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Control systems with increased potential of robust stability of nonlinear object in the class of two-parametric structurally stable mappings

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***Abstract* — This project outlines method of development of control system with increased potential of robust stability for spacecraft with nonlinear model in the class of two-parametric structurally stable mappings, where marginal increase of potential of robust stability was displayed. Such control is considered as one of the key factors under uncertain conditions which will ensure protection for the system from occurrence and development of deterministic chaos creating "strange attractors".**

Geometric interpretation of Lyapunov's theorem second method and definition of system stability allowed to present initial dynamic system in the form gradient system, whereas Lyapunov's function was presented in the form of potential function from catastrophe theory. Based on the above, universal approach is proposed for development and research of control system with increased potential of robust stability for spacecraft with nonlinear model.

Keywords— spacecraft; robust stability; Lyapunov's functions method; two-parametric structurally stable mappings.

I. INTRODUCTION

Modern development of automatic control theory is characterized by search of new methods and enhancement of available methods of analytical research of nonlinear dynamic control systems, functioning under some uncertainty of object's parameters and its characteristics drift within wide ranges. Currently robust stability is one of the most topical issues being of considerable practical interest [1].In general formulation, it presents modification limits of system parameters within which stability can be kept [2].

Researches of latest years also revealed that great diversity of nonlinear system dynamics led to one of the most important discovery of XX century in nonlinear systems – deterministic chaos [3]. Chaos is inner feature of any nonlinear deterministic dynamic system (object of control) [3,4,5,6].. Chaotic regimes may arise in many real nonlinear objects being sometimes harmful and useful in other cases, i.e. practically important classes of problem appeared when nonlinear objects should be controled by decreasing, eliminating or increasing the level of chaotic state [4,5,6]. It is to be noted that deterministic chaos appears in orientation and stabilization systems of spacecrafts as a result of loss of stability of existing stationary states, i.e. it is determined by indefinite parameters of the system going beyond robust stability boundaries. One of the approaches to deterministic chaos control may be extension of robustness range depending upon changes in indefinite parameters of the system, i.e. increase of potential for robust stability system [7,8,9]. Used concept for development of control system with increased potential of robust stability of spacecraft orientation and stabilization systems is new scientific trend in the sphere of control theory and is based on results of qualitative theory of dynamic system and catastrophe theory [10,11].

The present project considers research method and deterministic chaos control on an example of spacecraft orientation and stabilization systems. Research of spacecraft orientation and stabilization systems robust stability is conducted by universal new approach based on geometric interpretation of the second method of Lyapunov and definition of stability of dynamic systems [12,13,14,15]. At the same time dynamic system gradient nature, potential functions [10] – Lyapunov functions vector [16] were considered. In addition, main results reached by using by abovementioned methods were provided.

The organization of this paper is as follows. Section II presents the mathematical model and the control problem. Research of system stability in the stationary states is designed in section III. The section IV presents closing remarks and conclusions.

II. PROBLEM STATEMENT

Lets consider nonlinear system of spacecraft [17]:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = \dfrac{1}{I_x}\left(I_y - I_z\right)x_2 x_6 + \dfrac{1}{I_x}\left(-M_{xu} + M_{xf}\right) \\ \dfrac{dx_3}{dt} = x_4 \\ \dfrac{dx_4}{dt} = \dfrac{1}{I_y}\left(I_z - I_x\right)x_4 x_6 + \dfrac{1}{I_y}\left(-M_{yu} + M_{yf}\right) \\ \dfrac{dx_5}{dt} = x_6 \\ \dfrac{dx_6}{dt} = \dfrac{1}{I_z}\left(I_x - I_y\right)x_2 x_4 + \dfrac{1}{I_z}\left(-M_{zu} + M_{zf}\right) \end{cases} \quad (1)$$

where I_x, I_y, I_z - main central moments of spacecraft inertia relative to relevant axis; M_{xu}, M_{yu}, M_{zu} & M_{xf}, M_{yf}, M_{zf} - respectively projections of momentum control and disturbing moment relative to relevant axis.

