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A New Stability Analysis for Discrete-time Switched Time-delay Systems

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Abstract— **This paper addresses on the asymptotic stability of a class of discrete-time switched time-delay systems. Those systems to be studying are described by delay difference equations which are represented in the state from. Then, another transformation is made towards an arrow form. Therefore, by applying the Kotelyanski lemma and the M-matrix properties, new sufficient stability conditions are established under arbitrary switching. These obtained stability conditions correspond to a vector Lyapunov function. Finally, a numerical example is presented permitting to understand the application of the proposed approach.**

Index Terms— **Discrete-time switched time-delay systems, Global asymptotic stability, M-matrix, Kotelyanski lemma, Arrow matrix, Arbitrary switching.**

I. INTRODUCTION

Time delay is frequently viewed as a source of instability and encountered in various engineering systems such as aircraft stabilization, chemical engineering systems, inferred grinding model, manual control, neural network, nuclear reactor, population dynamic model, rolling mill, ship stabilization, and systems with lossless transmission lines [1–3]. Frequently, time delay is caused by finite speed in energy propagation and may destroy the stability or the performance of systems.

In recent years, switched systems have received growing attention. Switched systems are an important class of hybrid dynamical, which consist of a finite number of subsystems described by differential or difference equations and a switching signal that orchestrates switching between these sub-systems. The motivation for studying switched systems comes partly from the fact that switched systems have numerous applications in control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, and many other fields, many important and interesting results have been proposed in terms of all kinds of techniques [4-9]. Since time delay frequently appears in the real systems and is a source of the poor performance and even instability. Hence, it is for great importance to investigate switched time-delay systems.

The stability analysis switched linear time-delay systems nowadays present a theoretical challenge, which has attracted growing attention in the literature. For this, many methods have been employed and many interest results have been obtained [10-17].

Generally, stability under arbitrary switching remains a fundamental issue in practical switched systems. In this context, it is well known that, the existence of a common Lyapunov function for all the subsystems, is a sufficient condition to guarantee the stability of the switched system under arbitrary switching law. Despite that it has some attempts for construction to a common Lyapunov function [18, 19], by intruding in order the Lyapunov Krasovskii functionals and the linear matrix inequity (LMI) approach. Although, this method is usually very difficult to apply, even impossible in many instance.

Motivated by these gaps, as well in the sense of various methods and for getting a larger stability domain. This paper, tented to established new stability conditions for a class of discrete-time switched time-delay, by transforming the representation of the system to be studying into another specific form, and by using an appropriate Lyapunov function associated with the Kotelyanski conditions [20-27] and the $M-$matrix proprieties [28-30]. Therefore, the obtained stability conditions are simple to employ, explicit and allow us to avoid the search for a common Lyapunov function.

Within the frame of studying the stability analysis, the appropriate Lyapunov function combined with an $M-$matrix proprieties has already been introduced in [21, 22] in reference to the continuous-time time delay systems in a field related to the study of convergence.

In this work, the same approach will used for a class of discrete-time switched time-delay systems given by a set of subsystems, each one is modelled by the following delay difference equation:

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$$y(k+n)+\left(\sum_{j=0}^{n-1} a_i^{n-j} y(k+j)+\sum_{j=0}^{n-1} d_i^{n-j} y(k+j-\tau)\right)=0 \quad (1)$$

when $i=1,...,N$ represented the subsystems.

The remainder of this paper is organized as follows: in the next section, we present the description and the problem formulation of the studied switched systems. In section III, sufficient stability conditions of these discrete-time switched time-delay systems based on an appropriate Lyapunov function associate with an $M-$matrix properties are presented. In section IV, we show the efficiency of the obtained results, by applying them to systems modeled in (1). In section V, a validation on numerical example is drawn, and finally, some concluding remarks are summarized in section VI.

II. PROBLEM FORMULATION AND PRELIMINAIES

A. Preliminaries

The following notations will be used throughout. I is an identity matrix with appropriate dimension. Let $\Re^n$ denoted an n- dimensional linear vector space over the reals with the norm $\|.\|$. For any $u=(u_i)_{1\le i\le n}$, $v=(v_i)_{1\le i\le n}\in\Re^n$, we define the scalar product of the vector u and v as: $\langle u,v\rangle=\sum_{i=1}^{n}u_i v_i$.

