Parameters estimation of single-diode Photovoltaic module/array using least square estimator: A comparative study

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Abstract—In this paper, background on modelling PV module/array is established first; next, the combination of the optimization method least square estimator (LSE) algorithm and Newton Raphson resolution for identifying the five unknown parameters of single diode photovoltaic Module/Array is proposed. This predicted method is compared with other popular predictive and optimization methods such as Lambert solution, Villalva’s, modified Newton Raphson (NR), and nonlinear least square found in the literature. Here, two types of comparison are made: first, the dynamic variations of all five parameters values are carried out by graphs and compared in tables with the values found with the other methods (mentioned above); this evaluated the proposed method; secondly, the (I-V) and (P-V) curves are carried out at STC (Standard Test Conditions) and at1000W/m², 75°C, justifying the accuracy of the proposed method. The results proved the effectiveness of the least square estimator method, by accuracy parameters of the PV module/array. The accuracy of estimated parameters is sensitive to the initial parameters of trust region.

Keywords— Parameter estimation, least square estimator method, Newton Raphson resolution, single diode photovoltaic Module/Array.

1. Introduction
The world’s rapidly increasing energy demand has forced to focus on improving the efficiency of renewable energy resources. Improving the efficiency of these systems requires a precise modelling for performance evaluation. An accurate extraction and optimization of solar cells, modules and arrays parameters are very important in improving the device quality during fabrication and in device modelling. Precise parameters of mathematical model can play a key role in the simulation, performance evaluation, optimization, control and supervision of solar cells, modules and array systems. It is necessary to take into consideration the parameters identification with a feasible optimization method. The main drawback in accurate modelling is the lack of information about the precise values of the models parameters. In order to make a good agreement between experimental data and the models results, parameter identification with the help of an optimization technique is necessary. To extract the exact parameters of the PV model there are various methods, like analytical method, optimization method and iterative method. Among these methods, optimization methods are very effective to extract the parameters of PV model. Several methods for solar cell parameter extraction using the (I-V) characteristics have been proposed [1]. Some of them use (I-V) curve characteristics [2, 3]. The direct approaches are based on the use of the (I-V) curve features such as the axis intercepts and the gradients at selected points, to determine some of the cell, module or array parameters. The accuracy of these techniques is therefore limited by the accuracy of the measured data, the errors introduced by numerical differentiation and the simplified formulae used for parameter extraction. De Soto et al [4] and Boyd et al [5] used a specialized non-linear equation solver to get a solution for the single diode five parameters model. Despite having the advantage of simplicity, its drawback heavily depends on the selection of the initial value. Furthermore, a new analytical solution method based on Lambert W-function has been proposed to estimate parameters of solar cell model [6]-[7]-[8]-[9]. The Lambert solution method extracts successively the parameters. Villalva et al [10] explicitly defined one parameter, the diode ideality factor (n), and then solved the remaining parameters by minimizing the error in the maximum power prediction, extracting simultaneously two parameters Rp and Rs.

Townsend [11] simplified the model by assuming the shunt resistance to be infinite, which reduces the non-linearity of the system. He then solved the remaining parameters iteratively. Carrero et al [12] used an iterative procedure to find three parameters, series resistance (Rs), shunt resistance (Rp), and diode ideality factor (n), then two others, photo generated current (Iph) and saturation current (Io). The nonlinear least square curve fitting algorithm is used in [13]; the authors used at first the particle swarm optimization (PSO) method to enhance the search capability of the least square (LS) algorithm by considering the final PSO solution as an initial parameters vector to the least square (LS) algorithm; the photo
current $I_{ph}$ is assumed to be $I_{sc}$ at STC and the four other parameters are extracted by least square nonlinear curve fitting, using lsqcurvefit function of optimization toolbox in Matlab. Nonlinear least square method is also one of the recent optimization methods used for extracting all five unknown parameters [14]; Here, the authors use minimization of a single objective function (using lsqnonlin command of Matlab), which is the sum of multiple objective functions described on modified Newton-Raphson method [15]. Optimization methods for extraction of all parameters of PV module/array are generally different from algorithms and objective functions. Although good results were obtained through the above-mentioned methods, improving those approaches by using a practical and simplified objective function, and algorithm, can rapidly lead to desired solutions. This paper presents the least square estimator method for extraction of all five parameters of a PV module, which can be extended to PV array. The used method requires the Newton Raphson resolution of the model, the data found in datasheet as experimental data for (I-V) curve or (P-V). As precise parameters of mathematical model play a key role in the simulation or model prediction, comparison between experimental data (I-V) or (P-V) curves and predicted model is carried out for polycrystalline silicon photovoltaic MSX60 solar module at STC. The aim of this paper is to provide the reader with all necessary information to develop PV module/array models and to find all parameters in PV module/array with a good estimator algorithm like least square. The rest of the paper is organized as follows: Section 2 presents in detail the electrical model of PV cell/module/array and outlines the relationship between all parameters and environmental conditions; Section 3 presents the proposed identification approach; Section 4 discusses identification and experimental results found for polycrystalline silicon photovoltaic MSX60 solar module; finally, the last section 5 provides conclusions to the work.

