $|s(k)| < \varepsilon T / 1 - qT$ *, then* $|s(k+1)| < |s(k)|$, [13].

Remark 1. *If the reaching law in Eq (23) is respected in the design of a suitable control law, the reachability condition of*

$|s(k+1)| < |s(k)|$ *can be satisfied under the condition that* $|s(k)| < \varepsilon T / 1 - qT$.

Remark 2. $|s(k)| < \varepsilon T / 1 - qT$ *Define a switching boundary in which the state trajectory will cross the ideal switching surface s(k)=0 at the next sampling instance.*

Referring to the control law defined [13]: for unsaturated systems, we developed a control law for saturated systems, by integrating the term of saturation Ψ this integration is given by the following equation:

$$G^T\Phi\, x(k) + G^T\Gamma\Psi u(k) - G^T x(k) = -qT\, s(k) - \varepsilon T\, sig(s(k)) \tag{24}$$

The resolution in u(k) gives the control law expressed by:

$$u(k) = -\left(G^T\Gamma\Psi\right)^{-1}\left[\left(G^T\Phi - G^T + qTG^T\right)x(k) + \varepsilon T\, sig(s(k))\right] \tag{25}$$

By identification one obtains:

$$K = -\left(G^T\Gamma\Psi\right)^{-1}\left[G^T\Phi - G^T + qTG^T\right] \tag{26}$$

And

$$\beta = -\left(G^T\Gamma\Psi\right)^{-1}\varepsilon T \tag{27}$$

V. Numerical application

In this study, a quarter-vehicle MR suspension system is established to evaluate the control performance of the manufactured MR damper. Fig. 2 shows the quarter-vehicle model of the semi-active MR suspension system, which has two degrees of freedom. Here, m1 and m2 represent the sprung mass and unsprung mass respectively. The spring for the suspension is assumed to be linear and the tire is also modeled as linear spring component and MR damper. Now, by considering the dynamic relationship, the state-space control model is expressed for the quarter-vehicle MR suspension system as follows:

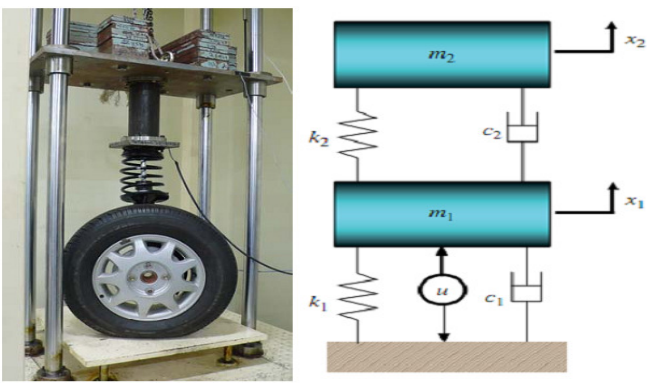

Figure 2: two degree of freedom vibrating system with one actuator

The state equation of the system is given by, [2]:

$$\dot{X}(t) = Ax(t) + Bu(t)$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1+k_2}{m_1} & \dfrac{k_2}{m_1} & -\dfrac{C_1+C_2}{m_1} & \dfrac{C_2}{m_1} \\ \dfrac{k_2}{m_2} & -\dfrac{k_2}{m_2} & \dfrac{C_2}{m_2} & -\dfrac{C_2}{m_2} \end{pmatrix},\ B \begin{pmatrix} 0 \\ 0 \\ \dfrac{1}{m_1} \\ 0 \end{pmatrix}$$

Simulation is achieved under the following condition:

$$m_1 = m_2 = 1kg,\ k_1 = k_2 = 1N/m$$

$$C_1 = C_2 = 0.01Ns/m,\ -1 \le u(k) \le 1$$

The initial condition is given by $x_0 = [0 \quad 0 \quad 0 \quad 1]'$

The discrete-time model is given as follows:

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$

With $\Phi = e^{AT}$, $\Gamma = \int_0^T e^{A(T-\tau)}Bd\tau$

In this section we have the results of simulations with a sampling period T=0.3, one obtains:

$$\Phi = \begin{bmatrix} 0.9119 & 0.0439 & 0.2902 & 0.0049 \\ 0.0439 & 0.9558 & 0.0049 & 0.2951 \\ -0.5756 & 0.2854 & 0.9061 & 0.0467 \\ 0.2854 & -0.2902 & 0.0467 & 0.9529 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.0442 \\ 0.0004 \\ 0.2902 \\ 0.0049 \end{bmatrix}$$

