3rd International Conference on Automation, Control, Engineering and Computer Science (ACECS'16)
Proceedings of Engineering & Technology (PET)
pp. 632-637

Integral higher order sliding mode control for MIMO uncertain systems: Application to chaotic PMSM

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***Abstract*—This paper presents an integral high order sliding mode controller for a class of multi-input, multi-output nonlinear uncertain system. After transforming the system into high order input-output dynamic system, the idea is to define an integral sliding surface based on a nominal control law designed to stabilize in finite time the unperturbed system. By imposing some conditions on the controller parameter, the proposed method aims to stabilize the system although the presence of bounded uncertainties. In order to reduce the chattering caused by the discontinuous sign function in the controller, we replace this function by an hyperbolic tangent trigonometric function which is smooth and continuous. Stability of the proposed controller is proved based on Lyapunov approach. An illustrative example of perturbed chaotic permanent magnet synchronous motor demonstrates the effectiveness of this controller.**

I. Introduction

Controlling physical systems, which are subjected to external disturbances and varying parameters, is quite challenging because such systems are difficult to model. Several control techniques have been reported in literature to tackle the uncertain systems with good robustness. Among them, we cite sliding mode control [15], [17] and variable structure control [6], [5]. Sliding mode control has been observed since 1930 [17]. It is known for its robustness and effectiveness that make it very attractive.

Conventional sliding mode control consists: first, on choosing a sliding manifold to steer the states of the system to move along it and then, on designing a discontinuous control law in such a way that the system trajectories reach and stay on the choosing manifold after a finite time. Unfortunately, the presence of high control switching frequency leads to undesirable and dangerous phenomenon called chattering. Various approaches have been suggested to attenuate this undesirable effect [1], [2], [3]. Such approach is to replace the discontinuous signum function by a continuous approximation as the saturation function [3] or the hyperbolic tangent [9] (the boundary layer solution). Another approach is the high order sliding mode. Indeed, the sign function acts on high order time derivative of the sliding surface instead of the first time derivative [16], [13]. Also, it doesn't require relative degree 1 of the system which is a restriction of the standard sliding mode control.

The approach of high order sliding mode is also used in [12], but the chattering is usually exists due to the large discontinuous gain especially when the system is strongly perturbed. In this context, this paper proposes a novel high order sliding mode controller for a class a multi-input multi-output decoupled systems. The decoupling allows that each subsystem can be treated independently so the discontinuous control gain is not the same and can be reduced for some subsystems. We adopt this method to control the chaotic behavior [8], [11], [4] exhibited by a smooth air-gap permanent magnet synchronous motor under some specific conditions. This kind of electric drives is widely used in various industrial applications owing to many features such as low inertia, high efficiency, low maintenance cost…etc [14].

The remainder of the work is organized as follows. In section 2, the problem under investigation is formulated. Section 3 is devoted to describe the design of the proposed controller. Simulation results for suppressing the chaotic behavior of the PMSM are presented in section 4. The fifth section includes some remarks.

A. *Problem statement*

A class of non linear multi-input, multi-output uncertain system is considered:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y_1 &= \sigma_1(x) \\ &\vdots \\ y_m &= \sigma_m(x) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector and $u = [u_1, \ldots, u_m]^T \in \mathbb{R}^m$ is the control vector. $f(x)$ and $g(x) = [g_1, \ldots, g_m]^T$ are smooth uncertain functions. The uncertainties are due to parameter variations, unmodeled dynamics or external disturbances. $\sigma(x) = [\sigma_1, \ldots, \sigma_m]^T \in \mathbb{R}^m$ is a smooth measurable output vector, known as the sliding variables.

ISSN: 2356-5608

Assumption 1. The relative degree vector $r = [r_1, \ldots, r_m]^T$ of system (1) with respect to $\sigma(x)$ is assumed to be constant and known and the associated zero dynamics are stable.

Definition 1. ([10], [12]) Consider the nonlinear system (1) and suppose that the time derivatives of the sliding variables $\sigma_i, \dot{\sigma_i}, \ldots, \sigma_i^{(r_i-1)}$ are considered continuous ($i = 1, \ldots, m$). The manifold defined by:

$$\Sigma^r = \{x : |\sigma_i(x) = \ldots = \sigma_i^{(r_i-1)}(x) = 0, i = 1, \ldots, m\} \tag{2}$$

is called 'r^{th}-order sliding set' which is non-empty and locally an integral set in the Filippov sense [7]. The motion on Σ^r is called 'r^{th}-order sliding mode' with respect to the sliding variable σ.

