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Research of systems with a high potential for robust stability by Lyapunov function

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ABSTRACT

The proposed paper presents a method of constructing SISO control system with increased potential of robust stability. The article suggests new approach to building control systems for objects with uncertain parameters in the form of two-parameter structurally stable mappings of catastrophe theory to synthesize highly efficient control system, which has an extremely wide field of robust stability.

Research of robust stability of control systems based on the method of Lyapunov functions. We offer a new approach to constructing a Lyapunov function on based on a geometric interpretation of theorems A.M. Lyapunov, using the equation of state of the system, the concept of antigradient Lyapunov function and Morse theory. Satisfies the necessary and sufficient conditions for stability. The effectiveness of control systems with increased potential the robust stability is clearly illustrated by the example of construction of control systems in the form of two-parameter structurally stable mappings for process drying materials in the textile industry.

Keywords — control system, robust stability, increased potential of robust stability structurally-stable mappings, the stability region, catastrophe theory.

I. INTRODUCTION

A particularly important place in modern control theory is the problem of robust stability which requires guidance restrictions on modifying indefinite system parameters that preserve stability. Uncertainty can be resulting from ignorance of the true values of the parameters of control objects and unpredictable changes in their time in exploitation process. This is typical of most real objects of control, operating in terms of uncertainty. Robustness of the control systems set limits on the variation of their parameters. These limits are determined by the region of stability for uncertain parameters of the control system. Solving this problem for linear control systems with parametric uncertainty dedicated to in many papers [1,2,3,4]. They considered the problem of robust stability of polynomials with parametric uncertainty [1,2,4]. The problem of robust stability in the matrix of probability and uncertainty within the μ-analysis considered in [2,3].

There are currently no scientific principles to develop control systems with increased potential of robust stability and no universal methods to research their robust stability.

Many well-known processes of self-development model in the physical-chemical, biological and socio-economic systems [5] presented a mathematical model in the form of structurally stable maps [6,7,8]. It is therefore of particular interest in conditions of high uncertainty to build a control system in a class of structurally-stable maps with protection from the regime of deterministic chaos and strange attractors.

The concept of development control system with increased potential of robust stability of dynamic objects [9,10,11,12,13] is based on the achievements of the catastrophes theory, where were investigated the main structural-stable mappings.

Research of robust stability of control systems based on the new approach to the construction of Lyapunov's vector functions to the velocity vector, which is based on the geometric interpretation of A. M. Lyapunov theorem about asymptotic stability and a necessary and sufficient condition of stability of the system [14,15,16,17]. Robust stability conditions are obtained in the form of inequalities on the parameters of the control object and selectable parameters control device[18,19].

Let the control system described by the equation of state

$$\dot{x} = Ax + bu\,,\ \ y = cx\,,\ \ x \in R^n\,,\ \ y \in R\,. \tag{1}$$

Control law is given in the form of two-parameter structurally stable maps [2,5]:

$$u_i = -x_i^4 - k_{1i}x_i^2 + k_{2i}x_i\,,\ i = 1,...,n\,. \tag{2}$$

System (1) with (2) can be written in expanded form:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2, \\ \dfrac{dx_2}{dt} = x_3, \\ \dots \dots \dots \dots \dots \dots, \\ \dfrac{dx_n}{dt} = -x_1^4 + k_1^{'} x_1^2 + (k_1 - a_n)x_1 - \\ - x_2^4 + k_2^{'} x_2^2 + (k_2 - a_{n-1})x_2 - \dots - x_n^4 + \\ + k_n^{'} x_n^2 + (k_n - a_1)x_n. \end{cases}$$

$$y = x_1. \tag{3}$$

Find the steady state of the system

$$x_{1s} = x_{2s} = \dots = x_{ns} = 0\,. \tag{4}$$

Other stationary states of the system (3) will be determined by the solution of the equation

$$-x_{is}^3 - k_i^{'} x_{is} + (k_i - a_{n-i+1}) = 0,\ i = 1,\dots,n$$

$$x_{is}^2 = 2\sqrt[3]{\frac{a_{n-i+1} - k_i}{2}}\ , x_{js} = 0\ , i \neq j\ , i = 1,\dots,n\,. \tag{5}$$

$$x_{is}^{3,4} = \sqrt[3]{\frac{a_{n-1+i} - k_i}{2}}\ , x_{js} = 0\ ,\ i \neq j\ , i = 1,\dots,n\,. \tag{6}$$

II. Stability of the Stationary States of the System

To investigate the robust stability of the steady states (5) and (6) use the basic provisions of the direct Lyapunov method [20,21], for the asymptotic stability of the equilibrium state of the system if and only if there exists a positive definite Lyapunov function $V(x)$ such that its total time derivative $\dot{V}(x)$ along the solutions of the differential equation of state (3) is a negative definite function.

