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On the stability and stabilization of discrete-time T-S fuzzy switched systems

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***Abstract*— This paper presents a new approach for dealing with stability and stabilization of discrete time T-S fuzzy switched systems under arbitrary switching. The proposed method applies the vector norms concept to a specific form of matrix called arrow form matrix. The application of Borne and Gentina practical criterion to the comparison system yields to simple algebraic stability conditions and thus avoids the search of a common Lyapunov function.**

I. INTRODUCTION

Switched systems are a special form of hybrid dynamical systems composed of a family of continuous-time or discrete-time subsystems and a rule that governs the switching among them.

Advances in the theory of both hybrid systems and fuzzy systems has given rise to fuzzy switched systems as an answer to more complicated real systems analysis and synthesis requirements such as multiple nonlinear systems, switched nonlinear systems and second-order nonholonomic systems. A Takagi-Sugeno fuzzy switched system is a combination of a hybrid system and multiple fuzzy models [1]. Originally inspired from the concept of sector nonlinearity, Takagi-Sugeno modeling idea consists of partitioning the nonlinear dynamics of a system into several locally linearized submodels so that the overall nonlinear system can be represented by a sufficiently accurate approximation [2]. Fuzzy modeling and control is a universal approximation tool and a reliable approach to handle complex and ill-defined systems.

Despite the considerable progress in the analysis of nonlinear systems, stability study of switched systems [3] and in particular of fuzzy switched systems is still complex [4]. In fact, the example of asymptotically stable subsystems which lead to an unstable behavior of the overall system, due to a specific switching sequence, is well known. Besides, the case of unstable subsystems that, via a particular switching law, yields to a stable global system also exists.

Stability analysis of T-S model-based switched systems has been conducted mainly on the basis of Lyapunov stability theory [5], [6], [7]. The first approach, based on common quadradic Lyapunov functions, requires the existence of a common symmetric positive definite matrix P to satisfy Lyapunov stability condition for all switching regions and local fuzzy systems. This method has proved to be conservative and even sometimes, P may not exist for many complex highly nonlinear systems. This conservativeness arises from the 'strict' feature of the Lyapunov function since it depends neither on the switching functions nor on the local fuzzy weighting functions. The switching Lyapunov function, considered as a piecewise quadratic Lyapunov function, represents an alternative to develop less conservative stability results and consists of local quadratic Lyapunov functions constructed in each switching region. The control design of T-S fuzzy switched systems is carried out by means of the so-called Parallel Distributed Compensation (PDC) scheme [6], [8], [9]. This model-based procedure consists of designing, for each local linear model of each subsystem, a linear feedback control. The resulting overall subcontroller is obtained by fuzzily blending of each individual linear controller.

To overcome limitations due to the existence of such Lyapuov functions, we propose, in this paper, to study the stability of T-S model-based switched systems under arbitrary switching through the study of the convergence of a regular vector norm, associated to a specific characteristic matrix form, called arrow form matrix [10]. The vector norm approach [11], [12], [13] [14], [15], based on the overvaluing principle [16], has a major advantage as it deals with a large class of systems since no restrictions are made on the matrices of states equations. The application of vector norms concept to switched systems has already been introduced in [17], [18], [19].

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The organization of the paper is as follows: section II gives a description of the considered class of discrete time T-S fuzzy switched systems. Proposed stability and stabilization conditions are given in section III. Section IV illustrates the obtained results through an example. Finally, some concluding remarks are given in section V.

II. DISCRETE TIME T-S FUZZY SWITCHED SYSTEMS-PROBLEM FORMULATION

Consider the following discrete-time switched system:

$$\begin{cases} x(k+1) = A_{\sigma(k)}(.)x(k) + B_{\sigma(k)}(.)u(k) \\ y(k) = C_{\sigma(k)}(.)x(k) \end{cases} \tag{1}$$

where $x(k) \in \Re^n$ is the state vector, $u(k) \in \Re^m$ is the control input, $y(k) \in \Re^q$ is the output vector, $A_{\sigma(k)}(.) \in \Re^{n\times n}$, $B_{\sigma(k)}(.) \in \Re^{n\times m}$, $C_{\sigma(k)}(.) \in \Re^{q\times n}$. $\sigma(k) : \Re^+ \to I = \{1,2,..,N\}$ is the switching signal assumed to be available in real time and N is the number of subsystems.