After 3 iterations, the algorithm gives the stabilizing gain F of the reduced discrete-time system:

F= [-3.5497 3.1844 -7.2307]

The figure 3 represents the poles of the reduced discrete-time system in an area defined by $\Omega(q,r)$ in the complex plan. Indeed, this area offers mainly a minimal damping of the poles and an absolute degree of stability minimum:

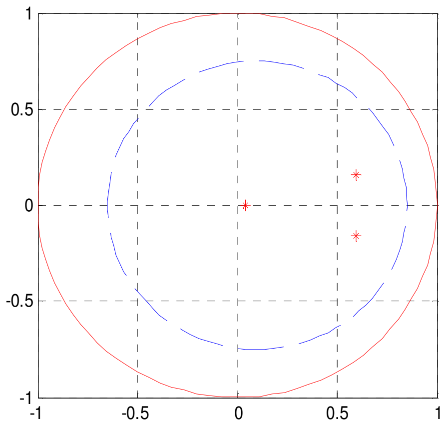

Figure 3: Poles of the reduced discrete system

The matrix G which definite the sliding surface is given by:

$$G = \begin{bmatrix} 3.1738 & 3.5543 & 0.4010 & 7.2908 \end{bmatrix}$$

The following results are obtained:

$$K = \begin{bmatrix} -2.2339 & 1.4510 & -1.3056 & -1.9059 \end{bmatrix}$$

$$\beta = -1.0217$$

Simulation enables us to obtain the results given in figure 4, figure 5 and figure 6 the switching surface, control input and state variables (system without saturation, system with saturation).