Here the total time derivative of the Lyapunov function, with the equation of state is defined geometrically as the scalar product of the gradient vector Lyapunov functions $(\partial V(x)/\partial x)$ for the velocity vector (dx/dt). The vector gradient of a scalar Lyapunov function is directed towards the growth of the largest, i.e., from the origin in the direction of the highest growth of Lyapunov functions. It should also be noted that in studies of stability of the system [20,21]. The origin corresponds to the given action or set the state of the system. State equations (1) or (3) are always in deviations Δx from the steady state $X_S\,(x = \Delta x = X - X_S)$. Therefore, the state equation (1) or (3) expresses the rate of change of a vector of deviations and we can assume that the velocity vector is directed deviations in a stable system to the origin.

If the Lyapunov function is specified as a vector function of $(V_1(x), V_2(x), \dots, V_n(x))$ a geometric interpretation of the components we choose antigradient Lyapunov functions $(-\partial V_i(x)/\partial x, i = 1,\dots,n)$ equal largest component of the velocity vector (dx/dt):

$$-\frac{dx_i}{dt} = \frac{\partial V_i^{(x)}}{\partial x_1} + \frac{\partial V_i(x)}{\partial x_2} + \dots + \frac{\partial V_i(x)}{\partial X_n},\ i = 1,\dots,n.$$

Then the total time derivative of the components of the vector Lyapunov function for the stability of the steady state (4) will be:

$$\begin{cases} \dfrac{dV_1(x)}{dt} = -x_2^2 \\ \dfrac{dV_1(x)}{dt} = -x_3^2 \\ \dots \quad \dots \\ \dfrac{dV_{n-1}(x)}{dt} = -x_n^2 \\ \dfrac{dV_n(x)}{dt} = \left[-x_1^4 + k_1^{'} x_1^2 + (k_1 - a_n)x_1 - x_2^4 + k_2^{'} x_2^2 + \right. \\ \left. + (k_2 - a_{n-1})x_2 -, \dots, - x_n^4 + k_n^{'} x_n^2 + (k_n - a_1)x_n \right]^2. \end{cases}$$

From this it follows that the total time derivative of the components of the vector Lyapunov function, there will always be sign-negative function.

Also, the total time derivative of the Lyapunov function can be represented as a scalar function of this we get

$$\begin{aligned} \frac{dV(x)}{dt} &= -x_2^2 - x_3^2 - \dots - x_n^2 - \\ &- \left[-x_2^4 + k_1^{'} x_1^2 + (k_1 - a_n)x_1 - x_2^4 + k_2^{'} x_2^2 + \right. \\ &\left. + (k_2 - a_{n-1})x_2 -, \dots, - x_n^4 + k_n^{'} x_n^2 + (k_n - a_1)x_n \right]^2. \end{aligned}$$

Here the components of the vector-function of Lyapunov $(V_i,\ i = 1,\dots,n)$ built in components of the gradient vector:

$$\frac{\partial V_1(x)}{\partial x_1} = 0,\ \frac{\partial V_1(x)}{\partial x_2} = -x_2,\ \frac{\partial V_1(x)}{\partial x_3} = 0,\dots,\ \frac{\partial V_1(x)}{\partial x_n} = 0$$

$$\frac{\partial V_2(x)}{\partial x_1} = 0,\ \frac{\partial V_2(x)}{\partial x_2} = 0,\ \frac{\partial V_2(x)}{\partial x_3} = -x_3,\dots,\ \frac{\partial V_2(x)}{\partial x_n} = 0$$