Provided that control law is given as two-parametric structurally stable mappings [10, 11]

$$\begin{cases} -M_{xu}+M_{xf} = -x_1^4 - k_1^1 x_1^2 + k_1^2 x_1 - x_2^4 - k_2^1 x_2^2 + k_2^2 x_2 \\ -M_{yu}+M_{yf} = -x_3^4 - k_3^1 x_3^2 + k_3^2 x_3 - x_4^4 - k_4^1 x_4^2 + k_4^2 x_4 \\ -M_{zu}+M_{zf} = -x_5^4 - k_5^1 x_5^2 + k_5^2 x_5 - x_6^4 - k_6^1 x_6^2 + k_6^2 x_6 \end{cases} \quad (2)$$

Equations (1) can be outlined as follows:

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = a\left(\frac{1}{b}-\frac{1}{c}\right)x_2x_6 - ax_1^4 - ak_1^1x_1^2 + ak_1^2x_1 - \\ -ax_2^4 - ak_2^1x_2^2 + ak_2^2x_2 \\ \frac{dx_3}{dt} = x_4 \\ \frac{dx_4}{dt} = b\left(\frac{1}{c}-\frac{1}{a}\right)x_4x_6 - bx_3^4 - bk_3^1x_3^2 + bk_3^2x_3 - \\ -bx_4^4 - bk_4^1x_4^2 + bk_4^2x_4 \\ \frac{dx_5}{dt} = x_6 \\ \frac{dx_6}{dt} = c\left(\frac{1}{a}-\frac{1}{b}\right)x_2x_4 - cx_5^4 - ck_5^1x_5^2 + ck_5^2x_5 - \\ -cx_6^4 - ck_6^1x_6^2 + ck_6^2x_6 \end{cases} \quad (3)$$

where $a = \frac{1}{I_x}$, $b = \frac{1}{I_y}$, $c = \frac{1}{I_z}$.

Stationary states of the system can be found by equation system:

$$\begin{cases} x_{2S} = 0 \\ a\left(\frac{1}{b}-\frac{1}{c}\right)x_{2S}x_{6S} - ax_{1S}^4 - ak_1^1x_{1S}^2 + ak_1^2x_{1S} - ax_{2S}^4 - \\ -ak_2^1x_{2S}^2 + ak_2^2x_{2S} = 0 \\ x_{4S} = 0 \\ b\left(\frac{1}{c}-\frac{1}{a}\right)x_{4S}x_{6S} - bx_{3S}^4 - bk_3^1x_{3S}^2 + bk_3^2x_{3S} - bx_{4S}^4 - \\ -bk_4^1x_{4S}^2 + bk_4^2x_{4S} = 0 \\ x_{6S} = 0 \\ c\left(\frac{1}{a}-\frac{1}{b}\right)x_{2S}x_{4S} - cx_{5S}^4 - ck_5^1x_{5S}^2 + ck_5^2x_{5S} - cx_{6S}^4 - \\ -ck_6^1x_{6S}^2 + ck_6^2x_{6S} = 0 \end{cases} \quad (4)$$

By equation (4) we identify stationary state of the system (3) which is:

$$x_{1S}=0,\ x_{2S}=0,\ x_{3S}=0,\ x_{4S}=0,\ x_{5S}=0,\ x_{6S}=0 \quad (5)$$

And other stationary states are identified by equations

$$x_{is}^3 - k_i^1 x_{is} + k_i^2 = 0,\ i = 1,\ldots,6 \quad (6)$$

It is known from catastrophe theory [10], that equation completion (6) have the following solutions:

$$x_{iS}^2 = 2\sqrt[3]{\frac{k_i^2}{2}},\ x_{iS}^{3,4} = \sqrt[3]{\frac{k_i^2}{2}}\ \&\ k_i^1 = 3\left(\frac{k_i^2}{2}\right)^{\frac{2}{3}},\ i = 1,\ldots,6 \quad (7)$$

