

Evaluation of the mechatronic systems reliability under parametric uncertainties

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Abstract—The aim of this paper is to evaluate the predicted reliability of mechatronic systems, by taking into account the epistemic uncertainties. The work reported here presents a new methodology based on integrating the belief functions in the Petri net (PN) model, in order to create a belief network, and to show how to propagate the parametric uncertainties in reliability models. Some notions of uncertainty related to systems reliability are presented; subsequently a brief definition of the belief function and its application in reliability studies are given and finally its integration in PN is detailed. In order to take into account the interactive aspect of mechatronic systems, we introduce the uncertainties associated to this interaction, by implementing the new method proposed by using belief network. Secondly, we study the propagation of these interaction uncertainties in system reliability. Finally, an industrial example of an "intelligent actuator" is developed, applying the proposed methodology.

Keywords— Reliability, predictive reliability, uncertainties, epistemic uncertainties, Belief Function, Petri Net (PN), mechatronics, interactions

Introduction

In reliability assessment studies of complex systems, the uncertainties represent an actual challenge, especially for mechatronics systems, basically the concept principle of the mechatronics manifests itself in exploiting to the maximum these multi domain couplings to offer a higher economic and technical performance; hence the complexity at all levels increases the risk of malfunction, the methods and the tools available to the designers which enable them to reliability are very various.

So the ultimate goal of any analysis is to predict reliability with a certain degree of confidence. Generally these uncertainties are divided in to two types: random uncertainties and epistemic uncertainties [1].

According to [2] and [3], the types of uncertainties are presented in Figure 1:

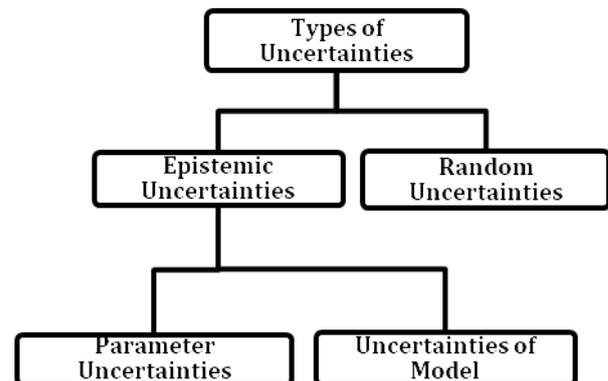


Figure 1: Types of uncertainties

The random uncertainties are characterized by variability, stochastic, and irreducible aspects, and random events [3], such as: the event initiator, failure at solicitation or failure in functioning, for which we associate Poisson models, binomial... The epistemic uncertainties result from imprecise, unobservable, cognitive and reducible knowledge, for example: imprecise failure rate, imprecise repair rate...

Furthermore, epistemic uncertainties can be categorized as either model uncertainties or parameter uncertainties. Model uncertainties are due to assumptions and simplifications related to the structure of the system, meaning an imprecision at the level of logical relations between the components, and representing the common failure modes.

Parameter uncertainties represent the lack of information for the input data of a reliability model; according to [3] they can be classified as follows:

- Components reliability data (failure rate, repair rate ...)
- Data related to the operating profile with determination of the status coefficients.

Many recent works ([4], [5], [6]) are treating the epistemic uncertainties for complex systems, by using belief function and Bayesian network.

Usually the reliability estimation of mechatronic system ([7],[8]) is done by qualitative and quantitative approaches. This study is based on a double qualitative analysis: functional and dysfunctional. The functional one allows obtaining an arborescent decomposition of the system by the method of Structured Analysis and Design Technique (S.A.D.T). Dysfunctional analysis is established using the Failure Mode and Effect Analysis (FMEA) method for failures. Both analyses will be used to build a PN model. An important thing to keep in mind is that mechatronic systems are dynamic, hybrid, interactive and reconfigurable systems [9]. We can notice the lack of studies on this topic, especially on evaluating the reliability of the multi domains interactions of mechatronic systems.

Therefore our contribution allows analyzing the Interactive aspect, as well as the propagation of the parameter uncertainties related to multi-domain interactions. Reliability is predicted by taking into account the overall uncertainty for each component of the system, and then computing the inference of the effects of those failures on the rest of the system.