$$\dots \quad \dots \quad \dots \quad \dots$$

$$\frac{\partial V_{n-1}(x)}{\partial x_1} = 0,\ \frac{\partial V_{n-1}(x)}{\partial x_2} = 0,\ \frac{\partial V_{n-1}(x)}{\partial x_3} = 0,\ \frac{\partial V_{n-1}(x)}{\partial x_n} = -x_n$$

$$-\frac{\partial V_n(x)}{\partial x_1} = -x_1^4 + k_1^{'} x_1^2 + (k_1 - a_n)x_1,$$

$$-\frac{\partial V_n(x)}{\partial x_2} = -x_2^4 + k_2^{'} x_2^2 + (k_2 - a_{n-1})x_2,\dots,$$

$$-\frac{\partial V_n(x)}{\partial x_n} = -x_n^4 + k_n^{'} x_n^2 + (k_n - a_1)x_n.$$

Lyapunov function to obtain the scalar form as:

$$V(x)=\frac{1}{5}x_1^5-\frac{1}{3}k_1^{'}x_1^3-\frac{1}{2}(k_1-a_n)x_1^2+\frac{1}{5}x_2^5-$$
$$-\frac{1}{3}k_2^{'}x_2^3-\frac{1}{2}(k_2-a_{n-1}-1)x_2^2+,...,+\frac{1}{5}x_n^5-\frac{1}{3}k_n^{'}x_n^3-$$
$$-\frac{1}{2}(k_n-a_1-1)x_n^2=\frac{1}{5}x_1^5-\frac{1}{3}k_1^{'}x_1^3-\frac{1}{2}(k_1-a_n)x_1^2+$$
$$+\frac{1}{5}x_2^5-\frac{1}{3}k_2^{'}x_2^3-\frac{1}{2}(k_2-a_{n-1}-1)x_2^2+,...,+\frac{1}{5}x_n^5-$$
$$-\frac{1}{3}k_n^{'}x_n^3-\frac{1}{2}(k_n-a_1-1)x_n^2. \quad (7)$$

Positive or negative definite function $V(x)$ from (7) is not obvious, so make use of Morse theorem from catastrophe theory.

Let the system in question (3) is in a state of equilibrium (stable or sustainable) i.e. in the steady state, where the velocity vector $(dx/dt=0)$ is zero, i.e. respectively, the gradient of the Lyapunov function $\nabla V(x)=0$ is zero, the stationary states of the system, we have the Morse theorem det $\nabla V(x)=0$, and guarantees the existence of a smooth change of variables, such that the Lyapunov function (7) can be represented locally quadratic form. It follows that it is necessary to calculate the Hessian matrix for the stationary states (5).

Stability matrix (Hessian) [2,5] for the steady state (5) that as a

$$V_{ij}(x_S)=\left\|\left.\frac{\partial^2 V(x)}{\partial x_i \partial x_j}\right|_{x_{S=0}}\right\|=\begin{Vmatrix}\mu_1 & 0 & 0 & \dots & 0\\ 0 & \mu_2 & 0 & \dots & 0\\ 0 & 0 & \mu_3 & \dots & 0\\ \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & \dots & \mu_m\end{Vmatrix},$$

where

$$\frac{\partial^2 V\ (x)}{\partial x_i \partial x_j}=0$$

at $i\neq j,$

$$\mu_1=-(k_1-a_n)$$
$$\mu_2=-(k_2-a_{n-1}-1)$$
$$\mu_3=-(k_3-a_{n-2}-1)$$
$$\dots\dots\dots$$
$$\mu_n=-(k_n-a_1-1)$$

Lyapunov function (7) in the neighborhood of the steady state (4) can be represented as a quadratic form

$$V(x)=-(k_1-a_n)x_1^2-(k_2-a_{n-1}-1)x_2^2-$$
$$-(k_3-a_{n-2}-1)x_3^2-,\cdots,-(k_n-a_1-1)x_n^2, \quad (8)$$

Terms of robust stability of the steady state (5) defined by the system of inequalities:

$$a_n>k_1, a_{n-1}>k_2-1,\ a_{n-2}>k_3-1,...,\ a_1>k_n-1, \quad (9)$$

Stability of steady states (5) and (6) based on Lyapunov functions obtained by:

$$a_n<k_1, a_{n-1}<k_2-\frac{1}{221},$$
$$a_{n-2}<k_3-\frac{1}{221},...,\ a_n<k_n-\frac{1}{221}, \quad (10)$$

$$a_n<k_1, a_{n-1}<k_2-\frac{1}{20},$$
$$a_{n-2}<k_3-\frac{1}{20},...,\ a_1<k_n-\frac{1}{20}, \quad (11)$$

Stationary state (5) and (6) at the same time can't exist. Only one of them exists in the moment and it will always be stable. With the loss of this state, there is a new steady state, and it will be stable. This makes it possible to build a management system that will be stable When these conditional states (10) and (11).