2. Review of modeling a photovoltaic cell/module/array

To reflect the solar cell/module/array performance as well as that of the real system, it is essential to obtain an accurate parameters’ identification that presents the characteristics of the solar cell/module/array. An accurate mathematical model describing the electrical characteristics of solar cell/module/array is needed in advance. So far, several models have been introduced and proved successful in representing the behavior of the solar cell/module/array systems by considering many physical variables. Among them, two models are practically used, namely, the double (equation 1) and single (equation 2) diode models [16]-[17]-[18].

$$I_{v,i} = I_{ph} - I_{sc} \exp\left(\frac{V_{oc} + R_{s}I_{v,i}}{n_{S}V_{oc}}\right) - 1.$$  \hspace{1cm} (1)

$$I_{v,i} = I_{ph} \left[ \exp\left(\frac{V_{oc} + R_{s}I_{v,i}}{n_{S}V_{oc}}\right) - 1 \right] - \left(\frac{V_{oc} + R_{s}I_{v,i}}{R_{sh}}\right).$$  \hspace{1cm} (2)

As we can see in equation 1, there are seven unknown parameters to be estimated for such a solar cell model, namely, the photo generated current ($I_{ph}$); saturation currents ($I_{a1}$ and $I_{a2}$); series resistance ($R_{s}$); shunt resistance ($R_{sh}$); diode ideality factors ($n_1$ and $n_2$). Due to the simplicity and accuracy, the single diode model is also used widely to represent the solar cell behavior. The concept of this model is inspired by combining both diode currents, under the introduction of a nonphysical diode ideality factor $n$. In recent years, it has been validated that the single diode model can fit the experimental data successfully to some extent. The representation of this model can be formulated as shown in equation 2. In this model, there are five unknown parameters to be identified, namely, the photo generated current ($I_{ph}$); saturation current ($I_{a}$); series resistance ($R_{s}$); shunt resistance ($R_{sh}$) and diode ideality factor ($n$). The double diode model significantly improves the accuracy but at the expense of additional parameter calculation. On the contrary, the single diode model has five unknown parameters, so it is much more common to use. In fact, both double and single diode models require the knowledge of all unknown parameters, which is usually not provided by manufactures.

The single diode model is known to have a reasonable tradeoff between simplicity and accuracy under normal weather conditions. Therefore, this paper employs the single diode model to identify the parameters of PV module/array. This model offers a good compromise between simplicity and accuracy [19] and has been used by several authors in previous works, sometimes with simplifications but always with the basic structure composed of a current source and a parallel diode [20, 21]. Assuming that the cells and modules are perfectly matched and uniformly illuminated, PV array is conventionally assembled by connecting PV modules in series forming a PV string to obtain the desired voltage, and these are connected in parallel to obtain the current of the load. Figure 1 shows the electrical circuit of a PV module, which is the association of $n_s$ series cells to form a string and $n_p$ parallel strings of cells. The equivalent electrical circuit of PV array, with is constructed by $N_s$ series modules in each string and $N_p$ parallel strings, is shown in figure 2.

Figure 1: Module composed of $n_s$ series and $p$ parallel cells

Figure 2: Series parallel combination in PV array or plant
The manufacturers of solar modules provide electrical characteristics to users under the STC \((G_{stc} = 1000W/m^2, T_{stc} = 25°C)\) or at NOCT (the Nominal operating condition test) with \(G_{noct} = 800W/m^2, T_{noct} = 20°C\), as described in table 1 [22], which include short-circuit current \(I_{sc}\), open-circuit voltage \(V_{oc}\), the current of maximum power point \(I_{mp}\), the voltage of maximum power point \(V_{mp}\), experimental maximum output power \(P_{max,e}\), temperature coefficient \(\alpha_v\) of voltage, temperature coefficient \(\alpha_i\) of current, and number of cells matched in module \(N_s\). Certain manufacturers provide also (I-V) or (P-V) curves data at STC or NOCT or at any operating conditions. From this known information, various extraction methods are used to deduce parameters of PV module/array.