In order to represent the multi-domain interactions, an organic analysis is implemented [10], and in our recent work [9] we introduced the multi domains interactions as influence factors associated to stress acceleration laws, such as Arrhenius law for temperature, or Cox law for several interactions. In the overview of the state-of-the art, we did not find a complete methodology to evaluate the uncertainties related to the mechatronic reliability together with the multi-domain interactions. In this paper we propose to assess these uncertainties by using the belief functions integrated to PN model.

We present a brief definition of the belief function, as well as of the belief mass, Belief function "BEL" and plausibility "PL", and then their implementation in reliability assessment. The choice of a suitable tool for modeling the uncertainties of complex systems has been well detailed in previous works ([1],[5]), belief functions are an effective tool and a unique framework for taking into account random and epistemic uncertainties concurrently.

I. THEORY OF BELIEF:

The Dempster-Shafer theory, also known as the theory of belief functions, is a generalization of the Bayesian theory of subjective probability. Whereas the Bayesian theory requires probabilities for each question of interest, belief functions allow basing degrees of belief for one question on probabilities for a related question [12]

A. Mass function

Let x be a discrete random variable taking values in the frame of discernment where all of its possible events are

mutually exclusive elementary propositions ($\Omega = \{x_1, x_2, \dots, x_n\}$). x can be also seen as a question and Ω as the set of possible answers to the question. Given a piece of evidence held by an agent, the state of belief about the actual value x_0 taken by x is represented by a Basic Belief Assignment (BBA). The BBA is defined as a mapping (also called a mass function $m(A)$) that assigns values to the elements of the power set 2Ω in the interval $[0, 1]$, such that:

$$\sum_{A \subseteq \Omega} m(A) = 1$$

$m(A)$ represents the part of belief assigned to the hypothesis that the truth lies in the subset A (i.e., the hypothesis $x_0 \in A$) without further dividing this belief to a strict subset of A .

There are two other important functions to represent knowledge: the belief function (bel) and the plausibility function (Pl). They are presented by the following expressions:

$$Bel(A) = \sum_{\phi \neq A \subseteq \Omega} m(B) \quad \forall A \subseteq \Omega$$

$$Pl(A) = \sum_{A \cap B \neq \phi} m(B) \quad \forall A \subseteq \Omega$$

$Bel(A)$ is the degree to which the evidence supports A , and $Pl(A)$ is the maximal degree of support that could be assigned to A if there were more available evidence. $Pl(A)$ may also be defined as the extent to which we fail to disbelieve the hypothesis of A .

II. EVIDENTIAL NETWORK WITH PETRI NETWORK:

Petri nets are considered as a powerful tool for modeling and predicting the reliability of a complex system. This choice is justified by the fact that this model is widely used in the modeling of dynamic hybrid systems [7]. In order to take into account the uncertainties, we use the belief functions, by implementing two functions BEL and PL, which have been associated with failure trees ([4], [13], [14]), and Bayesian networks. [5]

We propose a new method which consists in integrating uncertainties in the PN model. Each transition will be characterized by the interval of uncertainties $[\lambda_{\min}, \lambda_{\max}]$, considered as the minimum rate and the maximum rate, defined as variables Bel and PL, through the relations cited before.

To better understand the projection of the belief function, and the modeling the reliability of systems by using belief functions with serial configuration, let us consider a coherent system composed of two components [2] : $\mathbf{m} \Omega_1 \times \Omega_2 \times \Omega_3$ Série $\{(\mathbf{W1}, \mathbf{W2}, \mathbf{WS}), (\mathbf{W1}, \mathbf{F2}, \mathbf{FS}), (\mathbf{F1}, \mathbf{W2}, \mathbf{FS}), (\mathbf{F1}, \mathbf{F2}, \mathbf{FS})\} = 1$

In this approach, the Basic Probability Assignments (BPAs) of components are obtained by the two relations

$$W_i = \prod_i^n (1 - e^{-\lambda_{\max} \times t})$$

$$f_i = \prod_i^n e(-\lambda_{\max} \times t)$$

Thereafter, we affect the belief masses to each component:

$$\begin{aligned} m_{\Omega_i}(\{F_i\}) &= f_i \\ m_{\Omega_i}(\{w_i\}) &= w_i \\ m_{\Omega_i}(\{F_i, W_i\}) &= 1 - f_i - w_i \end{aligned}$$

Then the system reliability R_s is bounded by the interval: $[Bel(\{WS\}), Pl(\{WS\})]$.

