

# Numerical Simulation of Natural Convection in a Square Cavity with Partially Active Vertical and Horizontal Walls

Debbah Djoubeir<sup>#1</sup>, Kholai Omar<sup>#1</sup>, Chorfi Soufien<sup>#1</sup>, Boudebous Saadoun<sup>#1</sup>

<sup>#1</sup> *Laboratory of Transport and Environment Engineer, University Mentouri, Constantine, Algeria*

<sup>1</sup>debbah.djoubeir@gmail.com

**Abstract**—A numerical study is performed to analyze the steady natural convection phenomena of air in a square cavity with different locations of the heating portion. The heat sources parts in the left, right and bottom walls of the cavity are maintained at a higher temperature  $T_h$ , whereas the other parts of these sidewalls are kept at a lower temperature  $T_c$ . The enclosure's top wall is kept insulated. The coupled equations of continuity, momentum and energy are solved by a finite volume method. The SIMPLE algorithm is used to solve iteratively the pressure-velocities coupling. The numerical investigations in this analysis is made over a wide range of parameters, Rayleigh number ( $10^3 \leq Ra \leq 10^6$ ) and dimensionless heater lengths. The effect of three different heating locations on the vertical walls (bottom, Centre, and top) and the local heat source on the bottom wall was evaluated. Results are presented graphically in the form of streamlines, isotherms and also with a velocity profiles and average Nusselt numbers.

**Keywords**—Natural convection; Nusselt number; Square cavity; Numerical simulation; Finite volume method; Heat source; Streamline; Isotherm; Average Nusselt numbers

## I. INTRODUCTION

Natural convection in a two-dimensional square cavity with differentially heated walls has drawn considerable attention in many important engineering applications such as electronic equipment cooling [Incropera 1988], ventilation of buildings [Hoogendooren and Afgan 1978], solar ponds and solar collectors [Cha and Jauria 1984, Jauria and al 1989], and nuclear reactors. These discrete heaters are placed either on horizontal wall or vertical wall, representing certain interesting features of flow field and the heat transfer rate in a range of Rayleigh numbers. The other is its complex nature of the fluid flow and heat transfer characteristics due to discrete heat sources of different type, size, location and strength. To meet the increasing demand of engineering applications, main efforts have been focused on the approaches to enhance the heat transfer from the discrete heat sources.

A number of studies have been conducted to investigate the flow and heat transfer characteristics in closed cavities in the past. Natural convection flows in a square cavity with heat generating fluid and a finite size heater on the vertical wall have been investigated numerically by Rahman et al [2010]. Basak [2006] has studied effects of thermal boundary conditions on natural convection flows within a square cavity. It has been demonstrated that the formation of boundary

layers for both the heating cases occurs. It is also observed that thermal boundary layer develops over approximately 80% of the cavity for uniform heating whereas the boundary layer is approximately 60% for non-uniform heating when  $Ra=10^3$ .

Natural convection in air-filled 2D square enclosure heated with a constant source from below and cooled from above is studied [Cheikh and al 2007] numerically for a variety of thermal boundary conditions at the top and sidewalls. Simulations are performed for two kinds of lengths of the heated source.

Aydin and Yang [2000] treated numerically the convection of air in a rectangular enclosure that was locally heated from below and symmetrically cooled from the two vertical sides. Natural convection of air in square enclosures heated by a localized source from below and symmetrically cooled from the sides has been experimentally and numerically investigated by Calcagni [2005]. The experimental method uses the holographic interferometry technique in real-time and double-exposure, to obtain respectively the visualization of possible oscillation of the plume and steady-state temperature distribution inside the cavity. The same case has been studied [Che Sidik 2009] using finite difference double-distribution function thermal lattice Boltzmann model. The results obtained demonstrate that this approach is very efficient procedure to study flow and heat transfer in a differentially heated cavity flow.

Steady laminar natural convection in air-filled, 2-D rectangular enclosures heated from below and cooled from above is studied [Corcione 2003] numerically for a wide variety of thermal boundary conditions at the sidewalls.