III. RESEARCH OF SYSTEM STABILITY IN THE STATIONARY STATES

A. When researching robust stability of stationary states (5) & (7) of the system (3) developed method will be applied [12, 13], using geometric interpretation of main clause of Lyapunov's method. Based on geometric interpretation of theorem on asymptotic stability all gradient vectors from Lyapunov's vector-functions shall be found and expansion of system speed vector (3) components by coordinates respectively:

$$\frac{\partial V_1(x)}{\partial x_1} = 0,\ \frac{\partial V_1(x)}{\partial x_2} = -x_2,\ \ldots,\ \frac{\partial V_1(x)}{\partial x_6} = 0$$

$$\frac{\partial V_2(x)}{\partial x_1} = ax_1^4 + ak_1^1x_1^2 - ak_1^2x_1,\ \frac{\partial V_2(x)}{\partial x_2} = ax_2^4 + ak_2^1x_2^2 - ak_2^2x_2,$$

$$\frac{\partial V_2(x)}{\partial x_3} = 0,\ \ldots,\ \frac{\partial V_2(x)}{\partial x_6} = -a\left(\frac{1}{b}-\frac{1}{c}\right)x_2x_6$$

$$\frac{\partial V_3(x)}{\partial x_1} = 0,\ \ldots,\ \frac{\partial V_3(x)}{\partial x_4} = -x_4,\ \frac{\partial V_3(x)}{\partial x_5} = 0,\ \frac{\partial V_3(x)}{\partial x_6} = 0$$

$$\frac{\partial V_4(x)}{\partial x_1} = 0,\ \frac{\partial V_4(x)}{\partial x_2} = 0,\ \frac{\partial V_4(x)}{\partial x_3} = bx_3^4 + bk_3^1x_3^2 - bk_3^2x_3,$$

$$\frac{\partial V_4(x)}{\partial x_4} = bx_4^4 + bk_4^1x_4^2 - bk_4^2x_4,\ \ldots,\ \frac{\partial V_4(x)}{\partial x_6} = -b\left(\frac{1}{c}-\frac{1}{a}\right)x_4x_6$$

$$\frac{\partial V_5(x)}{\partial x_1} = 0,\ \ldots,\ \frac{\partial V_5(x)}{\partial x_6} = -x_6.$$

$$\frac{\partial V_6(x)}{\partial x_1} = 0,\ \frac{\partial V_6(x)}{\partial x_2} = -\frac{1}{2}c\left(\frac{1}{a}-\frac{1}{b}\right)x_2x_4,\ \frac{\partial V_6(x)}{\partial x_3} = 0,$$

$$\frac{\partial V_6(x)}{\partial x_4} = -\frac{1}{2}c\left(\frac{1}{a}-\frac{1}{b}\right)x_2x_4,\ \frac{\partial V_6(x)}{\partial x_5} = cx_5^4 + ck_5^1x_5^2 - ck_5^2x_5,$$

$$\frac{\partial V_6(x)}{\partial x_6} = cx_6^4 + ck_6^1x_6^2 - ck_6^2x_6$$

Projection of speed vector on coordinate axis is presented as follows

$$\left(\frac{dx_1}{dt}\right)_{x_1} = 0,\ \left(\frac{dx_1}{dt}\right)_{x_2} = x_2,\ \ldots,\ \left(\frac{dx_1}{dt}\right)_{x_6} = 0$$

$$\left(\frac{dx_2}{dt}\right)_{x_1} = -ax_1^4 - ak_1^1x_1^2 + ak_1^2x_1,$$

$$\left(\frac{dx_2}{dt}\right)_{x_2} = -ax_2^4 - ak_2^1x_2^2 + ak_2^2x_2, \left(\frac{dx_2}{dt}\right)_{x_3} = 0, \dots,$$

$$\left(\frac{dx_2}{dt}\right)_{x_6} = a\left(\frac{1}{b} - \frac{1}{c}\right)x_2x_6$$

$$\left(\frac{dx_3}{dt}\right)_{x_1} = 0, \dots, \left(\frac{dx_3}{dt}\right)_{x_4} = x_4, \left(\frac{dx_3}{dt}\right)_{x_5} = 0, \left(\frac{dx_3}{dt}\right)_{x_6} = 0$$

$$\left(\frac{dx_4}{dt}\right)_{x_1} = 0, \left(\frac{dx_4}{dt}\right)_{x_2} = 0, \left(\frac{dx_4}{dt}\right)_{x_3} = -bx_3^4 - bk_3^1x_3^2 + bk_3^2x_3,$$

$$\left(\frac{dx_4}{dt}\right)_{x_4} = -bx_4^4 - bk_4^1x_4^2 + bk_4^2x_4, \left(\frac{dx_4}{dt}\right)_{x_5} = 0,$$

$$\left(\frac{dx_4}{dt}\right)_{x_6} = b\left(\frac{1}{c} - \frac{1}{a}\right)x_4x_6$$