By defining a suitable discontinuous control action, the r^{th}-order sliding mode control approach consists on moving the states along the switching surfaces $\sigma_i(x) = 0$ and to keep its $(r_i - 1)$ first time derivatives $(\dot{\sigma_i}, \ldots, \sigma_i^{(r_i-1)})$ to a vicinity of zero. The r_i^{th} time derivative of each σ_i satisfies the following equation:

$$\left[\sigma_1^{(r_1)}(x), \ldots, \sigma_m^{(r_m)}(x)\right] = A(x) + B(x)u \tag{3}$$

where $A(x) = [L_f^{r_1}\sigma_1(x), \ldots, L_f^{r_m}\sigma_m(x)]^T$

$$B(x) = \begin{bmatrix} L_{g1}L_f^{r_1-1}\sigma_1(x) & \cdots & L_{gm}L_f^{r_1-1}\sigma_1(x) \\ \vdots & & \vdots \\ L_{g1}L_f^{r_m-1}\sigma_m(x) & \cdots & L_{gm}L_f^{r_m-1}\sigma_m(x) \end{bmatrix}$$

with L_f and L_g are the Lie derivatives of the smooth functions in (1). $B(x)$ is non singular and $L_{gj}L_f^k\sigma_i(x) = 0$, for $1 \leq i \leq m$, $1 \leq j \leq m$, and $0 \leq k \leq r_i - 1$.

Assumption 2. For the rest of this paper, we suppose that matrix $B(x)$ is diagonal i.e. $L_{gj}L_f^{r_i-1}\sigma_i(x) = 0$, for $1 \leq j \leq m$ and $j \neq i$.

$$B(x) = \begin{bmatrix} b_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & b_m \end{bmatrix}$$

Assumption 3. $B(x)$ and $A(x)$ are uncertain and partitioned into well known nominal parts ($\bar{A}$ and $\bar{B}$) and uncertain parts (Δ_A and Δ_B), i.e.

$$\begin{cases} A(x) = \bar{A}(x) + \Delta_A(x) \\ B(x) = \bar{B}(x) + \Delta_B(x) \end{cases}$$

Also, we define : $F(x) = [f_1(x), \ldots, f_m(x)]^T \in \mathbb{R}^m$ and $G(x) = diag(g_i(x)) \in \mathbb{R}^{m*m}, \{i = 1, \ldots, m\}$

where $\begin{cases} F(x) = \Delta_A(x) - \Delta_B(x)\bar{B}^{-1}(x)\bar{A}(x) \\ G(x) = \Delta_B(x)\bar{B}^{-1}(x) \end{cases}$

There are upper bounds nonlinear a priori known functions $\rho_i(x)$ and a priori known constants $0 < \vartheta_i \leq 1$ such that for all $\{i = 1, \ldots, m\}$ the uncertain functions satisfy the following inequalities:

$$\begin{cases} \|f_i(x)\| & \leq \rho_i(x) \\ \|g_i(x)\| & \leq 1 - \vartheta_i \end{cases}$$

where $\|.\|$ denotes a norm on $\mathbb{R}^n$.

To summarize, an r^{th}-order sliding mode controller of system (1) with respect to the sliding variable σ is equivalent to the finite time stabilization of the following multivariable uncertain system:

$$\begin{cases} \begin{cases} \dot{z}_{1,i} = z_{2,i} \\ \vdots \\ \dot{z}_{r_i-1,i} = z_{r_i,i} \end{cases} , \forall i = \{1, \ldots, m\} \\ \left[\dot{z}_{r_1,1}, \dot{z}_{r_2,2}, \ldots, \dot{z}_{r_m,m}\right]^T = A(x) + B(x)u \end{cases} \tag{4}$$

with $z_{l,i} = \sigma_i^{(l-1)}, 1 \leq l \leq r_i$.
Consider the following preliminary feedback:

$$u = \bar{B}^{-1}\{-\bar{A} + v\} \tag{5}$$

with $v = [v_1, \ldots, v_m]^T$ the vector of the auxiliary control input designed to stabilize the following new system inspite of the uncertainties:

$$\begin{cases} \begin{cases} \dot{z}_{1,i} = z_{2,i} \\ \vdots \\ \dot{z}_{r_i-1,i} = z_{r_i,i} \end{cases} , \forall i = \{1, \ldots, m\} \\ \left[\dot{z}_{r_1,1}, \ldots, \dot{z}_{r_m,m}\right]^T = [I_m + \Delta_B\bar{B}^{-1}]v - \Delta_B\bar{B}^{-1}\bar{A} + \Delta_A \end{cases} \tag{6}$$