### Table 1: Electrical characteristics of PV module

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_{mp})</td>
<td></td>
</tr>
<tr>
<td>(V_{mp})</td>
<td></td>
</tr>
<tr>
<td>(P_{max,e})</td>
<td></td>
</tr>
<tr>
<td>(I_{sc})</td>
<td></td>
</tr>
<tr>
<td>(V_{oc})</td>
<td></td>
</tr>
<tr>
<td>(\alpha_v)</td>
<td></td>
</tr>
<tr>
<td>(\alpha_i)</td>
<td></td>
</tr>
<tr>
<td>(N_s)</td>
<td></td>
</tr>
</tbody>
</table>

#### 3. Proposed identification approach

##### 4.1 Five-Parameter PV model Formulation

The five-parameter model formulation is obtained starting from the single-diode PV circuit scheme as described by equations 2. Five parameters \((I_{ph}, I_o, n, R_s, R_p)\) have to be determined to solve equation 2 and hence to yield the electrical characteristic of the PV source. Extraction process is illustrated in figure 3.

#### 4.2 Least square estimator, Newton Raphson resolution and optimization algorithm

The prediction approach based on least square estimator, as a mathematical optimization technique, is chosen for identification process. It can make the sum of squared errors between the predicted data and the measured data, and minimize it. The least square method is sensitive to gross errors, considering the parameter estimates contain considerable errors [23]. The generalised formula is given by (3), as an objective function before making an optimization process. The optimum parameters \(x^* = x_{opt}\) are those that satisfy equation (4).

\[
f'(x_k) = (1/2) f(x_k) f(x_k)^T \\
\nabla f = \left[ \frac{\partial f}{\partial x_i} \right]_{x=x^*} \approx 0 \\
\n\nabla f\text{ represents the cost function gradient.}
\]

\[
\min f(x_k) = \frac{1}{2} f(x_k) f(x_k)^T \\
\text{subject to } \begin{cases} x - x_{min} \geq 0 \\
\text{xmax} - x \geq 0 \end{cases} \\
x = [I_{ph}, I_o, R_s, n, R_p] \\
g(l) = I_{ph} - l - I_o \exp(V_R + R_I/nV_o) - (V/nV_o) - (V + R_I/l_R) \\
I_{i+1} = I_i - g(I_i)/g^*(I_i) \\
i = 0,1,...,4 \\
g^* \text{ is the first derivation of } g
\]

The optimization problem could then be formulated as given in equation (5). \(x_{max}\) and \(x_{min}\) are the upper and the lower bounds of the parameter vector \(x\), respectively. For the present work, it will be minimized using the optimization algorithm \textit{fmincon command on Matlab software} [24]. \(\varepsilon(x_k) = I_p(k) - I_{max}\) is the residual or error in the current prediction at known voltages. The trust region algorithm is used to find out the unknown parameters of PV module. Details on implementing identification parameters using least square method are shown as flowchart algorithm in figure 4. A vector, described by equation (6), defines each solution. The \(I_p\) expression is a nonlinear equation; in this work, the Newton Raphson resolution algorithm (described in equations 7, 8) is used to solve it, with five iterations.
4. Identification results and discussions

In this section, experimental results of least square estimator are presented, applied to one module under the STC conditions and at 1000 W/m$^2$, 75°C to test the effectiveness. The module under investigation is polycrystalline silicon photovoltaic MSX60 solar array; datasheet and experimental data are found in [10]-[14] and [25]. The specified electrical characteristics at STC are showed in table 3. The variation value of all estimated PV parameters at STC are computed and tabulated after running fmincon algorithm in Matlab software. In addition, the final obtained values of PV parameters are compared with the parameters obtained by the other mentioned methods at STC. In order to test the validity of the model, the corresponding predicted (I-V) and (P-V) curves are carried out and plotted in common graph for comparison with the experimental data curves, and also with the curves obtained by the other mentioned methods.

Table 2: Electrical characteristics of PV module MSX60 at STC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{mp}$</td>
<td>3.5 A</td>
</tr>
<tr>
<td>$V_{mp}$</td>
<td>17.1 V</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>59.85 W</td>
</tr>
<tr>
<td>$I_{sc}$</td>
<td>3.8 A</td>
</tr>
<tr>
<td>$V_{oc}$</td>
<td>21.1 V</td>
</tr>
<tr>
<td>$I_{α}$</td>
<td>-0.080 A/V</td>
</tr>
<tr>
<td>$α_s$</td>
<td>0.0032 A/K</td>
</tr>
<tr>
<td>$N_e$</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 3: Extracted five parameters of MSX60 solar array at 1000 W/m$^2$ by LSE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower bound $x_0$:</td>
<td>Rs=0.2Ω, Rp=50Ω, n=1.2, Iph=3.5 (A), Io=10$^{-10}$ (A)</td>
</tr>
<tr>
<td>Upper bound $x_∞$:</td>
<td>Rs=0.3Ω, Rp=600Ω, n=1.5, Iph=4 (A), Io=10$^{-9}$ (A)</td>
</tr>
</tbody>
</table>