Lattice Boltzmann method was employed for investigation the effect of the heater location on flow pattern, heat transfer and entropy generation in a cavity [Delavar and Sedighi 2011]. Results show that higher heat transfer was observed from the cold walls when the heater located on vertical wall. On the other hand, heat transfer increases from the heater surface when it is located on the horizontal wall.

Laminar natural convection in a two-dimensional square enclosure due to two and three source-sink pairs on the vertical side-walls was numerically investigated by Qi-Hong Deng [2008]. It was found that the total heat transfer was closely related with the number of eddies in the enclosure, the number of eddies in the enclosure would increase and hence

heat transfer was augmented. Natural convection heat transfer in a square air-filled enclosure with one discrete flush heater is examined numerically by Radhwan and Zaki [2000]. The optimum location over the range of Rayleigh number is for the heater mounted at the center of the wall, a result confirmed by previous experiments.

The phenomena of natural convection in an inclined square enclosure heated via corner heater have been studied numerically by Varol et al [2009]. One wall of the enclosure is isothermal but its temperature is colder than that of heaters while the remaining walls are adiabatic. It is observed that heat transfer is maximum or minimum depending on the inclination angle and depending on the length of the corner heaters. The effect of Prandtl number on mean Nusselt number is more significant for  $Pr < 1$ .

Laminar natural convection inside air-filled, rectangular enclosures heated from below and cooled from above, with the lower portions of both sidewalls maintained at the temperature of the bottom wall, and the remaining upper portions of the sidewalls maintained at the temperature of the top wall, is studied numerically [Caronna and al 2009]. It is found that when the heated portions of the two sidewalls are different in length, a steady-state solution is reached, with a basic three-cell flow pattern. In contrast, when the heated fractions of the sidewalls are the same, the asymptotic solution may be either stationary, with a flow field consisting of two pairs of superimposed roll cells, or periodic, with a flow pattern consisting of a primary cell and two secondary cells that pulsate about the center of the enclosure. Dimensionless heat transfer correlating equations are proposed.

In this context, the main aim of the present paper is to study the thermal behavior of tilted square enclosures that was locally one discrete heated from below and with different locations of the heating portion mounted symmetrically on the two vertical sides. The study is carried out numerically through a computational code based on the SIMPLE algorithm, which is used for the solution of the mass, momentum and energy transfer governing equations. Simulations are performed for different values of the Rayleigh number  $Ra$  in the range between 103 and 106, and of the different heater locations ( $0.25 \leq L_1/L \leq 0.5$ ) and Prandtl number, ( $0.01 \leq Pr \leq 10$ ). The influence of both parameters  $Ra$ ,  $Pr$  and location of heating portion on the flow pattern, on the local temperature distribution, and on the heat transfer rates across the enclosure are analyzed and discussed.

## II. MATHEMATICAL FORMULATION AND NUMERICAL COMPUTATION

The configuration of interest for the present study is shown in Fig. 1, which is two dimensional square enclosure with a side of length  $L$  and adiabatic top wall. The heat sources parts in the left, right and bottom walls of the cavity are maintained at a higher temperature  $T_h$ , whereas the other parts of this side walls are kept at a lower temperature  $T_c$ , with  $T_h > T_c$ . The length of the local heat source on the bottom wall is half the length of corresponding wall ( $L/2$ ), and the length of the partial heating on the vertical sides walls cavity is fourth ( $L/4$ ),

which is located between  $y=l_1$  and  $y=l_2$ . The position of the heat source of length  $\ell$  is controlled by  $s$  that varies from  $\ell/2$  to  $(L - \ell/2)$ .

Thermophysical properties of the fluid in the flow model assumed to be constant except the density variations causing a body force term in the momentum equation. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes, and to couple in this way the temperature field to the flow field.