In the next, we introduce several useful tools, including Kotelyanski lemma and definition of an M-matrix.

Kotelyanski lemma. [20] The real parts of the eigenvalues of matrix A, with non-negative off-diagonal elements, are less than a real number μ if and only if all those of matrix M, $M=\mu I_n - A$ are positive, with I the n identity matrix.

When successive principal minors of matrix $(-A)$ are positive, Kotelyanski lemma permits to conclude on stability property of the system characterized by A.

Theorem 1. [22]The matrix $A\in\Re^{n\times n}$ is called a $M-$matrix if following properties are verified:

- All the eigenvalues of A have a positive real part
- The real eigenvalues are positives
- Successive the principal minors of A are positive :

$$(A)\begin{pmatrix}1\ 2\ ...\ i\\ 1\ 2\ ...\ i\end{pmatrix}>0 \quad \forall i\in 1,...,n \tag{2}$$

- For any positive vector $x=(x_1,...,x_n)^T$ the algebraic equations Ax have a positive solution $w=(w_1,...,w_n)^T$

Remark 1. A discrete-time system characterized by a matrix A is stable if the matrix $(I-A)$ verified the Kotelyanski conditions, in this case $(I-A)$ is an $M-$matrix .

B. Problem formulation

Consider the following discrete-time switched systems time-delay formed by N subsystems represented in the state form:

$$\begin{cases} x(k+1)=\sum_{i=1}^{N}\zeta_i(k)\left(A_i x(k)+D_i x(k-\tau)\right) \\ x(s)=\phi(s) \quad s=-\tau,-\tau+1,...,0 \end{cases} \tag{3}$$

where $x(k)\in\Re^n$ is the state vector of the system at time k, τ is the time delay of state, $x(s)$ denote the initial states vector, $A_i\,(i=1,...,N)$ and $D_i\,(i=1...N)$ are matrices of appropriate dimensions denoting the subsystems, and $N\ge 1$ denotes the number of subsystems.

The switching sequence is defined through a switching vector: $\zeta(k)=\left[\zeta_1(k),...,\zeta_N(k)\right]^T$, whose components $\zeta_i(k)$ are exogenous functions that depend only on the time and not on the state, they are defined through:

$$\zeta_i(k)=\begin{cases}1 & \text{if } A_i \text{ and } D_i \text{ are active}\\ 0 & \text{otherwise}\end{cases} \quad \text{and} \quad \sum_{i=1}^{N}\zeta_i(k)=1 \tag{4}$$

III. MAIN RESULTS

The following theorem gives stability conditions for the discrete-time switched time-delay system (3).

Theorem 2. The system (3) is asymptotically stable under arbitrary switching rule (4) if the matrix $(I-T_c)$ is an $M-$matrix .

where:

$$T_c=\sup_{1\le i\le N}\left(|A_i|+|D_i|\right) \tag{5}$$

The matrices A_i, $i=1,...,N$ and D_i, $i=1,...,N$ are given as following:

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$$A_i = \begin{bmatrix} \left(a_i^{11}\right) & \dots & \dots & \left(a_i^{1n}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \left(a_i^{n1}\right) & \dots & \dots & \left(a_i^{nn}\right) \end{bmatrix} \tag{6}$$

$$D_i = \begin{bmatrix} \left(d_i^{11}\right) & \dots & \dots & \left(d_i^{1n}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \left(d_i^{n1}\right) & \dots & \dots & \left(d_i^{nn}\right) \end{bmatrix} \tag{7}$$

■

Proof: For any switched signal (4). Let $\left(w_l > 0, \forall l = 1,...,n\right)$, we consider the Lyapunov function defined such as:

$$v(k) = v_0(k) + \sum_{j=1}^{r} v_j(k) \tag{8}$$

where:

$$\begin{cases} v_0(k) = \left\langle \left|x(k)\right|, w \right\rangle \\ v_j(k) = \left\langle \left|D_i\right| \left|x(k-j)\right|, w \right\rangle,\ j = 1,...,r \end{cases} \tag{9}$$