It is then possible to write that (4): $RS \in [Bel(\{WS\}), Pl(\{WS\})]$ such as:

$$\begin{aligned} Bel(WS) &= \prod_i^n w_i \\ PL(WS) &= \prod_i^n (1 - f_i) \end{aligned}$$

Supposing the use of exponential law for the failure probabilities, we obtain:

$$\begin{aligned} Bel(WS) &= \prod_i^n w_i = \prod_i^n (1 - e(-\lambda_{\min} \times t)) \\ PL(WS) &= \prod_i^n (1 - f_i) = \prod_i^n (1 - e(-\lambda_{\max} \times t)) \end{aligned}$$

In order to create a Petri Network associated to Belief function, we associate to each failure transition in the PN model the interval $[\lambda_{\min}, \lambda_{\max}]$. The functions "Bel" and "PL" are computed using variables in PN model. This method will be used in the following application.

B. Application:

As an application example of this methodology, we choose the interaction coil/bearing as interaction between the electric domain and mechanical domain. An «intelligent actuator» [10], is a mechatronic system designed to carry out the unloading function of the wagons. Our objective is to evaluate the propagation of parameter uncertainty associated to this interaction, and its influence on the reliability of the whole system. The qualitative modeling (functional and dysfunctional) has been described in [10]:

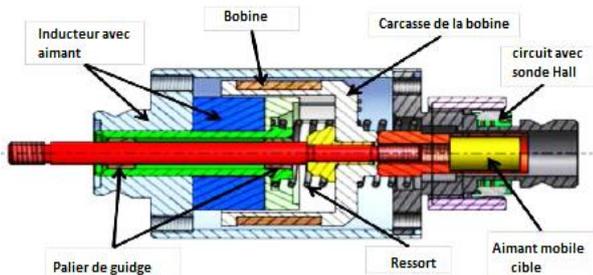


Figure 1. Intelligent actuator system cited in [10]

We considered a series configuration of the system, with the components: "the control and diagnostic card" of the actuator, the "conditioning card of the Hall effect probe",

"Coil", "inductor with magnet", and "guide bearing". The rest of the elements of the system are not taken into account.

In the PN model, we introduce the Probability of failure for each component as a transition between two places (working state to failure state), by affecting a probability law (exponential law to electronic field, Weibull law to mechanical field).

An imprecision concerning the failure rate of each component (six components) is given by the experts in the form of an interval $[\lambda_{\min}, \lambda_{\max}]$, with a confidence interval of 90% for failure time cited in [11]:

Component	failure rate min	failure rate max
Control Card	1.72E-07 1/h	2.22E-07 1/h
Packaging Card	1.16E-061/h	1.66E-061/h
Coil	5.10E-051/h	5.60E-051/h
Inductor	5.10E-051/h	5.60E-051/h
Guide Bearing	MTTF= 0.81E+03	MTTF= 1.3E+04
	Beta =1.5	Beta =1.5
Hall effet Hall	1.21E-071/h	1.7E-071/h

Table 1: Component failure rate

In order to represent the functions "BEL" and "PL", we use the previously presented equations of functions based on the table of failure rate.