The governing equations for the steady natural convection flow using conservation of mass, momentum and energy can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

(2)

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_c)$$

(3)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

(4)

With boundary conditions:

$$u(x, 0) = u(x, L) = u(0, y) = u(L, y) = 0,$$

$$v(x, 0) = v(x, L) = v(0, y) = v(L, y) = 0,$$

$$T(x, 0) = T(0, y) = T(L, y) = T_h, \text{ (on the heating portion)}$$

$T(x, 0) = T(0, y) = T(L, y) = T_c$ , (on the unheated portion)

$$\frac{\partial T}{\partial y}(x, L) = 0, 0 < x < L$$

(5)

here  $x$  and  $y$  are the distances measured along the horizontal and vertical directions, respectively;  $u$  and  $v$  are the velocity components in the  $x$ - and  $y$ -directions, respectively,  $T$  denotes the temperature;  $p$  is the pressure and  $\rho$  is the density;  $T_h$  and  $T_c$  are the temperature at hot and cold walls, respectively;  $L$  is the side of the square of cavity.

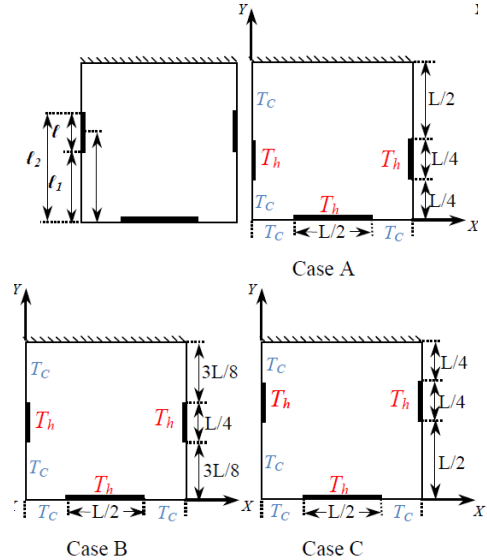


Fig. 1 Schematic of enclosure configurations

Using the following change of variables:

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha}, V = \frac{vL}{\alpha}, P = \frac{pL^2}{\rho\alpha^2},$$

$$\theta = \frac{(T-T_C)}{(T_h-T_C)}, \quad P_r = \frac{\rho}{\alpha}, \quad Ra = \frac{g\beta(T_h-T_C)L^3P_r}{v^2}$$

(6) After substitution of dimensionless variables (6) into equations (1)-(4) leads to:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + P_r \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + P_r \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + P_r Ra \theta$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{P_r} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)$$

with the boundary conditions

$$U(X, 0) = U(X, L) = U(0, Y) = U(L, Y) = 0,$$

$$V(X, 0) = V(X, L) = V(0, Y) = V(L, Y) = 0,$$

$$\theta(X, 0) = \theta(0, Y) = \theta(L, Y) = 1, \text{ (on the heating portion)}$$

$$\theta(X, 0) = \theta(0, Y) = \theta(L, Y) = 0, \text{ (on the unheated portion)}$$

$$\frac{\partial \theta}{\partial Y}(X, L) = 0, 0 < x < L$$

Here  $X$  and  $Y$  are dimensionless coordinates varying along horizontal and vertical directions, respectively;  $U$  and  $V$  are dimensionless velocity components in the  $X$ - and  $Y$ -directions, respectively;  $\theta$  is the dimensionless temperature;  $P$  is dimensionless pressure;  $Ra$  and  $Pr$  are Rayleigh and Prandtl numbers, respectively.

### III. STREAM FUNCTION AND NUSSELT NUMBER

The fluid motion is displayed using the stream function  $\Psi$  obtained from velocity components  $u$  and  $v$ . The relationships between stream function,  $\Psi$  and velocity components for two dimensional flows are defined as:

$$U = \frac{\partial \Psi}{\partial Y}, \quad V = -\frac{\partial \Psi}{\partial X}$$

This leads to a single equation:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$$

Using the above definition of the stream function, the positive sign of  $\Psi$  denotes anticlockwise circulation and the clockwise circulation are represented by the negative sign of  $\Psi$ .