To prove the stability of system (3) in the sense of Lyapunov, it suffices to show that:

$$\Delta v(k) = v(k+1) - v(k) < \left\langle \left(T_c\right) \left|x(k)\right|, w \right\rangle,\ r > 0 \tag{10}$$

where:

$$\Delta v(k) = \Delta v_0(k) + \sum_{j=1}^{r} \Delta v_j(k) \tag{11}$$

For any $r > 0$, we get from (9) that:

$$\Delta v_0 = \left\langle \left|x(k+1)\right|, w \right\rangle - \left\langle \left|x(k)\right|, w \right\rangle \tag{12}$$

and:

$$\begin{aligned} \Delta v_j = {} & \left\langle \left|D_i\right| \left|x(k-j+1)\right|, w \right\rangle \\ & - \left\langle \left|D_i\right| \left|x(k-j)\right|, w \right\rangle,\ j = 1,...,r \end{aligned} \tag{13}$$

Knowing that:

$$\begin{aligned} \left\langle \left|x(k+1)\right|, w \right\rangle &= \left\langle \left|A_i x(k) + D_i x(k-r)\right|, w \right\rangle \\ &< \left\langle \left|A_i\right| \left|x(k)\right| + \left|D_i\right| \left|x(k-r)\right|, w \right\rangle \\ &= \left\langle \left|A_i\right| \left|x(k)\right|, w \right\rangle + \left\langle \left|D_i\right| \left|x(k-r)\right|, w \right\rangle \end{aligned} \tag{14}$$

So, we have:

$$\begin{aligned} \sum_{j=1}^{r} \Delta v_j(k) = {} & \Delta v_1(k) \\ & + \Delta v_2(k) + ... + \Delta v_{r-1}(k) + \Delta v_r(k) \end{aligned} \tag{15}$$

Equation (14) yields:

$$\sum_{j=1}^{r} \Delta v_j(k) = \left\langle \left|D_i\right| \left|x(k)\right|, w \right\rangle - \left\langle \left|D_i\right| \left|x(k-r)\right|, w \right\rangle \tag{16}$$

Combining (11), (12) and (16), it follows that:

$$\Delta v(k) = \Delta v_0 + \left\langle \left|D_i\right| \left|x(k)\right|, w \right\rangle - \left\langle \left|D_i\right| \left|x(k-r)\right|, w \right\rangle \tag{17}$$

For $i = 1,...,N$, by (14) and (17), we have that:

$$\begin{aligned} \Delta v(k) < {} & \left\langle \left|A_i\right| \left|x(k)\right|, w \right\rangle + \left\langle \left|D_i\right| \left|x(k-r)\right|, w \right\rangle \\ & - \left\langle \left|x(k)\right|, w \right\rangle + \left\langle \left|D_i\right| \left|x(k)\right|, w \right\rangle - \left\langle \left|D_i\right| \left|x(k-r)\right|, w \right\rangle \end{aligned} \tag{18}$$

That is:

$$\begin{aligned} \Delta v(k) &< \left\langle \left|A_i\right| \left|x(k)\right|, w \right\rangle - \left\langle \left|x(k)\right|, w \right\rangle + \left\langle \left|D_i\right| \left|x(k)\right|, w \right\rangle \\ &= \left\langle \left(\left|A_i\right| + \left|D_i\right| - I\right) \left|x(k)\right|, w \right\rangle,\ i = 1,...,N \end{aligned} \tag{19}$$

and finally we obtain:

$$\Delta v(k) = \left\langle \left(T_c - I\right) \left|x(k)\right|, w \right\rangle \tag{20}$$

where the matrix T_c is defined in (5).