The simulation of this model was done using the Petri Net V12 tool of the GRIF software suite and is presented in the following:

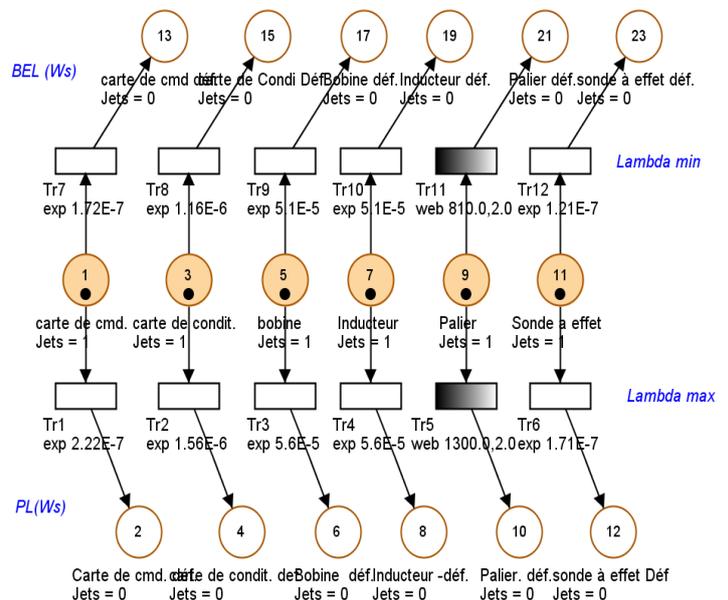


Figure 2. Belief Petri Network model under uncertainties

The results of this simulation are illustrated in Figure 3

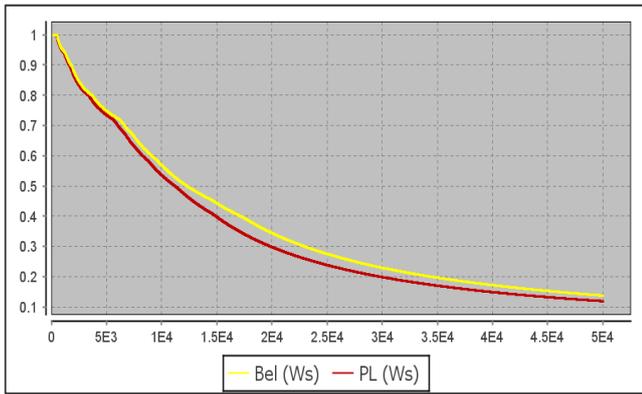


Figure 3. System reliability with taking into account the uncertainty

From a simulation up to $t = 50\,000$ hours, and by taking into account the parametric uncertainties, we find that the reliability of the intelligent actuator system is bounded by the credibility function Bel, and the plausibility PL. The reliability of the system is $R = [0.30, 0.35]$ for $t = 20000h$.

III. PROPAGATION OF THE UNCERTAINTIES OF MULTI-DOMAIN INTERACTIONS IN EVALUATING THE RELIABILITY.

As it was stated in the introduction, after modeling the multi-domain interaction in [9] as an influence factor, the purpose is to analyze the propagation of the parametric uncertainty related to this influence factor, (uncertainty of the failure rate ...) and its impact on the system reliability. The following figure represents the proposed approach:

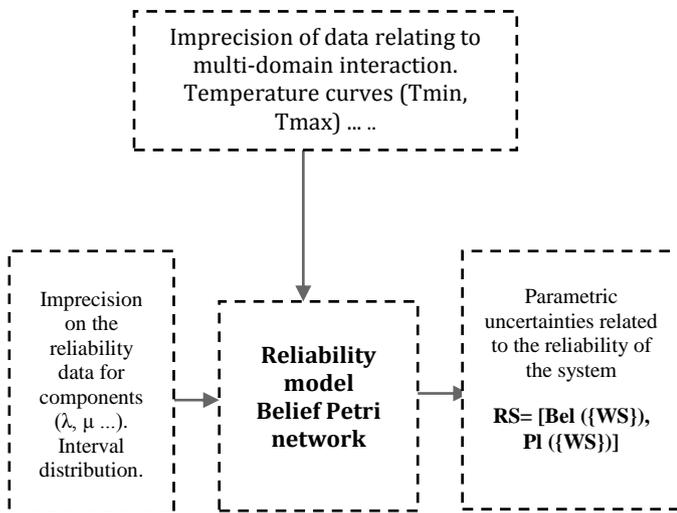


Figure 4. Reliability model Belief Petri network

A. Application:

In order to illustrate these notions let us consider the example of the mechatronic system "Intelligent actuator", for which the interaction taken into account is an interaction between a mechanical element "BEARING" and an electrical element "COIL", and for which the associated influence factor is the temperature influences on the operating state, by degradation of the bearing.