The local heat transfer rate along the heated section of the wall is obtained from the heat balance that gives an expression for the local Nusselt number as:

$$Nu_h(X) = \frac{\partial \theta}{\partial Y} \Big|_{Y=0}, \quad L/4 \leq X \leq 3L/4$$

$$Nu_h(Y) = \frac{\partial \theta}{\partial X} \Big|_{X=0, X=L}, \quad l_1 \leq Y \leq l_2$$

The average Nusselt number on the heat source  $Nu_h$  is calculated by:

$$\overline{Nu_h}(X) = \frac{1}{L/2} \int_{L/4}^{3L/4} -\frac{\partial \theta}{\partial Y} \Big|_{Y=0} dX \quad ; L/4 \leq X \leq 3L/4$$

$$\overline{Nu_h}(Y) = \frac{1}{l} \int_{l_1}^{l_2} -\frac{\partial \theta}{\partial X} \Big|_{X=0, X=L} dY \quad ; l_1 \leq Y \leq l_2$$

### IV. SOLUTION PROCEDURE

The governing equations were solved using the finite volume technique developed by Patankar, [1980]. This technique was based on the discretization of the governing equations using the central difference in space. Throughout this study, the number of grids ( $100 \times 100$ ) was used. To calculate Nusselt number, we use numerical differentiations. The discretization equations were solved by the Gauss-Seidel method. The iteration method used in this program is a line-by-line procedure, which is a combination of the direct method and the resulting Tri Diagonal Matrix Algorithm (TDMA).

### V. PROGRAM VALIDATION AND COMPARISON WITH PREVIOUS RESEARCH

In order to check on the accuracy of the numerical technique employed for the solution of the problem considered in the present study, it was validated by performing simulation for natural convection flow in a vertical square enclosure which reported by **Iwatsu et al [1993]**. Fig. 2 shows the plots the streamlines, isotherms distributions with  $Pr=0.7$ ,  $Re=10^3$  and  $Gr=10^2$ , (a) for present code and (b) for **R. Iwatsu et al**. It can be seen from the comparison that the present work is in a very good agreement with the previous works.

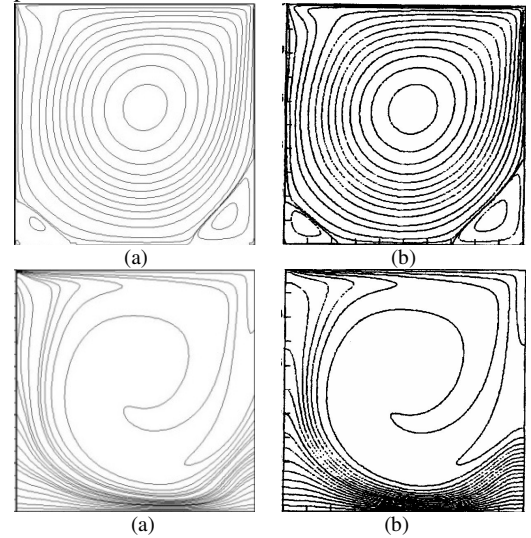


Fig. 2 Stream lines and isotherms for square cavity,  $Pr=0.7$ ,  $Re=103$ ,  $Gr=102$  ( $Ri=0.0001$ ) (a) present work (b) results of Iwatsu, et al [1993]

Another validation check was performed by comparing the results with the results for the benchmark solution of **De Vahl [1983]** for non-conjugate natural convection in an air filled square cavity. Table 1 shows the comparison between the results of two solutions.

TABLE I  
COMPARISON OF THE PRESENT NUMERICAL RESULTS WITH THE SOLUTION [DE VAHL 1983]

$Ra$	De Vahl Davis	Present study
	$ \Psi _{max}$	$ \Psi _{max}$
$10^5$	9.612	9.5898
$10^6$	16.750	16.719

Fig. 3 plots the predicted values for the streamlines, isotherms distributions and dimensionless velocity components in the  $X$ - and  $Y$ -directions, respectively over a range for Rayleigh number,  $Ra$ ,  $10^3$  to  $10^6$  for the present solution and the results published by *De Vahl* [1983]. It can be seen from the comparison that the present work is in a very good agreement with the previous works.