Table 4: Comparison of estimated five parameters of MSX60 at STC by the four other methods

<table>
<thead>
<tr>
<th>Parameters</th>
<th>n</th>
<th>Rs (Ω)</th>
<th>Rp (Ω)</th>
<th>Iph (A)</th>
<th>Io (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSE</td>
<td>1.0365</td>
<td>117.99</td>
<td>0.33</td>
<td>3.859</td>
<td>1.2654e-09</td>
</tr>
<tr>
<td>Modified NR</td>
<td>1.035</td>
<td>89.10</td>
<td>0.33</td>
<td>3.7926</td>
<td>9.7996e-10</td>
</tr>
<tr>
<td>Nonlinear LS</td>
<td>1.3000</td>
<td>99.05</td>
<td>0.33</td>
<td>3.8100</td>
<td>9.999e-10</td>
</tr>
<tr>
<td>Villalva’s method</td>
<td>1.0345</td>
<td>80.16</td>
<td>0.30</td>
<td>3.8165</td>
<td>9.360e-10</td>
</tr>
<tr>
<td>Lambert’s method</td>
<td>1.0345</td>
<td>81.52</td>
<td>0.31</td>
<td>3.8000</td>
<td>1.007e-09</td>
</tr>
</tbody>
</table>

The initial, lower, upper conditions and the final estimated five PV parameters by least square estimator are depicted in table 4. The dynamic evolution of these parameters, by iterations, are presented in figures 5a, 5b, 5c, 5d and 5e. As seen in these figures, the proposed method searches the optimal parameters and gets convergence to stable values up to 222 iterations. The saturation current is strongly dependent on the temperature noise; it is too small closed to zero and has not converged properly as shown in figure 5b. The final value of Io is the value obtained at final iteration that converge the four others parameters.

Table 5 carries out the comparison between the LS estimated parameters values and the same parameters found by Lambert solution, Villalva’s, modified Newton Raphson, and nonlinear least square methods. It has been observed that LSE can fail to find the optimal solution without a proper range and and initial parameters, particularly the case of saturation current.

![Figure 5: Convergence performance of Iph (a), Io (b), Rs (c), n (d), Rp (e) by LSE](image-url)
The obtained values are little different to the values obtained by the other mentioned methods. For the model validation process, the obtained parameters are utilized to carry out the (I-V) and (P-V) curves as seen in figure 10; the proposed method matches with a high precision the experimental data at STC.

In addition, the computed errors between the LSE and experimental data are also presented in figure 7, the mean error between experimental and predicted data is around zero. This result proves that the estimated model and experimental data are much closed. From figure 8, the errors from LSE is better closed to zero than means errors from others methods. This result proves that the proposed method and experimental data are much closed.
5. Conclusion
An efficient approach, the least square estimator algorithm method has been presented in this paper to estimate the single diode PV module/array parameters using the experimental data found in datasheet or collected by $(I-V-P)$ sensors during any operating conditions. The method is based on minimizing a simple objective function, constructed around the electrical current model of PV module/array. The estimation approach method is successfully applied on polycrystalline silicon photovoltaic MSX60 solar array. The advantages of the proposed method are simple and uncomplicated as a powerful tool to find quickly all five parameters of PV module/array. The obtained results for this PV module/array proved the validity and effectiveness of the proposed approach to find the all five parameters with good agreement between the experimental data and predicted curves. The dynamic evolution of all parameters are carried out and compared with other values (found by other popular methods) to show the capability of convergence of the proposed method after iterations. When compared to other popular methods like Villalva’s, Lambert’s, modified Newton Raphson and nonlinear least square methods, the proposed least square estimator method fits better the experimental data. However, the success of the least square method is affected by setting suitable initial parameters. It appears very suitable and easy to apply for the obtention of all five PV module parameters. The least square estimator algorithm can be efficiently applied to parameter identification of PV module/array model, which is useful for simulation, performance evaluation, optimization, control and supervision of the PV module/array system.

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Author Contributions
Albert Ayang is the main author of this work. This paper further elaborates on some of the results from his Ph.D. dissertation. René Wamkeue supervised and supported Albert Ayang’s scientific and technical expertise research. Noël Djongyang and Ndjakomo Essiane Salomé assisted in the results analysis and interpretation.

Competing Interests
The authors declare that there are no competing interests regarding the publication of this work.

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