III. The Robust Stability conditions for drying process of textile materials

Consider the process of drying of materials. Drying of materials can be described as a process of water evaporation from the material accelerated by the action of heated conductor (the agent of drying).

We can assume that the process of drying depends on the moisture accumulation in the surrounding steam and on the nature of the conductor, which serves as an acceleration link of first level [23].

Let the states of process control systems drying textile materials described by the equation:

$$\frac{dx}{dt}=Ax+Bu\ ,\ \ y=cx\ , \quad (12)$$

where

$$A=\begin{Vmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -T_1 & -T_0T_1\end{Vmatrix},\ b=\begin{Vmatrix}0\\ 0\\ k_0k_1\end{Vmatrix},$$

$$u=-x_1^4+k'x_1^2+k_px_1\ . \quad (13)$$

In expanded form the equation of state is written as

$$\begin{cases}\dfrac{dx_1}{dt}=x_2\\ \dfrac{dx_2}{dt}=x_3\\ \dfrac{dx_3}{dt}=-k_0k_1x_1^4+k_0k_1k'x_1^2+k_0k_1k_px_1-T_1x_2-T_0T_1x_3\end{cases}$$

Obtain the stationary state of the system

$$\begin{cases} x_{2S} = 0 \\ x_{3S} = 0 \\ -k_0 k_1 (x_1^4 - k' x_{1S}^2 - k_p x_{1S}) - T_1 x_{2S} - T_0 T_1 x_{3S} = 0 \end{cases} \quad . \qquad (14)$$

Find the stationary state of the system (14):

$$x_{1S} = 0, x_{2S} = 0, x_{3S} = 0. \qquad (15)$$

Other stationary states of the system (14) will be determined by solution of the equation

$$-x_{1S}^3 + k' x_{is} + k_p = 0. \qquad (16)$$

As is known from the theory of catastrophes [20,21,22], the solution of equation (16) corresponds to the critical points of catastrophe assembly given by the formula

$$f(x_{1S}, k', k_p) = -x_{is}^4 + k' x_{1S}^2 + k_p x_{1S} = 0. \qquad (17)$$

Critical, doubly degenerate critical and triply degenerate critical points catastrophe assembly (17) are determined by equating the respective first, second and third derivatives of expressions (17) to zero.

Condition (17) is performed at critical points

$$-4x_{1S}^3 + 2k' x_{1S} + k_p = 0, \qquad (18)$$

and

$$-12x_{1S}^2 + 2k' = 0, \qquad (19)$$

in double-degenerate critical points, and the conditions (8), (9)

$$-12x_{1S}^2 + 2k' = 0, \qquad (20)$$

in triply degenerate critical points [24,25].

Position in the parameter space (k', k_p) point that describes the function with three degenerate critical points defined as

$$(20) \Rightarrow x_{1S} = 0 \overset{(19)}{\Rightarrow} k' = 0 \overset{(18)}{\Rightarrow} k_p = 0. \qquad (21)$$

Corresponding function $f(x_{1S}, 0, 0) = x_{iS}^4$ has three degenerate points at the origin.

Points of the parameter space, which parameterize function with doubly degenerate critical points, are determined from the equations (19) and (18):

$$(19) \Rightarrow k' = 6x_{1S}^2 \overset{(18)}{\Rightarrow} k_p = 8x_{1S}^3. \qquad (22)$$

If the position of double-degenerate critical point denote x_{1S}, then formula (22) gives the values of parameters that describe the function of a doubly degenerate critical point x_{1S}.

Equations (22) define a parametric representation of the relationship for k' and k_p, which describe the function with doubly degenerate critical point x_{1S}.