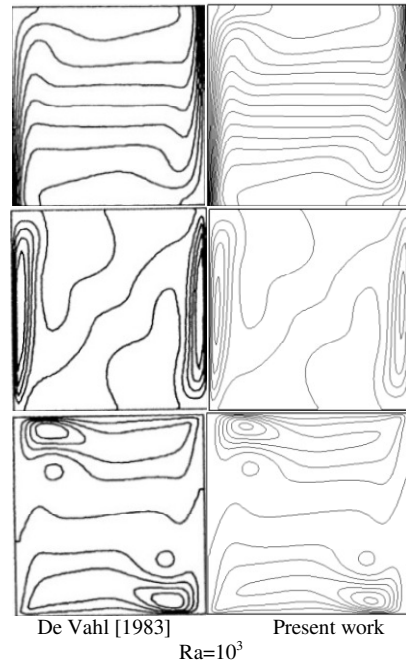
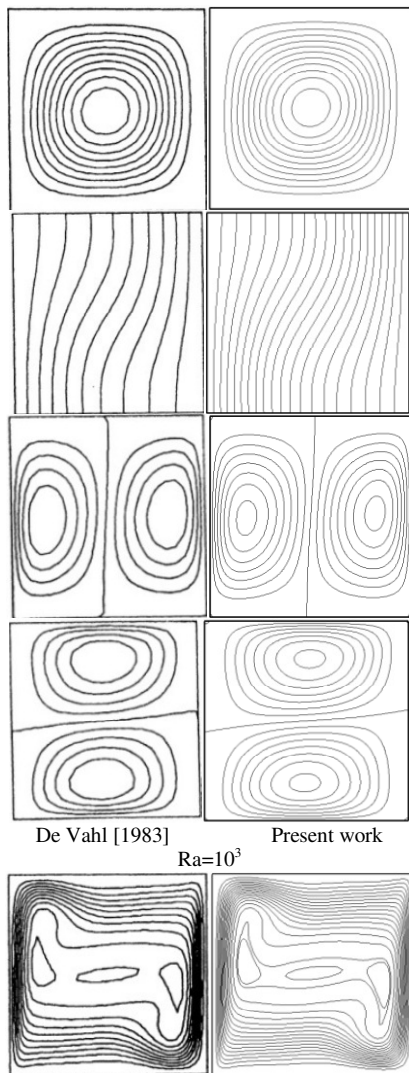
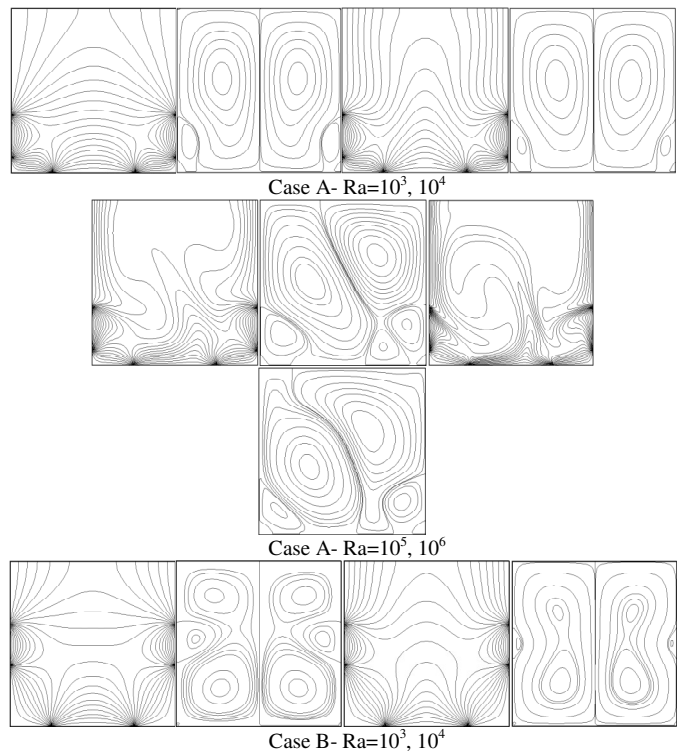


Fig. 3 Comparison of the streamlines, isotherms, calculated horizontal and vertical velocity with experimental data of [18].