B. Sliding mode controller design

In this section, we first propose a nominal control law to guarantee the finite time stabilization of an integrator chain system. In other words, a finite time stabilizing controller to nominal system (6) described by:

$$\begin{cases} \dot{z}_{1,i} = z_{2,i} \\ \vdots \\ \dot{z}_{r_i-1,i} = z_{r,i} \\ \dot{z}_{r_i,i} = v_{nom,i} \end{cases} , \forall i = \{1, \ldots, m\} \tag{7}$$

Then, we design a discontinuous control law in order to stabilize the perturbed system (6) in finite time.

1) Finite time stabilization of an integrator chain system: System (7) is composed of m single-input, single-output independent integrator chains. The control objective is to steer to $z = [z_1^T, \ldots, z_m^T] = 0$ the states of the system (7)(with $z_i = [z_{1,i}, \ldots, z_{r_i,i}]$, $i = \{1, \ldots, m\}$).

Theorem 1. [12]: Let the constants $\kappa_{1,i}, \kappa_{2,i}, \ldots, \kappa_{m,i}$ be positive such that the polynomial $\lambda^{r_i} + \kappa_{r_i,i}\lambda^{r_i-1} + \ldots + \kappa_{2,i}\lambda + \kappa_{1,i}$ is Hurwitz. There exists $\epsilon_i \in (0, 1)$ such that for every $\alpha_i \in (1 - \epsilon_i, 1)$, system (7) is stabilized at the origin in finite time under the following feedback:

$$\begin{aligned} v_{nom,i}(z_i) = &-\kappa_{1,i}\text{sign}(z_{1,i})|z_{1,i}|^{\alpha_{1,i}} - \kappa_{2,i}\text{sign}(z_{2,i})|z_{2,i}|^{\alpha_{2,i}} \\ &- \ldots - \kappa_{r_i,i}\text{sign}(z_{r_i,i})|z_{r_i,i}|^{\alpha_{r_i,i}} \end{aligned} \tag{8}$$

where $\alpha_{1,i}, \ldots, \alpha_{r_i,i}$ satisfy:

$$\alpha_{j-1,i} = \frac{\alpha_{j,i}\alpha_{j+1,i}}{2\alpha_{j+1,i} - \alpha_{j,i}}, j \in \{2, \ldots, r_i\} \tag{9}$$

with $\alpha_{r_i+1,i} = 1$ and $\alpha_{r_i,i} = \alpha_i$

2) Integral sliding mode controller design: After designing a finite time controller $v_{nom}(z)$ that guarantees the finite time stabilization, at the origin, of the nominal system (7), our aim in this section is to stabilize the perturbed system (6) at zero in finite time. To this end, we consider the following integral sliding surface:

$$s(z(t)) = [z_{r1,1}(t) - z_{r1,1}(t_0), \dots, z_{rm,m}(t) - z_{rm,m}(t_0)]^T - \int_{t_0}^{t} v_{nom}(\tau)d\tau \tag{10}$$

where

$$v_{nom} = [v_{nom,1}, \dots, v_{nom,m}]^T, \tag{11}$$
$$s(z(t)) = [s_1(t), \dots, s_m(t)]^T \tag{12}$$

and t_0 being the initial time. The state trajectories start on this sliding surface from the initial time t_0 $(s(t_0) = 0)$.

The time derivative of (10) along the system trajectories is given by:

$$\begin{aligned} \dot{s} &= [\dot{z}_{r1,1}, \dots, \dot{z}_{rm,m}]^T - v_{nom} \\ &= [I_m + G(x)]v + F(x) - v_{nom} \end{aligned} \tag{13}$$

In order to stabilize system (6) at the origin, we consider the following control law:

$$v(z) = v_{nom}(z) + v_{disc}(z) \tag{14}$$

where v_{nom} is defined in (8),(9),(11) and

$$v_{disc} = -K.sign(s) \tag{15}$$

with '.' is the dot product and the gain K defined by $K = [k_1, k_2, \dots, k_m]^T$ satisfy:

$$k_i \geq \frac{(1-\vartheta_i)\|v_{nom,i}\| + \rho_i + \xi_i}{\vartheta_i}, \quad \forall i = \{1, \dots, m\} \tag{16}$$

Theorem.2 Consider the non linear system (1) with assumptions $(1-3)$ fulfilled. The control law described by:

$$u = \bar{B}^{-1}\{-\bar{A} + v_{nom}(z) - K.sign(s)\} \tag{17}$$

where $v_{nom}(z)$ is given by (11) and K satisfies the condition (16), ensures the establishment of an r^{th} order sliding mode with respect to σ in finite time.