Therefore, the switched system is composed of N discrete-time subsystems which are expressed as:

$$\begin{cases} x(k+1) = A_i(.)x(k) + B_i(.)u(k) \\ y(k) = C_i(.)x(k) \end{cases}, \ i \in I \tag{2}$$

where $A_i(.)$, $B_i(.)$ and $C_i(.)$ are matrices of appropriate dimensions.

Using the Takagi-Sugeno fuzzy modeling method, the l^{th} fuzzy rule, associated with the i^{th} discrete-time subsystem i being active at instant k, can be defined as:

Rule l for subsystem i:

IF x_1 is M_{li}^1 and x_2 is M_{li}^2 and …and x_n is M_{li}^n

THEN

$$\begin{cases} x(k+1) = A_{li}x(k) + B_{li}u(k) \\ y(k) = C_{li}x(k) \end{cases}, \ i \in I \tag{3}$$

where $x_1(k)$, $x_2(k)$,…, $x_n(k)$ are the premise variables, M_{li}^j are the fuzzy sets, r and n are the number of fuzzy rules and state variables respectively.

By the product inference engine and the center of average defuzzification, the final output of the i^{th} subsystem is inferred as:

$$x(k+1) = \sum_{l=1}^{r} h_{li}(x(k))\left\{A_{li}x(k) + B_{li}u(k)\right\} \tag{4}$$

where:

$$h_{li}(x(k)) = \frac{\prod_{j=1}^{n} M_{li}^j(x_j(k))}{\sum_{l=1}^{r}\prod_{j=1}^{n} M_{li}^j(x_j(k))} = \frac{\omega_{li}(x(k))}{\sum_{l=1}^{r}\omega_{li}(x(k))} \tag{5}$$

$M_{li}^j(x_j(k))$ is the firing strength of membership function M_{li}^j. It is assumed that $\omega_{li}(x(k)) \ge 0$ and $\sum_{l=1}^{r}\omega_{li}(x(k)) > 0$ $\forall l = 1,2,..,r$. Hence, $h_{li}(x(k)) \ge 0$ and $\sum_{l=1}^{r} h_{li}(x(k)) = 1$.

By introducing $\zeta_i(k)$ as an exogenous function defining the switching law,

$$\zeta_i(k) = \begin{cases} 1 & \text{if subsystem } i \text{ is active} \\ 0 & \text{otherwise} \end{cases}, \ i = 1,2,.,N \tag{6}$$

switched fuzzy system can then be represented by:

$$x(k+1) = \sum_{i=1}^{N}\zeta_i(k)\sum_{l=1}^{r} h_{li}(x(k))\left\{A_{li}x(k) + B_{li}u(k)\right\} \tag{7}$$

A PDC controller scheme is employed to deal with the stabilization of fuzzy switched systems. The l^{th} rule of the switching PDC controller stabilizing subsystem (4) is given by:

Rule l for controller of subsystem i:

IF x_1 is M_{li}^1 and x_2 is M_{li}^2 and …and x_n is M_{li}^n

THEN

$$u(k) = -K_{li}x(k), \ \ l = 1,2,..,N \tag{8}$$

where $K_{li} = \begin{bmatrix} k_{li}^0 & k_{li}^1 & \cdots & k_{li}^{n-1} \end{bmatrix}$ is the local feedback gain vector.

The final output of controller (8) is represented by:

$$x(k+1) = -\sum_{l=1}^{r} h_{li}(x(k))K_{li}x(k) \tag{9}$$

Substituting (9) into (7), closed-loop representation of the T-S fuzzy switched system is:

$$x(k+1) = \sum_{i=1}^{N}\zeta_i(k)\sum_{l=1}^{r} h_{li}(x(k))\sum_{q=1}^{r} h_{qi}(x(k)) \\ (A_{li} - B_{li}K_{qi})x(k) \tag{10}$$

In the sequel, $h_{li}(x(k))$ and $h_{qi}(x(k))$ will be simplified to h_{li} and h_{qi}.

III. NEW STABILITY AND STABILIZATION CONDITIONS FOR T-S DISCRETE TIME SWITCHED SYSTEMS

A. *Main results*

First, consider system (1) in the autonomous mode (u=0):

$$x(k+1) = \sum_{i=1}^{N}\zeta_i(k)\sum_{l=1}^{r} h_{l\sigma(k)}A_{l\sigma(k)}x(k) \tag{11}$$

Theorem 1.