Equations (22) define a parametric representation of the relationship between k' and k_p can be obtained if we exclude x_{1S} from (22):

$$\left(\frac{k'}{6}\right)^{1/2} = x_{1S} = \left(\frac{k_p}{8}\right)^{1/3},$$

or

$$\left(\frac{k'}{3}\right)^{1/2} = x_{1S} = \left(\frac{k_p}{2}\right)^{1/3},$$

$$\left(\frac{k'}{3}\right)^3 = \left(\frac{k_p}{2}\right)^2.$$

Hence parametric relationship for k' and k_p, will be determined by the equation

$$\left(\frac{k'}{3}\right)^3 - \left(\frac{a_{n-i+1} k_p}{2}\right)^2 = 0. \qquad (23)$$

As it known from elemenatry algebra, the cubic equation (16) can have up to three real solutions of te form

$$x_{1S}^2 = A + B,$$

$$x_{1S}^{3,4} = -\frac{A+B}{2} \pm j \frac{A-B}{2}\sqrt{3},$$

where

$$A = \sqrt[3]{\frac{-k_p}{2} + Q}, B = \sqrt[3]{\frac{-k_p}{2} - Q},$$

$$Q = \left(\frac{k'}{3}\right)^3 - \left(\frac{k_p}{2}\right)^2.$$

Hence considering (23), equation (16) has the solution:

$$x_{1S}^2 = 2\sqrt[3]{\frac{k_p}{2}},\ x_{2S} = 0,\ x_{3S} = 0. \qquad (24)$$

$$x_{1S}^{3,4} = \sqrt[3]{\frac{k_p}{2}},\ x_{2S} = 0,\ x_{3S} = 0.$$

From (23) we can find

$$\left(\frac{k'}{3}\right)^3 = \left(\frac{k_p}{2}\right)^2, \frac{k'}{3} = \left(\frac{k_p}{2}\right)^{2/3}.$$

Hence we obtain

$$k' = 3\left(\frac{k_p}{2}\right)^{2/3}.$$

To investigate the robust stability of the steady state (15) system (13) using a develop method based on the main provisions of Lyapunov function method [20,21], we find the vector components of the Lyapunov function

$$\frac{\partial V_1(x_1,x_2,x_3)}{\partial x_1} = 0,\ \frac{\partial V_1(x_1,x_2,x_3)}{\partial x_2} = -x_2,\ \frac{\partial V_1(x_1,x_2,x_3)}{\partial x_3} = 0,$$

$$\frac{\partial V_2(x_1,x_2,x_3)}{\partial x_1} = 0,\ \frac{\partial V_2(x_1,x_2,x_3)}{\partial x_2} = 0,\ \frac{\partial V_2(x_1,x_2,x_3)}{\partial x_3} = -x_3$$

$$\frac{\partial V_3(x_1,x_2,x_3)}{\partial x_1} = k_0k_1x_1^4 - k_0k_1k'x_1^2 - k_0k_1k_px_1,$$

$$\frac{\partial V_3(x_1,x_2,x_3)}{\partial x_2} = T_1x_2,\ \frac{\partial V_3(x_1,x_2,x_3)}{\partial x_3} = T_0T_1x_3.$$

Full derivatives by time from components of Lyapunov vector-function equal

$$\frac{dV_1(x)}{dt} = -x_2^2,\ \frac{dV_2(x)}{dt} = -x_3^2,$$

$$\frac{dV_3(x)}{dt} = -k_0^2k_1^2(x_1^4 - k'x_1^2 - k_px_1)^2 - T_1^2x_2^2 - T_0^2T_1^2x_3^2.$$

Or the full derivatives by time from scalar Lyapunov function can be written

$$\frac{dV(x)}{dt} = -k_0^2k_1^2(x_1^4 - k'x_1^2 - k_px_1)^2 - (T_1^2+1)x_2^2 - (T_0^2T_1^2+1)x_3^2. \qquad (26)$$

Total time derivatives from Lyapunov function is sign-negative function.