Suppose now that $\left(I - T_c\right)$ is an $M-$matrix , according to the proprieties of the $M-$matrix given in Theorem 1, we can find a vector $\rho \in \Re_+^{*n} \left(\rho_l \in \Re_+^*\ \ l = 1,...,n\right)$ satisfying the relation:

$$\left(I - T_c\right)^T w = \rho, \forall w \in \Re_+^{*n} \tag{21}$$

On the other hand, we can write:

$$\left\langle \left(I - T_c\right) \left|x(k)\right|, w \right\rangle = \left\langle \left(I - T_c\right)^T w, \left|x(k)\right| \right\rangle = \left\langle \rho, \left|x(k)\right| \right\rangle \tag{22}$$

Combining (20), (21) and (22), it follows that:

$$\Delta v(k) \le \left\langle (T_c - I)\left|x(k)\right|, w \right\rangle \le -\sum_{l=1}^{n} \rho_l \left|x_l(k)\right| < 0 \qquad (23)$$

System (3) is asymptotically stable. The proof is completed.

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IV. APPLICATION TO DISCETE-TIME SWITCHED TIME-DELAY SYSTEMS DEFINED BY DIFFERANCE EQUATIONS

In this part, an application of the obtained results is given for discrete-time switched time delay systems governed by the following switched linear difference equation:

$$y(k+n) + \sum_{i=1}^{N} \zeta_i(k) \left(\sum_{p=0}^{n-1} a_i^{n-p} y(k+p) + \sum_{p=0}^{n-1} d_i^{n-p} y(k+p-\tau) \right) = 0 \qquad (24)$$

where $\zeta_i(k)$ are the components of the switching function $\zeta(k)$, $i = 1,...,N$, given in (4).

Therefore, the presence of delay-time terms makes the stability analysis of problem (24) difficult. Among solution, we will adopt the following change of variable:

$$x_{j+1}(k) = y(k+j) \quad j = 0,...,n-1 \qquad (25)$$

By (25), equation (24) becomes:

$$x_j(k+1) = x_{j+1}(k) \quad j = 1,...,n-1 \qquad (26)$$

$$x_n(k+1) = \sum_{i=1}^{N} \zeta_i(k) \left(-\sum_{j=0}^{n-1} a_i^{n-j} x_{j+1}(k) - \sum_{j=0}^{n-1} d_i^{n-j} x_{j+1}(k-\tau) \right) \qquad (27)$$

or under a matrix representation:

$$\begin{cases} x(k+1) = \sum_{i=1}^{N} \zeta_i(k) \left(A_i x(k) + D_i x(k-\tau) \right) \\ x(s) = \phi(s) \quad s = -\tau,...,-1,0 \end{cases} \qquad (28)$$

where $x(k)$ is the state vector.

The matrices A_i and D_i, $i = 1,...,N$ are defined as following:

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ -a_i^n & -a_i^{n-1} & \cdots & -a_i^1 \end{bmatrix} \qquad (29)$$

$$D_i = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ -d_i^n & -d_i^{n-1} & \cdots & -d_i^1 \end{bmatrix} \qquad (30)$$

So, we define two associated polynomials for the subsystem(i), $i = 1,...,N$ defined by:

$$p_{A_i}(\lambda) = \lambda^n + \sum_{q=0}^{n-1} a_i^{n-q} \lambda^q \qquad (31)$$

$$p_{D_i}(\lambda) = \sum_{q=0}^{n-1} d_i^{n-q} \lambda^q \qquad (32)$$

In [22], a change of base for the system (28) under the arrow form gives:

$$z(k+1) = \sum_{i=1}^{N} \zeta_i(k) \left(\tilde{A}_i z(k) + \tilde{D}_i z(k-\tau) \right) \qquad (33)$$

where $z = Px$, $\tilde{A}_i, (i = 1,...,N)$ and $\tilde{D}_i$ $(i = 1,...,N)$ are matrices in the arrow form represented as following :

$$\tilde{A}_i = P^{-1} A_i P = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \alpha_{n-1} & \beta_{n-1} \\ \gamma_i^1 & \cdots & \cdots & \gamma_i^{n-1} & \gamma_i^n \end{bmatrix} \qquad (34)$$

$$\tilde{D}_i = P^{-1} D_i P = \begin{bmatrix} 0_{n-1,n-1} & 0_{n-1,1} \\ \delta_i^1 \cdots \delta_i^{n-1} & \delta_i^n \end{bmatrix} \qquad (35)$$

and P is the corresponding passage matrix defined by:

$$P=\begin{bmatrix} 1 & 1 & \cdots & 1 & 0 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 0 \\ (\alpha_1)^2 & (\alpha_2)^2 & \cdots & (\alpha_{n-1})^2 & \vdots \\ \vdots & \vdots & \cdots & \vdots & 0 \\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \cdots & (\alpha_{n-1})^{n-1} & 1 \end{bmatrix} \quad (36)$$