B. Temperature Uncertainty:

We consider the parametric uncertainty related to the temperature of "Bearing" in its degraded state, in the hot phase. The two curves: Minimum temperature "T-min" and Maximum temperature "T-max" are presented in Figure 5.

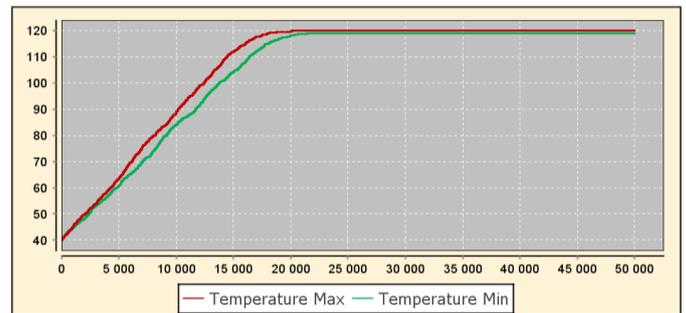


Figure 5. Temperature diagram of the coil

C. Uncertainty related to the coil failure rate:

The multi-domain interaction Bear /Coil has been treated in [9], as an influence factor described by the Arrhenius law.

1). Formulation of the Arrhenius Law

We consider the following model deterministically relating the lifetime of a component to its operating temperature:

$$v(T) = A_0 \cdot e^{-Ea / K * T}$$

$v(t)$: Component lifetime,

Boltzmann constant $k = 8,6171 \times 10^{-5} \text{ eV}/^\circ\text{C}$

Ea : activation energy parameter characterizing the kinetics of degradation electron volt (eV).

$A0$: constant associated with the component.

Af : The acceleration factor Af between two different temperatures T and T' :

$$A_f = T' / T * \exp[-E_a / K * (1/T - 1/T')]$$

By approximation [15] we obtain the relation failure rate which is used in the following:

$$\lambda(T, T_0, \lambda_0) \cong \lambda_0 \cdot e(-T_a / T)$$

D. Failure rates of the coil:

The relations of Arrhenius law indicate that failure rate is proportional to the parameters T-min, T-max. Therefore the λ_{\min} , λ_{\max} for the coil are defined as dynamic variables changed proportionally with these temperatures, including its parametric uncertainties.

$$\lambda \max(T, T_0, \lambda_0) \cong \lambda_{0 \max} \cdot e(-T_a / T \max)$$

$$\lambda \min(T, T_0, \lambda_0) \cong \lambda_{0 \min} \cdot e(-T_a / T \min)$$

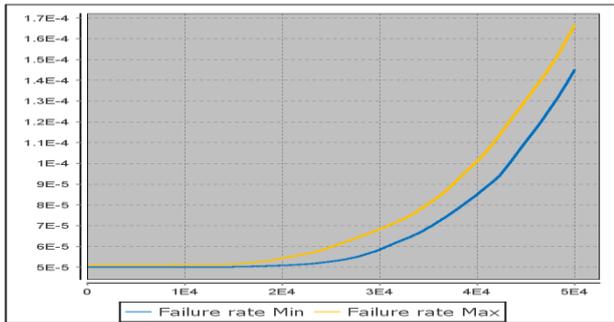


Figure 6. Evolution Failure rate of the coil

As presented in this figure, the failure rate (λ_{\min}) grows from the reference value of **5.1. 10⁻⁵/h** to **1.4510⁻⁵**, and (λ_{\max}) have a variation from the reference value to **1.7 10⁻⁴**. These two dynamic variables (failure rate) will be integrated in the PN model presented before in the transition corresponding to "Coil".