$$\left(\frac{dx_5}{dt}\right)_{x_1} = 0, \dots, \left(\frac{dx_5}{dt}\right)_{x_6} = x_6$$

$$\left(\frac{dx_6}{dt}\right)_{x_1} = 0, \left(\frac{dx_6}{dt}\right)_{x_2} = \frac{1}{2}c\left(\frac{1}{a} - \frac{1}{b}\right)x_2x_4, \left(\frac{dx_6}{dt}\right)_{x_3} = 0,$$

$$\left(\frac{dx_6}{dt}\right)_{x_4} = \frac{1}{2}c\left(\frac{1}{a} - \frac{1}{b}\right)x_2x_4, \left(\frac{dx_6}{dt}\right)_{x_5} = -cx_5^4 - ck_5^1x_5^2 + ck_5^2x_5,$$

$$\left(\frac{dx_6}{dt}\right)_{x_6} = -cx_6^4 - ck_6^1x_6^2 + ck_6^2x_6,$$

Full derivative of time from Lyapunov's vector function may be presented as follows:

$$\begin{aligned} \frac{dV(x)}{dt} &= \frac{\partial V(x)}{\partial t}\frac{dx}{dt} = \sum_{i=1}^{6}\sum_{j=1}^{6}\frac{\partial V_i(x)}{\partial x_j}\left(\frac{dx_i}{dt}\right)_{x_j} = -\left(ax_1^4 + \right. \\ &\left. + ak_1^1x_1^2 - ak_1^2x_1\right)^2 - \left(ax_2^4 + ak_2^1x_2^2 - ak_2^2x_2\right)^2 - x_2^2 - \\ &- a^2\left(\frac{1}{b} - \frac{1}{c}\right)^2 x_2^2x_6^2 - \left(bx_3^4 + bk_3^1x_3^2 - bk_3^2x_3\right)^2 - \\ &\left(bx_4^4 + bk_4^1x_4^2 - - bk_4^2x_4\right)^2 - - b^2\left(\frac{1}{c} - \frac{1}{a}\right)^2 x_4^2x_6^2 - \\ &- x_6^2 - \frac{1}{4}c^2\left(\frac{1}{a} - \frac{1}{b}\right)^2 x_2^2x_4^2 - \frac{1}{4}c^2\left(\frac{1}{a} - \frac{1}{b}\right)^2 x_2^2x_4^2 - \\ &- \left(cx_5^4 + ck_5^1x_5^2 - ck_5^2x_5\right)^2 - \left(cx_6^4 + ck_6^1x_6^2 - ck_6^2x_6\right)^2 \end{aligned} \tag{8}$$

Full derivative of time from vector function (8) is definitely negative function.

By gradient of Lyapunov's vector function, components of Lyapunov's vector function shall be constructed.

$$V_1(x) = -\frac{1}{2}x_2^2, V_2(x) = \frac{1}{5}ax_1^5 + \frac{1}{3}ak_1^1x_1^3 - \frac{1}{2}ak_1^2x_1^2 +$$
$$+ \frac{1}{5}ax_2^5\frac{1}{3}ak_2^1x_2^3 - \frac{1}{2}ak_2^2x_2^2 - \frac{1}{2}a\left(\frac{1}{b} - \frac{1}{c}\right)x_2x_6^2$$

$$V_3(x) = -\frac{1}{2}x_4^2, V_4(x) = \frac{1}{5}bx_3^5 + \frac{1}{3}bk_3^1x_3^3 - \frac{1}{2}bk_3^2x_3^2 +$$
$$+ \frac{1}{5}bx_4^5\frac{1}{3}bk_4^1x_4^3 - \frac{1}{2}bk_4^2x_4^2 - \frac{1}{2}b\left(\frac{1}{c} - \frac{1}{a}\right)x_4x_6^2$$

$$V_5(x) = -\frac{1}{2}x_6^2, V_6(x) = \frac{1}{5}cx_5^5 + \frac{1}{3}ck_5^1x_5^3 - \frac{1}{2}ck_5^2x_5^2 + \frac{1}{5}cx_6^5 +$$
$$+ \frac{1}{3}ck_6^1x_6^3 - \frac{1}{2}ck_6^2x_6^2 - \frac{1}{4}c\left(\frac{1}{a} - \frac{1}{b}\right)x_2^2x_4 - \frac{1}{4}c\left(\frac{1}{a} - \frac{1}{b}\right)x_2x_4^2$$