The T-S unforced fuzzy switched system (11) is globally asymptotically stable under an arbitrary switching law $\sigma(k) = i \in I$ if matrix $(I_n - M_D)$ is an M-matrix, where:

$$M_D = \max_{i\in I}\left(\left|\sum_{l=1}^{r} h_{l\sigma(k)}A_{l\sigma(k)}\right|\right) \tag{12}$$

Proof.

Let us consider system (11) under an arbitrary switching law $\sigma(k) = i \in I$ and let $\omega \in \Re_+^{*n}$ ($\omega_m > 0 \ \ \forall m = 1,..,n$).

Consider the following common radially unbounded Lyapunov functional for system (11):

$$V(x(k),k) = \left\langle |x(k)|, \omega \right\rangle \tag{13}$$

Thus, its difference is written as:

$$\begin{aligned}\Delta V(x(k),k) &= V(x(k+1),k+1) - V(x(k),k)\\ &= \langle |x(k+1)|, \omega\rangle - \langle |x(k)|, \omega\rangle\\ &= \left\langle \left|\sum_{l=1}^{r} h_{l\sigma(k)} A_{l\sigma(k)}\right| |x(k)|, \omega\right\rangle - \langle |x(k)|, \omega\rangle\\ &\le \left\langle \max_{i\in I}\left(\left|\sum_{l=1}^{r} h_{l\sigma(k)} A_{l\sigma(k)}\right|\right)|x(k)|, \omega\right\rangle - \langle |x(k)|, \omega\rangle\\ &\le \langle (M_D - I_n)|x(k)|, \omega\rangle\end{aligned}$$

and finally,

$$\Delta V(x(k),k) \le \langle -(I_n - M_D)|x(k)|, \omega\rangle \qquad (14)$$

On the other hand, we assume that $(I_n - M_D)$ is an M-matrix and according to the M-matrices properties, we can find a vector $\rho \in \Re_+^{*n}$ ($\rho_p > 0 \quad \forall p = 1,..,n$) such as $(I_n - M_D)^T \omega = \rho \quad \forall \omega \in \Re_+^{*n}$.

It comes:

$$\begin{aligned}\langle -(I_n - M_D)|x(k)|, \omega\rangle &= \langle -(I_n - M_D)^T \omega, |x(k)|\rangle\\ &= \langle -\rho, |x(k)|\rangle = -\sum_{p=1}^{n} \rho_p |x_p(k)| < 0\end{aligned} \qquad (15)$$

This completes the proof of theorem 1.

Theorem 2.

The T-S fuzzy switched system (10) is globally asymptotically stable with a PDC switching controller (9) under arbitrary switching law $\sigma(k) = i \in I$ if matrix $(I_n - M_D^c)$ is an M-matrix, with:

$$M_D^c = \max_{i\in I}\left(\left|A_{\sigma(k)}^c\right|\right) \qquad (16)$$

and

$$A_{\sigma(k)}^c = \sum_{l=1}^{r} h_{li}(x(k)) \sum_{q=1}^{r} h_{qi}(x(k))(A_{li} - B_{li}K_{qi}) \qquad (17)$$

Proof. The same proof as theorem 1.

B. Extension of the results to the case of systems defined by difference equations

Let us consider discrete-time T-S model-based switched systems described by the following difference equations:

$$y(k+n) + \sum_{i=1}^{N} \zeta_i(k) \sum_{l=1}^{r} h_{li} \sum_{p=0}^{n-1} a_{li}^p y(k+p) = u(k) \qquad (18)$$

A change of variable of the form $x_{p+1}(k) = x_p(k) = y(k+p)$ allows the system to be represented under its controllable form:

$$x(k+1) = \sum_{i=1}^{N} \zeta_i(k) \sum_{l=1}^{r} h_{li}(x(k))\left(A_{li}x(k) + B_{li}u(k)\right) \qquad (19)$$

with:

$$A_{li} = \begin{bmatrix} 0 & 1 & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & \cdots & 1\\ -a_{li}^0 & \cdots & -a_{li}^{n-1} & -a_{li}^n \end{bmatrix} \text{ and } B_{li} = B = \begin{bmatrix} 0\\ \vdots\\ 0\\ 1\end{bmatrix} \qquad (20)$$