The elements of the matrix $\tilde{A}_i$ are defined by:

$$\beta_j = \prod_{\substack{q=1 \\ q\neq j}}^{n-1} \left(\alpha_j - \alpha_q\right)^{-1} \ \forall\ j=1,...,n-1 \quad (37)$$

$$\begin{cases} \gamma_i^j = -P_{A_i}\left(\alpha_j\right) \ \forall\ j=1,...,n-1 \\ \gamma_i^n = -a_i^1 - \sum_{j=1}^{n-1} \alpha_j \end{cases} \quad (38)$$

and the elements of the matrix $\tilde{D}_i$ are:

$$\begin{cases} \delta_i^j = -p_{\tilde{D}_i}\left(\alpha_j\right) \ j=1,...,n-1 \\ \delta_i^n = -d_i^1 \end{cases} \quad (39)$$

where $\alpha_j \left(j=1,...n-1\right)$ are free real parameters, distinct in pairs, that can be chosen arbitrary.

After this formulation, we can deduce the following theorem for the stability of system (28).

Theorem 3. The discrete-time switched time-delay system (28) is globally asymptotically stable if there exist $\alpha_j \ \ (j=1,2,...,n-1)$, $\alpha_j \neq \alpha_q \ \forall\ j\neq q$, such as:

i) $1-\left|\alpha_j\right| > 0 \ \forall\ j=1,2,...n-1 \quad (40)$

ii) $$1-\left(\sup_{1\le i\le N}\left(\left|\gamma_i^n\right|+\left|\delta_i^n\right|\right)\right) - \sum_{j=1}^{n-1}\left(\sup_{1\le i\le N}\left(\left|\gamma_i^j\right|+\left|\delta_i^j\right|\right)\right)\left|\beta_j\right|\left(1-\left|\alpha_j\right|\right)^{-1} > 0 \quad (41)$$

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Proof: It suffices to verify that the matrix $\left(I-T_c\right)$ is an $M-$matrix.

where:

$$T_c = \sup_{1\le i\le N}\left(\left|\tilde{A}_i\right|+\left|\tilde{D}_i\right|\right) \quad (42)$$

Taking into account the previous matrix value; we obtain the matrix T_c as follows:

$$T_c = \begin{pmatrix} |\alpha_1| & & & & |\beta_1| \\ & |\alpha_2| & & & |\beta_2| \\ & & \ddots & & \vdots \\ & & & |\alpha_{n-1}| & |\beta_{n-1}| \\ t_c^1 & t_c^2 & \cdots & t_c^{(n-1)} & \left(\sup_{1\le i\le N}\left(\left|\gamma_i^n\right|+\left|\delta_i^n\right|\right)\right) \end{pmatrix} \quad (43)$$

where:

$$t_c^j = \sup_{1\le i\le N}\left(\left|\gamma_i^j\right|+\left|\delta_i^j\right|\right) \ \forall j=1,...,n-1 \quad (44)$$

Since the elements $\alpha_j \ j=1,...,n-1$ can be arbitrarily selected, the choice $\left|\alpha_j\right|\in\left]0,1\right[$ with $\alpha_j\neq\alpha_q \ \forall j=1,...,n-1$, The matrix T_c has all positive elements. Thus, the conditions of Theorem 2 can be deduced from the Kotelyanski conditions in the discrete-time case applied to the matrix $\left(I-T_c\right)$.