Lyapunov's vector function in scalar form is presented as follows:

$$\begin{aligned} V(x) &= \frac{1}{5}ax_1^5 + \frac{1}{3}ak_1^1x_1^3 - \frac{1}{2}ak_1^2x_1^2 + \frac{1}{5}ax_2^5\frac{1}{3}ak_2^1x_2^3 - \\ &- \frac{1}{2}\left(ak_2^2 + 1\right)x_2^2 - \frac{1}{2}a\left(\frac{1}{b} - \frac{1}{c}\right)x_2x_6^2 + \frac{1}{5}bx_3^5 + \frac{1}{3}bk_3^1x_3^3 - \\ &- \frac{1}{2}bk_3^2x_3^2 + \frac{1}{5}bx_4^5 + \frac{1}{3}bk_4^1x_4^3 - \frac{1}{2}\left(bk_4^2 + 1\right)x_4^2 - \\ &- \frac{1}{2}b\left(\frac{1}{c} - \frac{1}{a}\right)x_4x_6^2 + \frac{1}{5}cx_5^5 + \frac{1}{3}ck_5^1x_5^3 - \frac{1}{2}ck_5^2x_5^2 + \\ &+ \frac{1}{5}cx_6^5 + \frac{1}{3}ck_6^1x_6^3 - \frac{1}{2}\left(ck_6^2 + 1\right)x_6^2 - \frac{1}{4}c\left(\frac{1}{a} - \frac{1}{b}\right)x_2^2x_4 - \\ &- \frac{1}{4}c\left(\frac{1}{a} - \frac{1}{b}\right)x_2x_4^2 \end{aligned} \tag{9}$$

Function (9) satisfied all conditions of Morse theorem from catastrophe theory, therefore function (9) may be replaced by quadratic form. Omitting time-consuming operations of function expansion (8) around stationary state (5) and identification of Hessian matrix elements, quadratic form is described as follows:

$$\begin{aligned} V(x) &\approx -\frac{1}{2}ak_1^2x_1^2 - \frac{1}{2}\left(ak_2^2 + 1\right)x_2^2 - \frac{1}{2}bk_3^2x_3^2 - \\ &- \frac{1}{2}\left(bk_4^2 + 1\right)x_4^2 - ck_5^2x_5^2 - \frac{1}{2}\left(ck_6^2 + 1\right)x_6^2 \end{aligned} \tag{10}$$

Condition on existing of positive Lyapunov's function can be identified by inequality:

$$\begin{aligned} &k_1^2 < 0, \ k_2^2 < -\frac{1}{2a}, \ k_3^2 < 0, \\ &k_4^2 < -\frac{1}{2b}, \ k_5^2 < 0, \ k_6^2 < -\frac{1}{2c} \end{aligned} \tag{11}$$

B. Lets research robust stability of stationary state (7). For this equation (3) lets consider in deviations relative to stationary state (7). Omitting time-consuming formal expansion operations and identification of derivatives in stationary point of different orders, equation of this state can be described in deviations as follows:

$$\begin{cases}\dfrac{dx_1}{dt}=x_2\\ \dfrac{dx_2}{dt}=-ax_1^4-4a\sqrt[3]{\dfrac{k_1^2}{2}}x_1^3-9a\sqrt[3]{\left(\dfrac{k_1^2}{2}\right)^2}x_1^2-4ak_1^2x_1-\\ -ax_2^4-4a\sqrt[3]{\dfrac{k_2^2}{2}}x_2^3-9a\sqrt[3]{\left(\dfrac{k_2^2}{2}\right)^2}x_2^2-\left[4ak_2^2-a\left(\dfrac{1}{b}-\dfrac{1}{c}\right)\times\right.\\ \left.\times\sqrt[3]{\dfrac{k_6^2}{2}}\right]x_2+a\left(\dfrac{1}{b}-\dfrac{1}{c}\right)\sqrt[3]{\dfrac{k_2^2}{2}}x_6\\ \dfrac{dx_3}{dt}=x_4\\ \dfrac{dx_4}{dt}=-bx_3^4-4b\sqrt[3]{\dfrac{k_3^2}{2}}x_3^3-9b\sqrt[3]{\left(\dfrac{k_3^2}{2}\right)^2}x_3^2-4bk_3^2x_3-\\ -bx_4^4-4b\sqrt[3]{\dfrac{k_4^2}{2}}x_4^3-9b\sqrt[3]{\left(\dfrac{k_4^2}{2}\right)^2}x_4^2-\left[4bk_4^2-b\left(\dfrac{1}{c}-\dfrac{1}{a}\right)\times\right.\\ \left.\times\sqrt[3]{\dfrac{k_6^2}{2}}\right]x_4+b\left(\dfrac{1}{c}-\dfrac{1}{a}\right)\sqrt[3]{\dfrac{k_4^2}{2}}x_6\\ \dfrac{dx_5}{dt}=x_6\\ \dfrac{dx_6}{dt}=-cx_5^4-4c\sqrt[3]{\dfrac{k_5^2}{2}}x_5^3-9c\sqrt[3]{\left(\dfrac{k_5^2}{2}\right)^2}x_5^2-4ck_5^2x_5+\\ +c\left(\dfrac{1}{a}-\dfrac{1}{b}\right)\sqrt[3]{\dfrac{k_4^2}{2}}x_2+c\left(\dfrac{1}{a}-\dfrac{1}{b}\right)\sqrt[3]{\dfrac{k_2^2}{2}}x_4-cx_6^4-\\ -4c\sqrt[3]{\dfrac{k_6^2}{2}}x_6^3-9c\sqrt[3]{\left(\dfrac{k_6^2}{2}\right)^2}x_6^2-4ck_6^2x_6\end{cases}\qquad(12)$$

Identify all components of gradient vector from vector-function and expand of speed vector components on coordinate axes based on a geometric interpretation of theorem on asymptotic stability [12, 13, 14].

Full derivative of time from Lyapunov's vector function from state equation (12) can be identified by:

$$\frac{dV(x)}{dt}=\frac{\partial V(x)}{\partial t}\frac{dx}{dt}=\sum_{i=1}^{6}\sum_{j=1}^{6}\frac{\partial V_i(x)}{\partial x_j}\left(\frac{dx_i}{dt}\right)_{x_j}=$$

$$-\left(ax_1^4+4a\sqrt[3]{\frac{k_1^2}{2}}x_1^3+9a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^2+4ak_1^2x_1\right)^2-$$

$$-\left\{ax_2^4+4a\sqrt[3]{\frac{k_2^2}{2}}x_2^3+9a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^2+\left[4ak_2^2-a\left(\frac{1}{b}-\frac{1}{c}\right)\times\right.\right.\qquad(13)$$

$$\left.\left.\times\sqrt[3]{\frac{k_6^2}{2}}\right]x_2\right\}^2-a^{2^2}\left(\frac{1}{b}-\frac{1}{c}\right)^2\sqrt[3]{\left(\frac{k_2^2}{2}\right)}x_6^2-\left(bx_3^4+4b\sqrt[3]{\frac{k_3^2}{2}}x_3^3+\right.$$

$$\left.+9b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^2+4bk_3^2x_3\right)^2-\left\{bx_4^4+4b\sqrt[3]{\frac{k_4^2}{2}}x_4^3+\right.$$

$$\left.+9b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^2+\left[4bk_4^2-b\left(\frac{1}{c}-\frac{1}{a}\right)\times\sqrt[3]{\frac{k_6^2}{2}}\right]x_4\right\}^2-$$

$$-b^2\left(\frac{1}{c}-\frac{1}{a}\right)^2\sqrt[3]{\frac{k_4^2}{2}}x_6^2-c^2\left(\frac{1}{a}-\frac{1}{b}\right)^2\sqrt[3]{\frac{k_4^2}{2}}x_2^2-c^2\left(\frac{1}{a}-\frac{1}{b}\right)^2\sqrt[3]{\frac{k_2^2}{2}}x_4^2-$$

$$-\left(cx_5^4+4c\sqrt[3]{\frac{k_5^2}{2}}x_5^3+9c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^2+4ck_5^2x_5\right)^2-$$

$$-\left(cx_6^4+4c\sqrt[3]{\frac{k_6^2}{2}}x_6^3+9c\sqrt[3]{\left(\frac{k_6^2}{2}\right)^2}x_6^2+4ck_6^2x_6\right)^2$$

Full derivative of time (13) from vector function is definitely is negative function.