Components of Lyapunov functions can be obtained as:

$$V_1(x) = -\frac{1}{2}x_2^2,\ V_2(x) = -\frac{1}{2}x_3^2,$$

$$V_3(x) = \frac{1}{5}k_0k_1x_1^5 - \frac{1}{3}k_0k_1k'x_1^3 - \frac{1}{2}k_0k_1k_px_1^2 + \frac{1}{2}T_1x_2^2 + \frac{1}{2}T_0T_1x_3^2. \qquad (27)$$

Robust stability terms of the stationary state of (15), obtain by using negative definite function (26) of the conditions for positive definiteness of the function (27) and based on Morse theorem of catastrophe theory, function (27) represented as a quadratic form.

From (27) find the matrix of stability

$$V_{ij} = \left\|\frac{\partial^2 V(x)}{\partial x_i \partial x_j}\right|_0\Bigg\| = \begin{Vmatrix} -k_0k_1k_p & 0 & 0 \\ 0 & T_1 - 1 & 0 \\ 0 & 0 & T_0T_1 - 1 \end{Vmatrix},$$

it follows that the Lyapunov function (27) in the vicinity of state (15) can be represented as a quadratic form:

$$V(x) = -k_0k_1k_px_1^2 + (T_1-1)x_2^2 + (T_0T_1-1)x_3^2. \qquad (28)$$

Conditions for positive definiteness of the quadratic form, respectively, of the Lyapunov function (27) at point (15) is determined by the inequality

$$-k_0k_1k_p > 0,\ T_1 - 1 > 0,\ T_0T_1 - 1 > 0 \Rightarrow k_p < 0,$$
$$k_0 > 0,\ k_1 > 0,\ T_1 > 0,\ T_0 > 0. \qquad (29)$$

Zero steady state (15) is stable at negative values $k_p\,(k_p < 0)$ and loses stability at positive values $k_p\,(k_p > 0)$. Herewith time there are new stationary state at $k_p > 0$ (24) and (25) and we can find the conditions for the stability of steady states.

For this equation of state (14) represents the relative deviation $\Delta x = X - X_S = x$ from the stationary state $(x_{1S}^2, 0, 0)$ (24):

$$\left.\frac{\partial F_1}{\partial x_1}\right|_{x_S} = 0,\ \frac{\partial F_1}{\partial x_2} = 1,\ \frac{\partial F_1}{\partial x_3} = 0,\ \frac{\partial F_2}{\partial x_1} = 0,\ \frac{\partial F_2}{\partial x_2} = 0,\ \frac{\partial F_3}{\partial x_3} = 1,$$

$$\left.\frac{\partial F_3}{\partial x_1}\right|_{x_S} = -4k_0k_1x_1^3|_{x_S} + 2k'k_0k_1x_1|_{x_S} + k_0k_1k_p =$$

$$= \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \ldots\ldots\ldots\ldots \\ \dot{x}_3 = -k_0k_1\left(x_1^4 + 8\sqrt[3]{\frac{k_p}{2}}x_1^3 + 21\sqrt[3]{\left(\frac{k_p}{2}\right)^2}x_1^2 + \right. \\ \left. + 9k_pk_1\right) - T_1x_2 - T_0T_1x_3 \end{cases}. \qquad (30)$$

Total time derivative of Lyapunov vector-function $V(x)$ with the equation of state in the deviations (31) concerning the steady state (24) is determine

$$\frac{dV(x)}{dt} = -x_2^2 - x_3^2 - \left(k_0k_1x_1^4 + 8k_0k_1\sqrt[3]{\frac{k_p}{2}}x_1^3 + 21k_0k_1\sqrt[3]{\left(\frac{k_p}{2}\right)^2}x_1^2 + 9k_0k_1k_px_1 + T_1x_2 + T_0T_1x_3\right)^2. \qquad (31)$$

Function (31) is the sign-negative function.

Find the components of gradient for vector-function:

$$\frac{\partial V_1(x)}{\partial x_1}=0\,,\frac{\partial V_1(x)}{\partial x_2}=-x_2\,,\frac{\partial V_1(x)}{\partial x_3}=0\,,$$

$$\frac{\partial V_2(x)}{\partial x_1}=0\,,\ \frac{\partial V_2(x)}{\partial x_2}=0\,,\frac{\partial V_2(x)}{\partial x_3}=-x_3\,,$$

$$\frac{\partial V_3(x)}{\partial x_1}=k_0k_1x_1^4+8k_0k_1\sqrt[3]{\frac{k_p}{2}}x_1^3+$$

$$+21k_0k_1\sqrt[3]{\left(\frac{k_p}{2}\right)^2}\,x_1^2+9k_0k_1k_px_1\,,$$

$$\frac{\partial V_3(x)}{\partial x_2}=T_1x_2\,,\ \frac{\partial V_3(x)}{\partial x_3}=T_0T_1x_3\,.$$