Proof: Choose the following Lyapunov function:

$$V = \frac{1}{2}s^T s \tag{18}$$

The time derivative of V along the system trajectories is given by:

$$\begin{aligned} \dot{V} &= s^T([I_m + G]v + F - v_{nom}) \\ &= s^T([I_m + G]v_{disc} + F + Gv_{nom}) \end{aligned} \tag{19}$$

If we choose v_{disc} as in (15), the gains k_i as in (16) and under the assumptions $(2-3)$, one obtains:

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{m}(-k_i(1+g_i)s_i sign(s_i) + s_i(f_i + g_i v_{nom,i})) \\ &\leq \sum_{i=1}^{m}\|s_i\|(-k_i(1+g_i) + \|f_i\| + \|g_i\|\|v_{nom,i}\|) \\ &\leq \sum_{i=1}^{m}(-k_i\|s_i\| + k_i(1-\vartheta_i)\|s_i\| + \rho_i\|s_i\| + (1-\vartheta_i)\|s_i\| \\ &\quad \|v_{nom,i}\|) \\ &\leq \sum_{i=1}^{m}\|s_i\|(-\vartheta_i k_i + (1-\vartheta_i)\|v_{nom,i}\| + \rho_i + \xi_i) \\ &\leq \sum_{i=1}^{m} -\xi_i\|s_i\| \\ &\leq \sum_{i=1}^{m} -min\xi_i\|s_i\| \\ &\leq -min\xi_i\sqrt{2V} \end{aligned} \tag{20}$$

Inequality (20) implies that $V = 0$ in finite time and therefore the manifold $s(z(t))$ becomes zero in some finite time.

Remark1: Although the use of high order sliding mode control, the increase of uncertainties and perturbations requires a greater control effort so the chattering is usually exist. Therefore, to alleviate this phenomenon, we replace the sign function by the following hyperbolic tangent function:

$$\tanh(\mu s) = \frac{1-\exp(-2\mu s)}{1+\exp(-2\mu s)} \tag{21}$$

where μ is a positive parameter defining the smoothing degree of the function.

C. Application to a chaotic PMSM

In order to evaluate the efficiency of the proposed control law, simulation studies are carried out on the chaotic perturbed permanent magnet synchronous motor. Our aim is to suppress the chaotic behavior of the motor, in the presence of uncertainties and external perturbations, by applying the constructed high order integral sliding mode controller.

1) System Description: The mathematical model of a smooth-air-gap uncertain chaotic permanent magnet synchronous motor is described as follows:

$$\begin{aligned} \dot{i}_d &= -i_d + \omega i_q + u_d \\ \dot{i}_q &= -i_q + \omega i_d + (\overline{\gamma} + \delta\gamma)\omega + u_q \\ \dot{\omega} &= (\overline{\sigma} + \delta\sigma)(i_q - \omega) - T_l \end{aligned} \tag{22}$$

Where i_d and i_q are the direct-axis and quadrature-axis currents respectively (A), ω is the electrical rotor angular speed (rad/s), u_d, u_q and T_l stand for the direct-axis, quadrature-axis voltages (v) and the load torque (Nm),$\overline{\gamma} = 20$, $\overline{\sigma} = 5.46$, $\delta\gamma$ and $\delta\sigma$ are the uncertain parts of γ and σ respectively and the dot is the derivation with respect to t.

By denoting $x_1 = i_d$, $x_2 = i_q$ and $x_3 = \omega$, the PMSM model (22) has the form of system (1) with:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},\ f(x) = \begin{bmatrix} -x_1 + x_2 x_3 \\ -x_2 - x_1 x_3 + (\overline{\gamma} + \delta\gamma) x_3 \\ (\overline{\sigma} + \delta\sigma)(x_2 - x_3) - T_l \end{bmatrix},$$

$$g(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ u = \begin{bmatrix} u_q \\ u_d \end{bmatrix}$$