## VI. RESULTS AND DISCUSSION

The numerical results for the streamline and isotherms contours for various values of thermal Rayleigh number and the heater location, will be presented and discussed. In addition, the results for both average Nusselt, and velocity profiles, at various conditions will be presented and discussed.



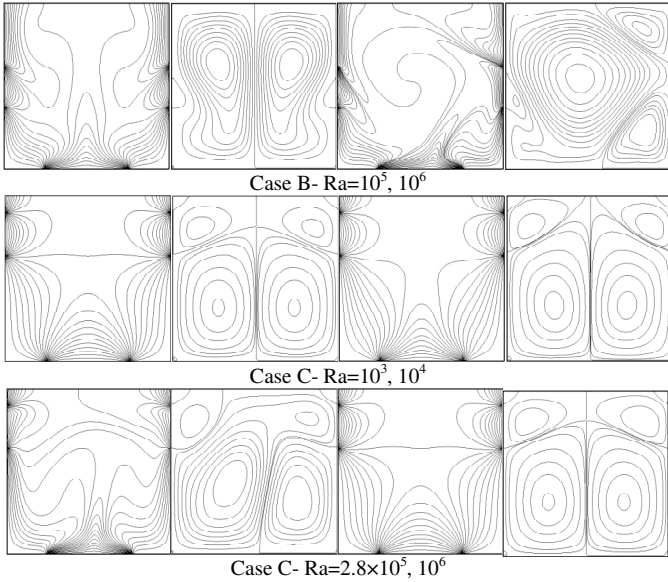


Fig. 4 The effect of Rayleigh number on Stream line and Isotherms

Fig. 4 shows the effect of Rayleigh number on Stream functions and Isothermal lines for  $Pr=0.71$  and for three locations of partial thermally active vertical walls as Rayleigh number varies from  $10^3$  to  $10^6$ .

It is clear that, for the case A (Lower\_Lower heat source location), the flow consists of a four symmetric cellular stream function with ( $\Psi_{max} = 0.10885$  at  $Ra=10^3$  and  $\Psi_{max} = 2.2594$  at  $Ra=10^4$ ). The magnitudes of stream function are very low and the heat transfer is primarily due to conduction. Owing to the symmetrical boundary conditions on the vertical walls, the flow and temperature fields are symmetrical about the mid-length of the enclosure. The symmetrical boundary conditions in the vertical direction result in a two pair of counter-rotating cells in the left and right halves of the enclosure for all the parametric values considered. Owing to the symmetry, the flows in the left and right halves of the enclosure are identical except for the sense of rotation.

Each big cell ascends through the symmetry axis, the faces the upper adiabatic wall through which it moves horizontally toward the corresponding cold wall and finally it descends along the corresponding cold wall. The cold part of the vertical wall above the heater, suppresses this buoyancy effect. While each small cell in the two lower corner of enclosure ascends toward the heat source mounted at the lower vertical wall, then it descends along the big cell. The isotherms are clustered near the partial heating at the lower surface, and the symmetrical heat source in the vertical wall.

The analysis of Streamlines and isotherms for the second case (case B: Middle\_Middle heat source location) indicates that when the heating source is located at the center,  $s = L/2$ , various differences occur in the flow pattern according to the change in  $Ra$  number. It can be seen that for  $Ra=10^3$  two strong eddies appear inside the enclosures. They circulate counterclockwise at the left of the enclosure and clockwise at the right, where the intensity of both eddies is identical

because the symmetry in boundary conditions in the vertical walls, flow and heat transfer are controlled by the local heat source and the difference in temperature on the vertical walls has a negligible influence, this influence become significant with the increasing of  $Ra$  number. For  $Ra = 10^6$  the enclosures have two very weak clockwise eddy at the upper and lower corner of the right side and the only larger eddy is a counterclockwise flow moving upwards along the right wall and downwards along the left wall. Note that the increase in temperature differences between the vertical walls ( $Ra$  number) affects the fluid dynamic behavior, increasing the intensity of the flow in the enclosure.