Then, relation (19) becomes :

$$\begin{aligned}x(k+1) &= \sum_{i=1}^{N} \zeta_i(k) \sum_{l=1}^{r} \sum_{q=1}^{r} h_{li}h_{qi}(A_{li} - B_{li}K_{qi})x(k)\\ &= \sum_{i=1}^{N} \zeta_i(k) \sum_{l=1}^{r} \sum_{q=1}^{r} h_{li}h_{qi}(A_{li} - BK_{qi})x(k)\\ &= \sum_{i=1}^{N} \zeta_i(k)(\sum_{l=1}^{r} h_{li} \sum_{q=1}^{r} h_{qi}A_{li}x(k) -\\ &\qquad \sum_{q=1}^{r} h_{qi} \sum_{l=1}^{r} h_{li}BK_{qi}x(k))\\ &= \sum_{i=1}^{N} \zeta_i(k)(\sum_{l=1}^{r} h_{li}A_{li}x(k) - \sum_{q=1}^{r} h_{qi}BK_{qi}x(k))\end{aligned}$$

Hence,

$$\begin{aligned}x(k+1) &= \sum_{i=1}^{N} \zeta_i(k) \sum_{l=1}^{r} h_{li}(A_{li} - BK_{li})x(k)\\ &= \sum_{i=1}^{N} \zeta_i(k) \sum_{l=1}^{r} h_{li}A_{li}^c x(k)\end{aligned} \qquad (21)$$

A change of base $z(k) = Px(k)$ of (21) under the arrow form gives :

$$z(k+1) = \sum_{i=1}^{N} \zeta_i(k) \sum_{l=1}^{r} h_{li}M_{li}z(k) \qquad (22)$$

where matrix $M_{li} = P^{-1}A_{li}^c P$ is in the arrow form and P is the corresponding passage matrix:

$$P = \begin{bmatrix} 1 & 1 & \dots & 1 & 0\\ \alpha_1 & \alpha_2 & \dots & \alpha_{n-1} & 0\\ (\alpha_1)^2 & (\alpha_2)^2 & \dots & (\alpha_{n-1})^2 & \vdots\\ \vdots & \vdots & \dots & \vdots & 0\\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \dots & (\alpha_{n-1})^{n-1} & 1\end{bmatrix} \qquad (23)$$

and

$$M_{li} = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 & \beta_1\\ 0 & \ddots & \ddots & \vdots & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots\\ 0 & \cdots & 0 & \alpha_{n-1} & \beta_{n-1}\\ \gamma_{li}^{c(1)} & \dots & \dots & \gamma_{li}^{c(n-1)} & \gamma_{li}^{c(n)}\end{bmatrix} \qquad (24)$$

with:

$$\begin{cases}\beta_j = \prod_{\substack{q=1\\ q\ne j}}^{n-1}\left(\alpha_j - \alpha_q\right)^{-1}, \ \forall\ j = 1,...,n-1\\ \gamma_{li}^{c(j)} = -P_{A_{li}^c}\left(\alpha_j\right), \ \forall\ j = 1,...,n-1\\ \gamma_{li}^{c(n)} = -(a_{li}^{n-1} + k_{li}^{n-1}) - \sum_{j=1}^{n-1}\alpha_j\end{cases} \qquad (25)$$

In such conditions, if $p(w)$ denotes a vector norm of w such that $p(w) = \left[|w_1|, |w_2|, .., |w_n|\right]^T$, it is possible by the use of the aggregation techniques to define a discrete time comparison system $y(k+1) = M_D^c y(k)$ such that the pseudo-overvaluing matrix M_D is computed as follows:

$$M_D^c=\begin{bmatrix}|\alpha_1| & 0 & \cdots & 0 & |\beta_1|\\ 0 & \ddots & \ddots & \vdots & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots\\ 0 & \cdots & 0 & |\alpha_{n-1}| & |\beta_{n-1}|\\ \max_{i\in I}\left|\sum_{l=1}^{r}h_{li}\gamma_{li}^{c(1)}\right| & \cdots & \cdots & \max_{i\in I}\left|\sum_{l=1}^{r}h_{li}\gamma_{li}^{c(n-1)}\right| & \max_{i\in I}\left|\sum_{l=1}^{r}h_{li}\gamma_{li}^{c(n)}\right|\end{bmatrix} \quad (26)$$