By Lyapunov's vector function gradient, lets develop components of Lyapunov's vector function.

$$V_1(x)=-\frac{1}{2}x_2^2$$

$$V_2(x)=\frac{1}{5}ax_1^5+a\sqrt[3]{\frac{k_1^2}{2}}x_1^4+3a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^3+2ak_1^2x_1^2+\frac{1}{5}ax_2^5+a\sqrt[3]{\frac{k_2^2}{2}}x_2^4+$$

$$+3a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^3+\frac{1}{2}\left[4ak_2^2-a\left(\frac{1}{b}-\frac{1}{c}\right)\sqrt[3]{\frac{k_6^2}{2}}\right]x_2^2-\frac{1}{2}a\left(\frac{1}{b}-\frac{1}{c}\right)\sqrt[3]{\frac{k_2^2}{2}}x_6^2$$

$$V_3(x)=-\frac{1}{2}x_4^2$$

$$V_4(x)=\frac{1}{5}bx_3^5+b\sqrt[3]{\frac{k_3^2}{2}}x_3^4+3b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^3+2bk_3^2x_3^2+\frac{1}{5}bx_4^5+b\sqrt[3]{\frac{k_4^2}{2}}x_4^4+$$

$$+3b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^3+\frac{1}{2}\left[4bk_4^2-b\left(\frac{1}{c}-\frac{1}{a}\right)\sqrt[3]{\frac{k_6^2}{2}}\right]x_4^2-\frac{1}{2}b\left(\frac{1}{c}-\frac{1}{a}\right)\sqrt[3]{\frac{k_4^2}{2}}x_6^2$$

$$V_5(x)=-\frac{1}{2}x_6^2$$

$$V_6(x)=-\frac{1}{2}c\left(\frac{1}{a}-\frac{1}{b}\right)\sqrt[3]{\frac{k_4^2}{2}}x_2^2-\frac{1}{2}c\left(\frac{1}{a}-\frac{1}{b}\right)\sqrt[3]{\frac{k_2^2}{2}}x_4^2+\frac{1}{5}cx_5^5+c\sqrt[3]{\frac{k_5^2}{2}}x_5^4+$$

$$+3c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^3+2ck_5^2x_5^2+\frac{1}{5}cx_6^5+c\sqrt[3]{\frac{k_6^2}{2}}x_6^4+3c\sqrt[3]{\left(\frac{k_6^2}{2}\right)^2}x_6^3+2ck_6^2x_6^2$$

Lets present Lyapunov's vector function in a scalar form:

$$V(x)=\frac{1}{5}ax_1^5+a\sqrt[3]{\frac{k_1^2}{2}}x_1^4+3a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^3+2ak_1^2x_1^2+\frac{1}{5}ax_2^5+a\sqrt[3]{\frac{k_2^2}{2}}x_2^4+$$

$$+3a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^3+\frac{1}{2}\left[4ak_2^2-a\left(\frac{1}{b}-\frac{1}{c}\right)\sqrt[3]{\frac{k_6^2}{2}}-1-c\left(\frac{1}{a}-\frac{1}{b}\right)\sqrt[3]{\frac{k_4^2}{2}}\right]x_2^2+\qquad(14)$$

$$+\frac{1}{5}bx_3^5+b\sqrt[3]{\frac{k_3^2}{2}}x_3^4+3b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^3+2bk_3^2x_3^2+\frac{1}{5}bx_4^5+b\sqrt[3]{\frac{k_4^2}{2}}x_4^4+$$

$$+3b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^3+\frac{1}{2}\left[4bk_4^2-b\left(\frac{1}{c}-\frac{1}{a}\right)\sqrt[3]{\frac{k_6^2}{2}}-c\left(\frac{1}{a}-\frac{1}{b}\right)\sqrt[3]{\frac{k_2^2}{2}}-1\right]x_4^2+$$

$$+\frac{1}{5}cx_5^5+c\sqrt[3]{\frac{k_5^2}{2}}x_5^4+3c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^3+2ck_5^2x_5^2+\frac{1}{5}cx_6^5+c\sqrt[3]{\frac{k_6^2}{2}}x_6^4+$$

$$+3c\sqrt[3]{\left(\frac{k_6^2}{2}\right)^2}x_6^3+\frac{1}{2}\left[4ck_6^2-a\left(\frac{1}{b}-\frac{1}{c}\right)\sqrt[3]{\frac{k_2^2}{2}}-b\left(\frac{1}{c}-\frac{1}{a}\right)\sqrt[3]{\frac{k_4^2}{2}}-1\right]x_6^2$$