Hence the Lyapunov function to obtain the scalar form

$$V(x)=\frac{1}{5}k_0k_1x_1^5+2k_0k_1\sqrt[3]{\frac{k_p}{2}}x_1^4+7k_0k_1\sqrt[3]{\left(\frac{k_p}{2}\right)^2}\,x_1^3+$$

$$+\frac{9}{2}k_0k_1k_px_1^2+\frac{1}{2}(T_1-1)x_2^2+\frac{1}{2}(T_0T_1-1)x_3^2\,. \qquad (32)$$

Using the Morse theorem, obtain a matrix in the form of stability:

$$V_{ij}=\left\|\left.\frac{\partial^2V(x)}{\partial x_i\partial x_j}\right|_0\right\|=\left\|\begin{matrix}9k_0k_1k_p & 0 & 0\\ 0 & T_1-1 & 0\\ 0 & 0 & T_0T_1-1\end{matrix}\right\|.$$

Hence we obtain Lyapunov function in the neighborhood of the stationary state (24) as a quadratic form

$$V(x)=9k_0k_1x_1^2+(T_1-1)x_2^2+(T_0T_1-1)x_3^2\ . \qquad (33)$$

Conditions for positive definiteness of the quadratic form

$$k_0k_1k_p>0,\ T_1-1>0, T_0T_1>0,\ k_p>0,$$
$$k_0>0,\ k_1>0,\ T_0>0,\ T_1>0. \qquad (34)$$

Let research the robust stability of steady state (25). Equation of state in the deviation in the form of

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=x_3\\ \ldots\ldots\ldots\ldots\\ \dot{x}_3=-k_0k_1(x_1^4+4\sqrt[3]{\frac{k_p}{2}}x_1^3+3\sqrt[3]{\left(\frac{k_p}{2}\right)^2}\,x_1^2+2k_px_1)-\\ -T_1x_2-T_0T_1x_3\end{cases}$$

Hence we can find full derivatives by time from Lyapunov vector-function

$$\frac{dV(x)}{dt}=-x_2^2-x_3^2-(k_0k_1x_1^4+4k_0k_1\sqrt[3]{\frac{k_p}{2}}x_1^3+$$

$$+3k_0k_1\sqrt[3]{\left(\frac{k_p}{2}\right)^2}\,x_1^2+2k_px_1+T_1x_2+T_0T_1x_3)^2\,. \qquad (35)$$

Function (35) is the sign-negative function.
Lyapunov function in a quadratic form represent as

$$V(x)=k_0k_1k_px_1^2+(T_1-1)x_2^2+(T_0T_1-1)x_3^2\ . \qquad (36)$$

Terms of robust stability condition:

$$k_0k_1k_p>0,\ T_1-1>0, T_0T_1-1>0,\ k_p>0,$$
$$k_0>0,\ k_1>0,\ T_0>0,\ T_1>0. \qquad (37)$$

IV. CONCLUSIONS

In this paper substantiated approach to building control systems with high potential for robust stability of objects with uncertain parameters with the choice of the control law in the two-parameter class of structurally stable maps. Displaying that the system has an asymptotically stable stationary state and in the negative and positive change in the field of uncertain parameters of the control object. When passing through zero uncertain parameters bifurcation occurs, and new stable branch. In this case zero steady state loses stability. These stationary states at the same time do not exist and there are opportunities to build a stable system with any change of uncertain parameters.

Using two-parameter structurally stable mappings for the construction of the drying process control shows that the control system is unstable for any value of uncertain parameter not only stabilized, but is not limited to changes in uncertain parameters of the drying process.

Proposed and substantiated a new approach to the construction of Lyapunov functions in vector-function form, which is a defined antigradient component of the velocity vector (the right part of the equation of state) of the system. Using the proposed universal approach to the construction of Lyapunov function allows you to get the region of stability of the stationary states of any dimension in the form of simple inequalities for uncertain parameters of the control object.

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