Matrix $(I_n - M_D^c)$ is then given by:

$$(I_n - M_D^c)=\begin{bmatrix}1-|\alpha_1| & 0 & \cdots & 0 & -|\beta_1|\\ 0 & \ddots & \ddots & \vdots & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots\\ 0 & \cdots & 0 & 1-|\alpha_{n-1}| & -|\beta_{n-1}|\\ -t_D{}^1 & \cdots & \cdots & -t_D{}^{n-1} & 1-t_D{}^n\end{bmatrix} \quad (27)$$

$$\begin{cases} t_D{}^j = \max_{i\in I}\left|\sum_{l=1}^{r}h_{li}\gamma_{li}^{c(j)}\right| & \forall j=1,..n-1\\ t_D{}^n = \max_{i\in I}\left|\sum_{l=1}^{r}h_{li}\gamma_{li}^{c(n)}\right| \end{cases} \quad (28)$$

The application of Borne and Gentina criterion to $(I_n - M_D^c)$ leads to the new sufficient stability conditions of the closed loop system (21) stated by theorem 3.

Theorem 3.

The T-S fuzzy switched system (21) is globally asymptotically stable under arbitrary switching law $\sigma(k)=i\in I$ if there exist α_j $(j=1,..,n-1)$, $\alpha_j \neq \alpha_q$ $\forall\, j\neq q$ such as:

i) $$1-|\alpha_j|>0, \quad j=1,..,N \quad (29)$$

ii) $$1-t_D{}^n-\sum_{j=1}^{n-1}t_D{}^j\,|\beta_j|\left(1-|\alpha_j|\right)^{-1}>0 \quad (30)$$

Theorem 3. can be reduced to the corollary below.

Corollary.

The closed-loop T-S fuzzy system (21) is globally asymptotically stable under arbitrary switching $\sigma(k)=i\in I$ if the following conditions are met $\forall \alpha_j \in]0,1[$, $\alpha_j\neq\alpha_q$, $\forall j\neq q$, for each $i\in I$, $j=1,..,n-1$ and $l=1,..,r$:

i) $$\beta_j\sum_{l=1}^{r}h_{li}P_{A_{li}^c}(\alpha_j)<0 \quad (31)$$

ii) $$\sum_{l=1}^{r}h_{li}\gamma_{li}^{c(n)}>0 \quad (32)$$

iii) $$1-\max_{i\in I}\left(\sum_{l=1}^{r}h_{li}\gamma_{li}^{c(n)}\right)-\sum_{j=1}^{n-1}\left(\max_{i\in I}\sum_{l=1}^{r}h_{li}\gamma_{li}^{c(j)}\right)\beta_j(1-\alpha_j)^{-1}>0 \quad (33)$$

IV. Illustrative Example

As a numerical example, let us consider the two second-order systems arbitrarily switching between each other and given by:

$$x(k+1)=\sum_{i=1}^{2}\zeta_i(k)\sum_{l=1}^{2}h_{li}(x(k))A_{li}x(k)$$

where:

$$A_{11}=\begin{bmatrix}0 & 1\\ -0.6703 & 1.703\end{bmatrix},\ A_{21}=\begin{bmatrix}0 & 1\\ -0.6703 & 1.77\end{bmatrix}$$

$$A_{12}=\begin{bmatrix}0 & 1\\ -0.905 & 1.905\end{bmatrix},\ A_{22}=\begin{bmatrix}0 & 1\\ -0.819 & 1.819\end{bmatrix}$$

By means of switching PDC control with state feedback gain vectors $K_{11}=\left[k_{11}^1\ \ k_{11}^2\right]$, $K_{21}=\left[k_{21}^1\ \ k_{21}^2\right]$, $K_{12}=\left[k_{12}^1\ \ k_{12}^2\right]$ and $K_{22}=\left[k_{22}^1\ \ k_{22}^2\right]$.