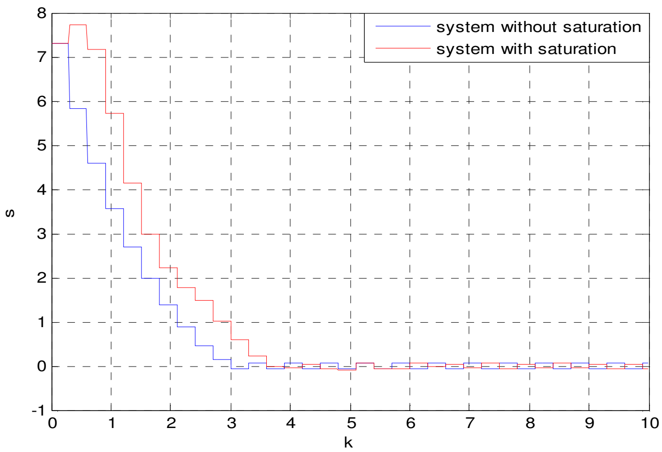

Figure4: Evolution of switching surface

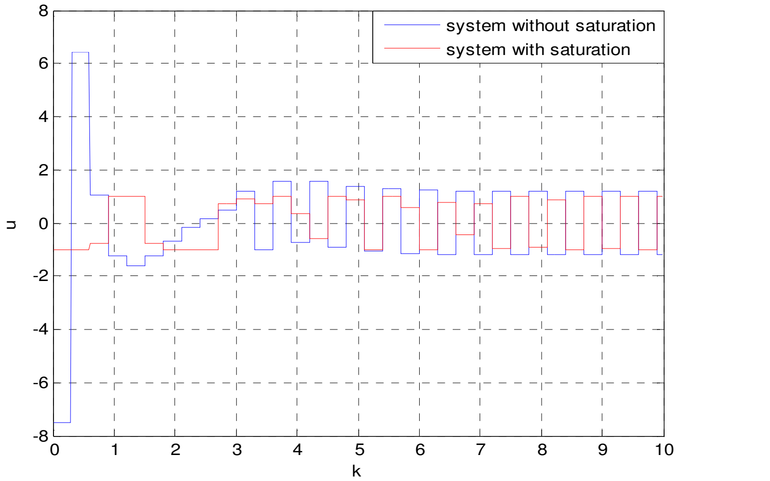

Figure5: Evolution of the control law

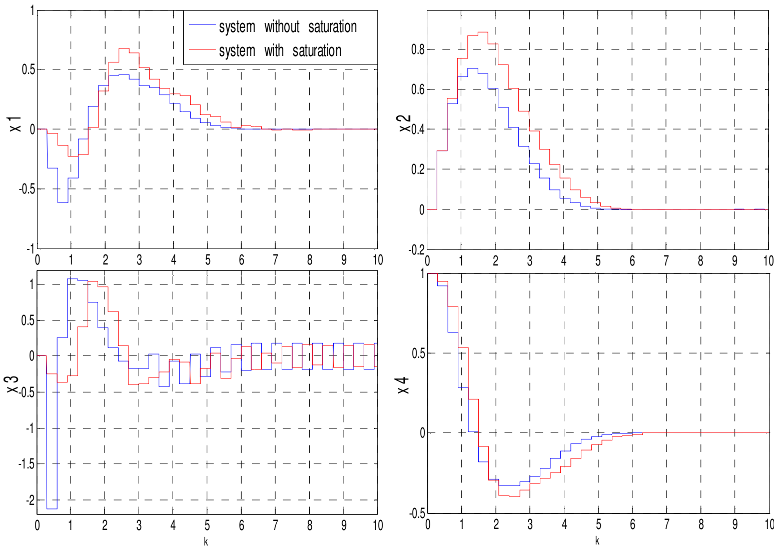

Figure6: Evolution of state variables

These figures show a typical stable sliding mode convergence of the system in the two cases. However, that the introduction of saturation level of the control law is slightly degrade system performance. As consequence, the state variables dynamics of the saturated system have a more slowly transient mode than that of the system without saturation. The control is saturated and always inferior to its maximal value, and able to reach S in a small time.

The purpose of the second simulations is to illustrate the performance of the proposed control scheme, and in particular the effect of the uncertainty.

To analyze the robustness of the control techniques to matching condition uncertainty, we repeated the same simulation in the previous case but we have introduced the uncertainty in damping coefficient.

Figure (7) and figure (8) present, respectively, the evolution of the state variables and control input of the ideal system ( ) uncertain system( ).

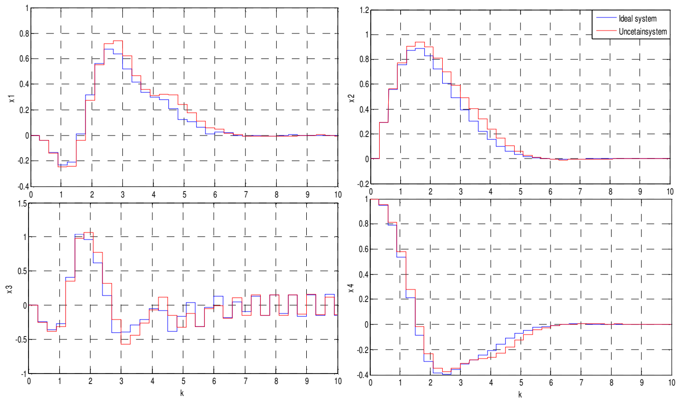

Figure7: Evolution of state variables

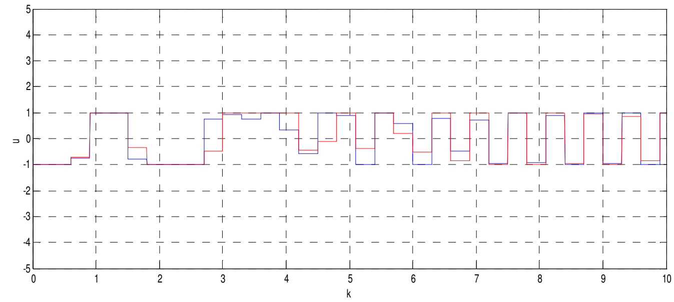

Figure8: Evolution of the control law

The positive effect of the proposed control structure on the behavior of the uncertain system can be seen from Figures 7 and 8. In fact the performance of the closed-loop system, in the presence of the uncertainty are almost similar is close to that of the ideal system. This confirms the robustness of the sliding mode control.

The dynamics behavior of the system is totally invariant with respect to a subset of uncertainties called matched uncertainties.

VI. CONCLUSION

In this paper, we presented a new design approach for discrete-time sliding mode of a class of linear time invariant saturated system. The control input is saturated and is being of constant limitations in amplitude. In the first step, we have exposed the design of the stable sliding surface by solving linear matrix inequalities by means of the LMI. In the second step, a non-linear control schema is introduced, which drives and maintain system state trajectories in to a switch band in limit time. Numerical simulations have been presented showing the applicability, the efficiency, and the robustness of the proposed method.

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