The $n-1$ first conditions are checked because $\left|\alpha_j\right|\in\left]0,1\right[\ j=1,...,n-1$, however the last condition however the last condition yields to:

$$\det\left(I-T_c\right)= \begin{vmatrix} 1-|\alpha_1| & & & & -|\beta_1| \\ & 1-|\alpha_2| & & & -|\beta_2| \\ & & \ddots & & \vdots \\ & & & 1-|\alpha_{n-1}| & -|\beta_{n-1}| \\ -t_c^1 & -t_c^2 & \cdots & -t_c^{(n-1)} & 1-\left(\sup_{1\le i\le N}\left(\left|\gamma_i^n\right|+\left|\delta_i^n\right|\right)\right) \end{vmatrix} > 0 \quad (45)$$

$$= 1-\left(\sup_{1\le i\le N}\left(\left|\gamma_i^n\right|+\left|\delta_i^n\right|\right)\right) - \sum_{j=1}^{n-1}\left(\sup_{1\le i\le N}\left(\left|\gamma_i^j\right|+\left|\delta_i^j\right|\right)\right)\left|\beta_j\right|\left(1-\left|\alpha_j\right|\right)^{-1} > 0$$

To simplify the application of the obtained stability conditions, Theorem 2 can be simplified in the following corollary.

Corollary 1. The discrete-time switched time-delay system (28) globally asymptotically stable if there exist $\alpha_j\in\left]0\ 1\right[\ \left(j=1,...,\ n-1\right)$, $\alpha_j\neq\alpha_q \ \forall\ j\neq q$ such as in:

i) $\beta_j\left(P_{A_i}\left(\alpha_j\right)+P_{D_i}\left(\alpha_j\right)\right)<0$, $\forall i=1,...,N \quad (46)$

ii) $\left(P_{A_i}(\lambda=1)+P_{D_i}((\lambda=1))\right)>0$, $\forall i=1,...,N$ (47)

iii) $\gamma_i^n+\delta_i^n>0$, $\forall i=1,...,N$ (48)

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V. A NUMERICAL EXAMPLE

In this section, a numerical example is studied to show the effectiveness of the proposed method.

Example. Consider a discrete-time switched time-delay system described by the recurrence equation given by:

$$y(k+2)+\sum_{i=1}^{2}\zeta_i\left(\sum_{j=0}^{1}a_i^{2-j}y(k+j)+\sum_{j=0}^{1}d_i^{2-j}y(k+j-1)\right)=0$$

The time-delay is fixed to $\tau=1$.

Now, by (25), (26), (27) and (28); this system will be give under the following matrix representation:

$$x(k+1)=\sum_{i=1}^{2}\zeta_i\left(\begin{pmatrix}0 & 1\\ -a_i^2 & -a_i^1\end{pmatrix}x(k)+\begin{pmatrix}0 & 0\\ -d_i^2 & -d_i^1\end{pmatrix}x(k-1)\right)$$

where $i=1,2$, and the matrices are listed below:

$$A_1=\begin{bmatrix}0 & 1\\ 0.01 & -0.02\end{bmatrix},\qquad D_1=\begin{bmatrix}0 & 0\\ 0.01 & -0.08\end{bmatrix}$$

and:

$$A_2=\begin{bmatrix}0 & 1\\ 0.07 & -0.02\end{bmatrix},\qquad D_2=\begin{bmatrix}0 & 0\\ 0.05 & -0.04\end{bmatrix}$$

Therefore, according to (33), (34), (35), (36), (37), (38) and (39) a change of base for the discrete-time switched time-delay system under the arrow form gives the following parameters and matrices:

$$\tilde{A}_1=\begin{bmatrix}\alpha & 1\\ \gamma_1^1 & \gamma_1^2\end{bmatrix},\ \tilde{A}_2=\begin{bmatrix}\alpha & 1\\ \gamma_2^1 & \gamma_2^2\end{bmatrix}$$

and:

$$\tilde{D}_1=\begin{bmatrix}0 & 0\\ \delta_1^1 & \delta_1^2\end{bmatrix},\ \tilde{D}_2=\begin{bmatrix}0 & 0\\ \delta_2^1 & \delta_2^2\end{bmatrix}.$$

Then, the stability conditions deduced from theorem 3 are:

i) $|\alpha|<1$

ii) $1-\sup\left(|\gamma_1^2|+|\delta_1^2|,|\gamma_2^2|+|\delta_2^2|\right)$
$-\left(\sup\left(|\gamma_1^1|+|\delta_1^1|,|\gamma_2^1|+|\delta_2^1|\right)\right)|\beta|(1-|\alpha|)^{-1}>0$

For a particular choice, $\alpha=0.05$, $\beta=1$.