The closed loop system can then be written as follows:

$$A_{11}{}^c=\begin{bmatrix}0 & 1\\ -0.6703-k_{11}^1 & 1.703-k_{11}^2\end{bmatrix},$$

$$A_{21}{}^c=\begin{bmatrix}0 & 1\\ -0.6703-k_{21}^1 & 1.77-k_{21}^2\end{bmatrix}$$

$$A_{12}{}^c=\begin{bmatrix}0 & 1\\ -0.905-k_{12}^1 & 1.905-k_{12}^2\end{bmatrix}$$

$$A_{22}{}^c=\begin{bmatrix}0 & 1\\ -0.819-k_{22}^1 & 1.819-k_{22}^2\end{bmatrix}$$

A change of base under the arrow form matrix gives:

$$F_{11}=P^{-1}A_{11}{}^cP=\begin{bmatrix}\alpha & 1\\ \gamma_{11}^1 & \gamma_{11}^2\end{bmatrix},\ F_{21}=P^{-1}A_{21}{}^cP=\begin{bmatrix}\alpha & 1\\ \gamma_{21}^1 & \gamma_{21}^2\end{bmatrix},$$

$$F_{12}=P^{-1}A_{12}{}^cP=\begin{bmatrix}\alpha & 1\\ \gamma_{12}^1 & \gamma_{12}^2\end{bmatrix},\ F_{22}=P^{-1}A_{22}{}^cP=\begin{bmatrix}\alpha & 1\\ \gamma_{22}^1 & \gamma_{22}^2\end{bmatrix}$$

with:

$$\begin{cases}\gamma_{11}^1=-P_{A_{11}{}^c}(\alpha)=-\left[\alpha^2+(-1.703+k_{11}^2)\alpha+0.6703+k_{11}^1\right]\\ \gamma_{11}^2=1.703-k_{11}^2-\alpha\end{cases},$$

$$\begin{cases}\gamma_{21}^1=-P_{A_{21}{}^c}(\alpha)=-\left[\alpha^2+(-1.77+k_{21}^2)\alpha+0.6703+k_{21}^1\right]\\ \gamma_{21}^2=1.77-k_{21}^2-\alpha\end{cases},$$

$$\begin{cases}\gamma_{12}^1=-P_{A_{12}{}^c}(\alpha)=-\left[\alpha^2+(-1.905+k_{12}^2)\alpha+0.905+k_{12}^1\right]\\ \gamma_{12}^2=1.905-k_{12}^2-\alpha\end{cases},$$

and

$$\begin{cases}\gamma_{22}^1=-P_{A_{22}{}^c}(\alpha)=-\left[\alpha^2+(-1.819+k_{22}^2)\alpha+0.819+k_{22}^1\right]\\ \gamma_{22}^2=1.819-k_{22}^2-\alpha\end{cases}$$

The minimal pseudo-overvaluing matrix corresponding to the comparison system that will allow us to conclude to the stability of the initial system is:

$$M_D^c = \begin{bmatrix} |\alpha| & 1 \\ \max\limits_{1\le i\le 2}\left|\sum_{l=1}^{2} h_{li}\gamma_{li}^1\right| & \max\limits_{1\le i\le 2}\left|\sum_{l=1}^{2} h_{li}\gamma_{li}^2\right| \end{bmatrix}$$

For an arbitrary choice of $\alpha = 0.1$ and $K_{11} = [-1\ 1.35]$, $K_{21} = [-1\ 1.35]$, $K_{12} = [-1.105\ \ k_{22}^2]$ and $K_{22} = [-0.919\ \ k_{22}^2]$ and by applying the corollary, we can deduce the following stability conditions:

- $\sum_{l=1}^{2} h_{li} P_{A_{li}^c}(\alpha_j) < 0 \quad \forall i \in \{1,2\}$
- $\sum_{l=1}^{2} h_{li}\gamma_{li}^2 > 0 \quad \forall i \in \{1,2\}$
- $1 - \max\limits_{i\in I}\left(\sum_{l=1}^{2} h_{li}\gamma_{li}^2\right) - \max\limits_{i\in I}\left(\sum_{l=1}^{2} h_{li}\gamma_{li}^1\right)(1-\alpha_j)^{-1} > 0$

Stability domain is given by the adjustable parameter k_{22}^2 as a function of h_{11}:

$$0.186h_{11} + 0.919 < k_{22}^2 < 0.153h_{11} + 1.399$$

i)