The control objective is to suppress the chaotic phenomenon of the motor and drive it to track the following references asymptotically:

$$\begin{cases} x_{3ref}(x) = 5 \\ x_{1ref}(x) = 2 \end{cases}$$

Suppose that the sliding surfaces are defined as:

$$\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} x_3 - x_{3ref} \\ x_1 - x_{1ref} \end{bmatrix} \tag{23}$$

Thus, the relative degree of system (22) versus the sliding variable σ is $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. From equation (3), we obtain:

$$\begin{bmatrix} \ddot{\sigma}_1 \\ \dot{\sigma}_2 \end{bmatrix} = (\bar{A} + \Delta A) + (\bar{B} + \Delta B) \begin{bmatrix} u_q \\ u_d \end{bmatrix} \tag{24}$$

with $\bar{A} = \begin{bmatrix} \bar{\sigma}((-x_2 - x_3 x_1 + \bar{\gamma} x_3) - \bar{\sigma}(x_2 - x_3)) \\ \\ -x_1 + x_3 x_2 \end{bmatrix}$,

$$\Delta A = \begin{bmatrix} \delta\sigma(-x_2 - x_3 x_1 + (\bar{\gamma} + \delta\gamma) x_3 - (\bar{\sigma} + \delta\sigma)(x_2 \\ -x3) + T_l) + \bar{\sigma}\delta\gamma x_3 - \bar{\sigma}\delta\sigma(x_2 - x_3) - \dot{T}_l \\ \\ 0 \end{bmatrix},$$

$\bar{B} = \begin{bmatrix} \bar{\sigma} & 0 \\ 0 & 1 \end{bmatrix}$, $\Delta B = \begin{bmatrix} \delta\sigma & 0 \\ 0 & 0 \end{bmatrix}$

Then, by using the following control law derived from (5):

$$\begin{bmatrix} u_q \\ u_d \end{bmatrix} = \bar{B}^{-1}\{-\bar{A} + v\} \tag{25}$$

system (24) becomes

$$\begin{bmatrix} \ddot{\sigma}_1 \\ \dot{\sigma}_2 \end{bmatrix} = \left[I_2 + \Delta B \bar{B}^{-1}\right] v - \Delta B \bar{B}^{-1} \bar{A} + \Delta A \tag{26}$$

whith v reading as:

$$v = \begin{bmatrix} v_{nom,1} \\ v_{nom,2} \end{bmatrix} - \begin{bmatrix} k_1 sign(s_1) \\ k_2 sign(s_2) \end{bmatrix} \tag{27}$$

The nominal control laws are obtained from (8) and (9) as

$$v_{nom,1} = -\kappa_{1,1}\mathrm{sign}(\sigma_1)|\sigma_1|^{\alpha_{1,1}} - \kappa_{2,1}\mathrm{sign}(\dot{\sigma}_1)|\dot{\sigma}_1|^{\alpha_{2,1}}$$
$$v_{nom,2} = -\kappa_{1,2}\mathrm{sign}(\sigma_2)|\sigma_2|^{\alpha_{1,2}}$$

The integral sliding surfaces are chosen as:

$$s_1 = \dot{\sigma}_1(t) - \dot{\sigma}_1(t_0) - \int_{t_0}^{t} v_{nom,1} d\tau$$
$$s_2 = \sigma_2(t) - \sigma_2(t_0) - \int_{t_0}^{t} v_{nom,2} d\tau$$

Remark.2 The time derivative of the first sliding variable ($\dot{\sigma}_1$) is calculated using Euler method and the high frequencies caused by the derivation were suppressed by a low pass filter with a small constant time.

2) Simulation results: A comparative study between high order sliding mode controller with discontinuous sign function, high order sliding mode controller with continuous hyperbolic tangent function and the nominal controller is carried out. The integration was executed according to the Euler method with the integration step $\tau = 1ms$ and following initial conditions: $(\omega(0), i_q(0), i_d(0)) = (-0.2685, 0.5, 8.7)$
The constant time of the low pass filter is chosen $\zeta = 10ms$. A constant load torque of $T_l = 5Nm$ is applied between seconds 10 and 16. The parameters of the nominal control are chosen as: $\kappa_{1,1} = 25$, $\kappa_{2,1} = 10$, $\kappa_{1,2} = 20$, $\alpha_{1,1} = 0.9048$, $\alpha_{2,1} = 0.95$, $\alpha_{1,2} = 0.99$. The parameters σ and γ are uncertain, we take a variation of 25%. Thus $\delta\gamma = -5$ and $\delta\sigma = 1.365$. One gets $\rho_1 = 15, \rho_2 = 10$ and $\vartheta_1 = 0.7, \vartheta_2 = 0.5$. Finally ,the gains k_1 and k_2 are tuned according to (16) ($k_1 = 200, k_2 = 50$).