E. PN Model with interaction:

After representing the knowledge on the uncertainties related to influence factors (temperatures), and after observing its propagation on the coil failure rate, we adapt the PN model, considering failure transition associated to the coil as a dynamic transition related to the variable failure rate (λ_{\min} , λ_{\max}). Then we infer the influence of the uncertainty propagation of this multi-domain interaction and its effect on the reliability of the system. The sampling technique used here is to generate the boundaries for the reliability functions Bel and Plausibility

$$\text{Bel}(Ws) = \prod_i^n (1 - e(-\lambda_{\min} \times t))$$

$$\text{PL}(Ws) = \prod_i^n (1 - e(-\lambda_{\max} \times t))$$

Finally we simulate the PN model using the simulator of GRIF software with these new parameters. The results are represented in Figure 7.

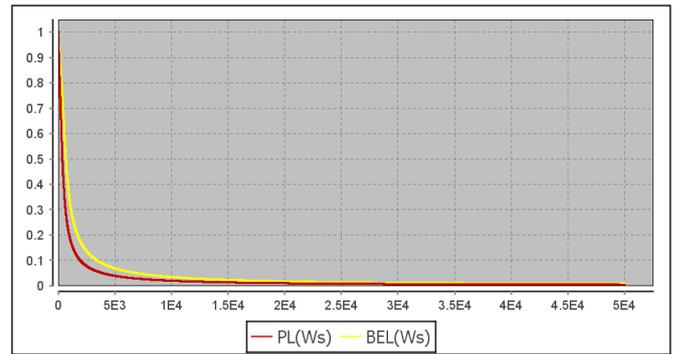


Figure 7. System reliability with taking into account of the uncertainty and of the interactive multi domain

The reliability of the "Intelligent Actuator" system, taking into account the bearing / coil interaction is bounded between the two curves "PL" and "Bel" approximately 7 months:

$$\text{BEL}(t=5000 \text{ hours}) = \mathbf{0,066}$$

$$\text{PL}(t=5000 \text{ hours}) = \mathbf{0,117}$$

F. Comparison (with/without Interaction)

Within this framework, it is worth noticing that the comparison of the reliability of the system "actuator" without/with interaction, was made in [9], but without making a representation of the parametric uncertainties. In this paper we present the Bel and PL functions, illustrated for both cases.

Therefore, we find from these curves in Figure 8:

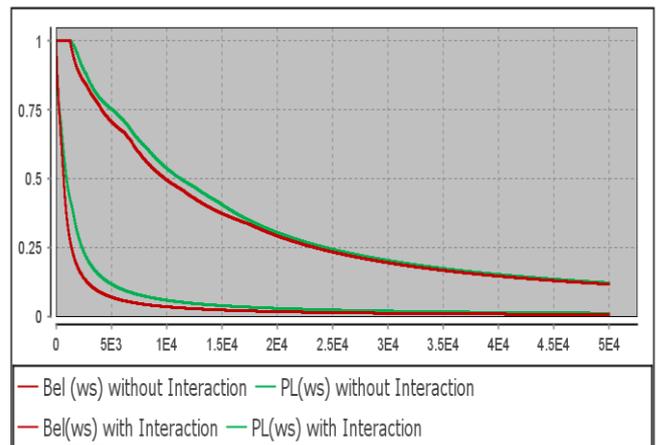


Figure 8. Comparison of system reliability curves with taking into account of the uncertainty for multi domain interactions

For $t = 5000$ hours (approximately 7 months) of operation:

$$\mathbf{R_{with\ interaction} = [0,066 - 0,117]}$$

$$\mathbf{R_{without\ interaction} = [0,703; 0,754]}$$

IV. CONCLUSIONS

Considering the results obtained, we can conclude that the integration of belief function in the PN model is an effective method for dealing with parametric uncertainties in assessing the reliability of mechatronic systems.

The proposed approach for taking into account the parametric uncertainty related to multi-domain interactions has been applied in the "Intelligent Actuator" system, through which we analyzed the propagation of this uncertainty to system reliability.

In the case of a mechatronic system with several multi-domain interactions, we can apply the same approach; by taking account the uncertainties related to each interaction, based on others stress acceleration laws, as for example Cox's law.

As a perspective, the random uncertainties represent the uncertainty of the logical relations between the components, especially for a mechatronic system as a reconfigurable system. Moreover, a more precise study of "common failure modes" and also of model dependencies between components seems to us an interesting way for futures works.

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