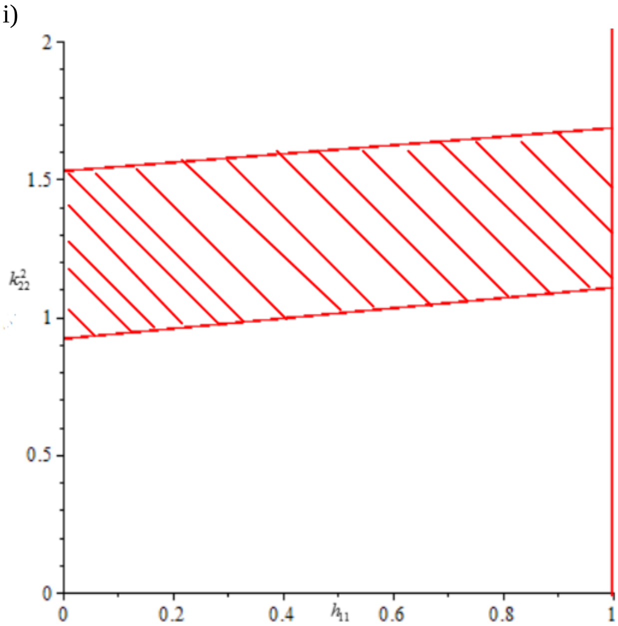

Fig1. Stability domain given by h_{11} function of k_{22}^2 obtained from corollary

In the case $h_{11} = 0.5$, $k_{22}^2 = 1.2$, sampling time $T_e = 0.2s$, the initial state vector $x(0) = [2\ \text{-}1]^T$ and the switching sequence given in Figure 2 The simulation result of the system are shown in Figures 3 and 4**,** respectively which correspond to the evolution of states with respect to time and the norm state.

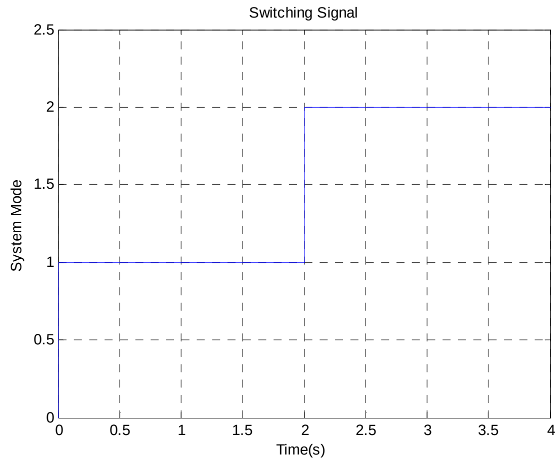

Fig 2. Switching function between subsystems

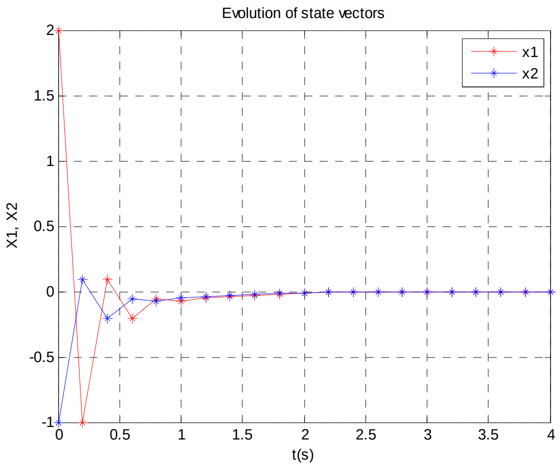

Fig 3. State responses of the system

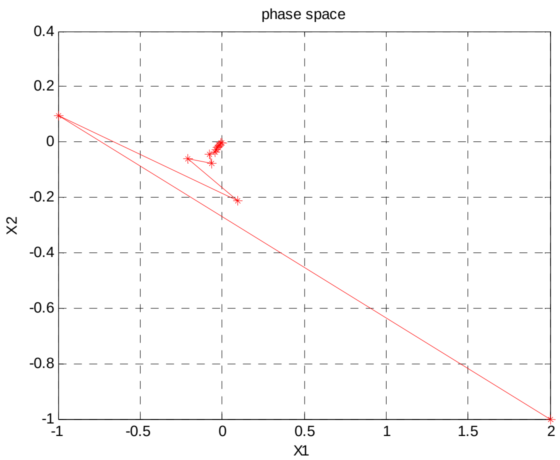

Figure 4. State's norm

V. Conclusion

This paper proposes an alternative to common problems encountered when constructing common Lyapunov functions.

In fact, this method may need to solve a large number of Linear Matrix Inequalities especially when the number of fuzzy rules required to fully describe the system is high. Stability criteria presented are applicable under arbitrary switching which is a great benefit particularly when the switching law is unknown or uncontrollable and are appropriate to a large class of nonlinear systems.

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