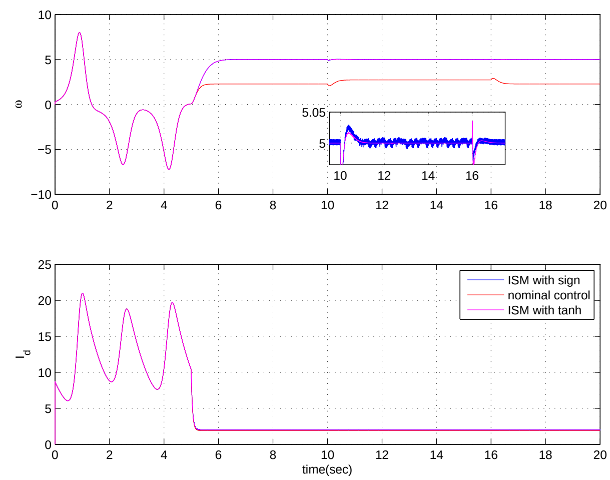

Fig. 1. ω and I_d versus time (sec)

The controller is applied at $t = 5s$. It is clear from Fig.1 that when the proposed controller (pink and blue curves) is applied, tracking of the reference signals is achieved in finite time despite the presence of perturbation and parameters uncertainties. But, the nominal control law (red curve) is unable to tackle the perturbed system and achieve the convergence of the system states. The control inputs and the sliding surfaces are depicted in Figs. 2-3. We remark the chattering attenuation when using the hyperbolic tangent function with the same tracking performance.

II. CONCLUSION

In this study, an integral higher order sliding mode controller is proposed for a class of multi-input, multi-output nonlinear uncertain systems. The design of higher order sliding mode control is equivalent to the finite time stabilization of higher order input-output system with bounded uncertainties. Due to the continuous hyperbolic tangent which replaces the signum function, the proposed control law becomes chattering free in the control input. Stability of the controlled system is proved based on Lyapunov approach. Simulation results demonstrates

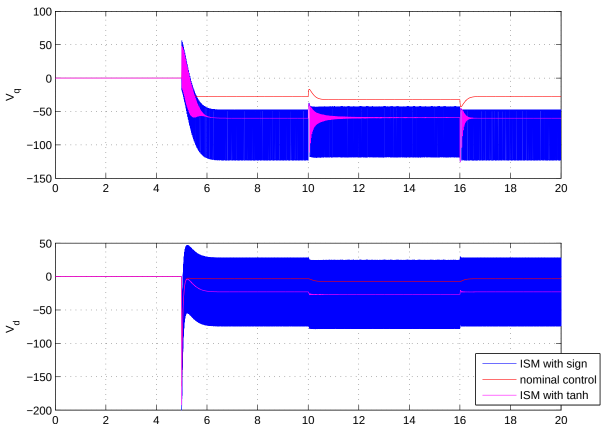

Fig. 2. V_q and V_d versus time (sec)

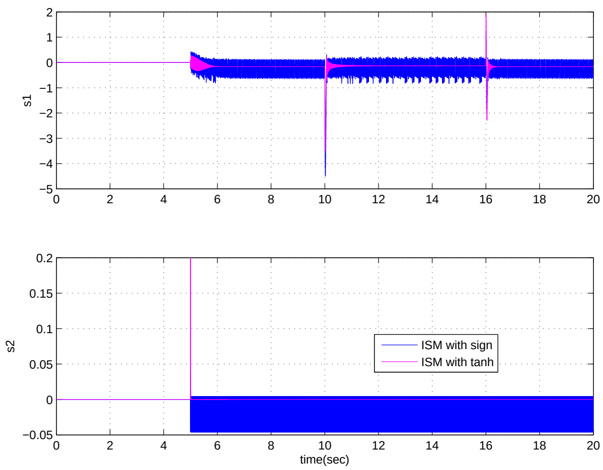

Fig. 3. Sliding surfaces versus time (sec)

that the proposed controller can eliminates the chaotic behavior of a permanent magnet synchronous motor subjected to parameter uncertainties and load torque disturbances. In practice, the discontinuous control gain is difficult to find because the upper bound of the system uncertainty is often unknown, so as perspective, we propose an adaptive tuning law to estimate this gain.

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