As  $Ra$  increases to  $10^5$  then  $10^6$ , the buoyancy forces become strong and the heat transfer is dominated by convection for  $Pr=0.71$ . It is interesting to observe that the fluid flows in four symmetric circulation cells along with tiny circulations at the central region of the side walls (for case B), at the lower (case A) and at the upper (case C) corners.

Fig. 6 shows the average Nusselt number as a function of the Rayleigh number along the horizontal mid-plane of the center of the heat source mounted in the vertical wall for  $s = 3L/8$  (case A),  $s = L/2$  (case B) and for  $s = 5L/8$  (case C).

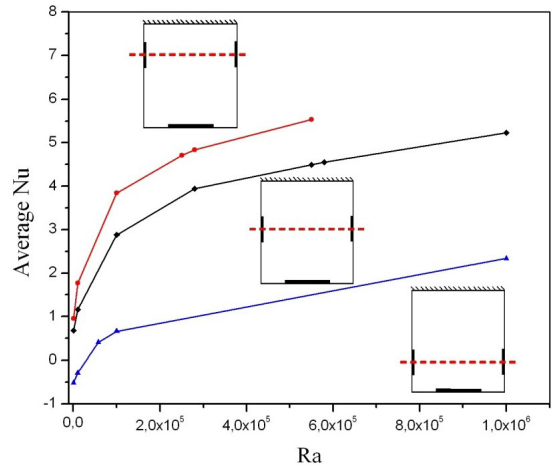


Fig. 6 The average Nusselt number versus Rayleigh number

It is noted that when  $Ra$  is smaller than  $10^3$ , the  $Nu$  nearly tends to unity, which indicates that heat conduction prevails over convection for the case of very small  $Ra$ , on the other words, very small scale. It is also found from Fig. 6 that the  $Nu$  increases more quickly with  $Ra$  in the range of  $10^3$  to  $10^4$  than that in the range of  $10^5$  to  $10^6$ . It is also important to underline a difference between the values recorded for  $s = 3L/8$  and for  $s = 5L/8$ . Larger average Nusselt numbers were recorded for the case C (the heat source are mounted in upper of the vertical wall) compared with the numbers found for the other locations of heat source (case A and case B). Therefore there is a more efficient heat transfer for  $s = 5L/8$  than for  $s = 3L/8$  and for  $s = L/2$ . These results can be clearly explained under the views of the isotherms given in Fig. 4.

## VII. CONCLUSIONS

A steady, two-dimensional natural convection phenomena of air in a square cavity with different locations of the heating portion have been simulated numerically using the finite volume method. A parametric study was undertaken and the effects of the position of active portions on the side walls and Rayleigh number on the heat transfer inside the cavity were investigated. The results show that, for all the Rayleigh numbers considered, the minimum average Nusselt number occurred for the case when the heat source located in the lower of the side walls (Case A), but the maximum average Nusselt number occurred for the upper position case.

The flow and temperature fields are symmetrical about the mid-length of the enclosure due to the symmetry of the boundary conditions in the vertical direction, excepting for the case A at  $R_{ac}=5.8 \times 10^4$  and case B at  $R_{ac}=10^6$ , this values of  $R_{ac}$  made to get the unsymmetrical steady solution, because it is very unstable and with using of some disturbances it is transitioned to unsteady oscillatory solution. For small Ra, the heat transfer is dominated by conduction across the fluid layer, while for high Ra the process is primarily one of convection, and the effect of conduction vanishes.

It is observed that the heat transfer is an increasing function of Rayleigh number. The influence of position of heat source is particularly strong in the range of Ra of  $10^3$  to  $10^4$ .

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