In this case, condition (ii) is verified such as:

$$1-\sup\left(|-0.03|+|0.04|,|-0.03|+|0.08|\right)$$
$$-\sup\left(|0.0065|+|0.006|,|0.0665|+|0.048|\right)(1.052)$$
$$=1-(0.03-0.08)-(0.0665+0.048)(1.052)=0.769546>0$$

The stability conditions for the example given by corollary 1 are the following:

i) $|\alpha|<1$

ii) $\beta\left(P_{A_1}(\alpha)+P_{D_1}(\alpha)\right)<0$

iii) $\beta\left(P_{A_2}(\alpha)+P_{D_2}(\alpha)\right)<0$

iv) $P_{A_1}(1)+P_{D_1}(1)>0$

v) $P_{A_2}(1)+P_{D_2}(1)>0$

vi) $\gamma_1^2+\delta_1^2>0$

vii) $\gamma_2^2+\delta_2^2>0$

For the same values of $\alpha=0.05$, then condition (ii), (iii), (iv), (v), (vi), (vii) are verified such as:

i) $-0.0125<0$, ii) $-0.1145<0$, iii) $0.36>0$, iv) $1.04>0$, v) $0.94>0$, vi) $0.05>0$ and vii) $0.01>0$

With fixed the sampling time $T_e=0.2s$, $t_f=kT_e=10s$ the switched time $t_1=k_1T_e=5s$ and the original state vector $\phi(l)=[-1\ \ 1]^T$.

Then, the evolution of the states and the state space are given in figure 1 and figure 2, respectively.

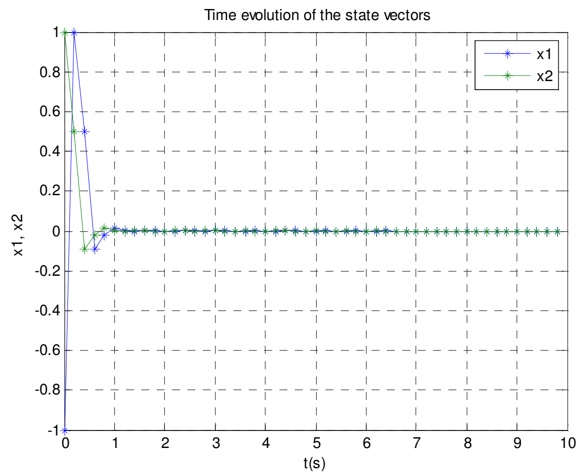

Fig. 1. Time evolution of the state vector for the example

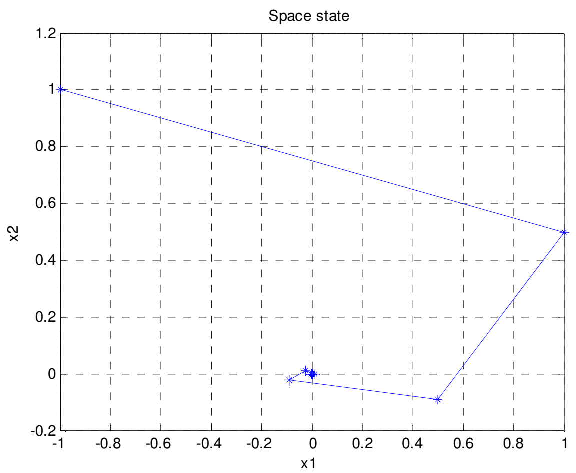

Fig. 2. Space state for the example

VI. CONCULSION

This paper has investigated new delay-independent explicit stability conditions for discrete-time switched time-delay systems under arbitrary switching.

These conditions were deduced from an appropriate Lyapunov function associated with the Kotelyanski conditions and the $M-$matrix proprieties. The main benefit of this technique that it avoids the problem of existence of Lyapunov functions. Finally, the effectiveness of the proposed method is illustrated by a numerical example.

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