Function (14) satisfies all conditions of Morse theorem [10,11], therefore function (14) may be presented in a quadratic form:

$$V(x)\approx\frac{1}{2}\left[4ak_2^2-a\left(\frac{1}{b}-\frac{1}{c}\right)\sqrt[3]{\frac{k_6^2}{2}}-c\left(\frac{1}{a}-\frac{1}{b}\right)\sqrt[3]{\frac{k_4^2}{2}}-1\right]x_2^2+2ak_1^2x_1^2+$$

$$+2bk_3^2x_3^2+\frac{1}{2}\left[4bk_4^2-b\left(\frac{1}{c}-\frac{1}{a}\right)\sqrt[3]{\frac{k_6^2}{2}}-c\left(\frac{1}{a}-\frac{1}{b}\right)\sqrt[3]{\frac{k_2^2}{2}}-1\right]x_4^2+$$

$$+2ck_5^2x_5^2+\frac{1}{2}\left[4ck_6^2-a\left(\frac{1}{b}-\frac{1}{c}\right)\sqrt[3]{\frac{k_2^2}{2}}-b\left(\frac{1}{c}-\frac{1}{a}\right)\sqrt[3]{\frac{k_4^2}{2}}-1\right]x_6^2$$

Condition on existing of positive Lyapunov's function can be identified by inequality:

$$\begin{cases} k_1^2>0{,}4ak_2^2-a\left(\frac{1}{b}-\frac{1}{c}\right)\sqrt[3]{\frac{k_6^2}{2}}-c\left(\frac{1}{a}-\frac{1}{b}\right)\sqrt[3]{\frac{k_4^2}{2}}-1>0 \\ k_3^2>0{,}4bk_4^2-b\left(\frac{1}{c}-\frac{1}{a}\right)\sqrt[3]{\frac{k_6^2}{2}}-c\left(\frac{1}{a}-\frac{1}{b}\right)\sqrt[3]{\frac{k_2^2}{2}}-1>0 \\ k_5^2>0{,}4ck_6^2-a\left(\frac{1}{b}-\frac{1}{c}\right)\sqrt[3]{\frac{k_2^2}{2}}-b\left(\frac{1}{c}-\frac{1}{a}\right)\sqrt[3]{\frac{k_4^2}{2}}-1>0 \end{cases} \quad (15)$$

Further to above it was revealed that control system with increased potential of robust stability of nonlinear object in the class of two-parametric structurally stable mappings for nonlinear spacecraft model remains stable in case of any changes of indefinite parameters, therefore it rules out appearance of deterministic chaos in dynamic system and guarantees operability and reliability of control system under uncertain conditions.

Stationary state (5) of the system (1) remains stable in case of changes in spacecraft parameters within range (11), whereas stationary states (7) acquires stability feature when losing stability state (5). These states are not stable when simultaneous. Stationary state (11) will be stable onle when inequalities followed (15).

IV. CONCLUSION

Control system with increased potential of robust stability for nonlinear spacecraft with indefinite parameters was developed based on the approach of control system in the class of two-parametric structurally stable mappings from catastrophe theory with displayed marginal increase of robust stability potential preventing occurrence of deterministic chaos process.

For a research purpose of robust stability for control system with increased potential of robustness new approach was applied in development of Lyapunov's vector-function based on a geometric interpretation of theorem on asymptotic stability in a space. Conditions of robust stability of control system with increased potential of spacecraft with usage Morse theorem from catastrophe theory were managed as a form of a simple inequalities system which defines conditions of Lyapunov's vector function. System ensures stability under any changes of indefinite parameters of spacecraft and allows to manage deterministic chaos process in the system. This will guarantee prevention of occurrence and development of deterministic chaos regime under uncertain conditions in the system.

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