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Actuator and sensor faults reconstruction for a class of output time-delay system based on Sliding Mode Observer

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Abstract— **In this paper we present a design method of Sliding Mode Observer (SMO) applied to reconstruct both actuator and sensor faults for a class of output time-delay systems. To guarantee the quadratically stability of the estimation error dynamics, sufficient conditions are developed with the Lyapunov-Krasovskii approach arising a set of Linear Matrix Inequality (LMI) optimization. Applying the equivalent output error injection, the reconstruction of actuator and sensor faults are obtained. To show the efficacy of the SMO design and faults reconstruction methods, a numerical example is proposed.**

keywords- Sliding mode observer; actuator fault reconstruction; sensor fault reconstruction; time-delay system; LMI technique

I. INTRODUCTION

Fault Detection and Isolation (FDI) has received a considerable attention in the last decades. Many solutions have been proposed [1-5] to solve this problem using different approaches such as the mathematical models [6-7]. The FDI based on SMO has attracted great attention due to its robustness properties against perturbations and parameters variations [8-10]. Consequently, many design methods based on SMO are successfully applied to solve this problem [11-17].

Its well known that time-delay can causes some problem such as the instability. Furthermore, due to its robustness, various researchers have applied this observer for time-delay systems based on control [18] and state estimation [19]. In addition, the analysis and the synthesis of SMO for time-delay systems becomes an important issue of many works [20]. Then, many authors considered the effect of state or/and input time delay in the design of SMO for reconstruction and fault detection [21-24]. In this context, an observer for fault detection for a class of two-level distributed networked control systems with time-delay is presented in [21]. Moreover, in [22] a robust delay- derivative-dependant SMO for fault reconstruction for linear uncertain time-varying delay systems is proposed. Then, in [23] a scheme for estimating the actuator and sensor fault for Lipschitz nonlinear systems using SMO is presented. Later, in [24], a SMO for fault detection and minimization of computation time-delay effect is proposed, where the time-delay is treated as a fault.

However, compared with the rich results in FDI based SMO of linear systems, few research results are addressed on the FDI for output time-delay systems, and this motivates our work. In [25], SMO design method for robust fault reconstruction in the presence of sampled output information proposed. Also, For a class of certain system with output time-delay a FDI approach using SMO is presented in [26].

The main idea of this paper is to present an extended design method of a new SMO for actuator and sensor faults reconstruction in the presence of output time-delay system. To guarantee the stability of the estimation error and computing the SMO gains, a Lyapunov-Krasvskii functional and the LMI technique are used. after this, the designed SMO is used to obtain a robust actuator and sensor faults reconstruction.

The rest of this paper is organized as follows: section II describes the SMO design method. In section III, a reconstruction fault formulation is proposed. An example is given to illustrate the theoretical concept in section IV. Finally, section V presents some conclusions.

Notation: The notation used throughout this paper $\Re$ denotes the field of real numbers. $\|.\|$ represents the Euclidean norm for vectors.

II. SLIDING MODE OBSERVER DESIGN

We consider the following linear certain output time-delay system :

$$\dot{x}(t) = Ax(t) + Bu(t) + Df_i(t) \tag{1}$$

$$y(t) = Cx(t-\tau) + f_s(t) \tag{2}$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$ are the state vector, the input vector and the output vector respectively. $f_a(t)$ represents the function of the actuator fault satisfies $\|f_a(t)\| \leq \beta$ where β is a constant positive scalar. $f_s(t)$ represents the sensor fault where is bounded $\|f_s(t)\| \leq \beta$. τ is the delay which assumed to be constant and bounded. A, B, C and D are constant matrices with appropriate dimensions. We assume that the matrices D and C are full rank and the following tow assumptions are satisfied for the existing of the sliding mode observer [3] :

A1. $\mathrm{rank(CD) = q}$.

A2. The system (1)-(2) is minimum phase.

The assumption A1 is verified by the calculation of the rank and A2 is assured if the system (A, D, C) is minimum phase.

Lemma 1 [23]: since C has full rank, then there exists a nonsingular change of coordinates $\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = T_0 x$ such that $T_0 = \begin{bmatrix} N_c & C^T \end{bmatrix}^T$. Where N_c spans the null space of C.

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Furthermore, if these conditions hold, then there exists a linear change of coordinate $\bar{x} = Tx$ such that the matrices (A,D,C) from (1)-(2) in the new coordinates are:

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \bar{D} = \begin{bmatrix} 0 \\ \bar{D}_2 \end{bmatrix}, \bar{C} = \begin{bmatrix} 0 & I_p \end{bmatrix} \quad (3)$$

Consider initially that $f_s(t) = 0$ and $f_a(t) \neq 0$.

Therefore, the original system (1)-(2) is written in the new coordinate system as :

$$\dot{\bar{x}}_1(t) = \bar{A}_{11}\bar{x}_1(t) + \bar{A}_{12}\bar{x}_2(t) + \bar{B}_1 u(t) \quad (4)$$

$$\dot{\bar{x}}_2(t) = \bar{A}_{21}\bar{x}_1(t) + \bar{A}_{22}\bar{x}_2(t) + \bar{B}_2 u(t) + \bar{D}_2 f_i(t) \quad (5)$$

$$y(t) = \bar{x}_2(t-\tau) \quad (6)$$

where $\bar{x}_1 \in \Re^{n-p}$, $\bar{x}_2 \in \Re^{p}$, $\bar{A}_{11} \in \Re^{(n-p)\times(n-p)}$ has stable eigenvalue and $\bar{D}_2 \in \Re^{p\times q}$ is non-singular.

A SMO for the system (4), (5) and (6) in the new coordinate is:

$$\dot{\hat{\bar{x}}}_1(t) = \bar{A}_{11}\hat{\bar{x}}_1(t) + \bar{A}_{12}\hat{\bar{x}}_2(t) + \bar{B}_1 u(t) - \bar{A}_{12}\bar{e}_2(t-\tau) \quad (7)$$

$$\begin{aligned} \dot{\hat{\bar{x}}}_2(t) \quad = \quad & \bar{A}_{21}\hat{\bar{x}}_1(t) + \bar{A}_{22}\hat{\bar{x}}_2(t) + \bar{B}_2 u(t) \\ & -\tfrac{1}{\mu}(\bar{A}_{22} - \bar{A}_{22}^s)\bar{e}_2(t-\tau) + \nu(t-\tau) \end{aligned} \quad (8)$$

$$\hat{y}(t) = \hat{\bar{x}}_2(t-\tau) \quad (9)$$

A suitable choice for the observer gain matrices $\bar{G}_l \in \Re^{n\times p}$ and $\bar{G}_n \in \Re^{n\times p}$ in the new coordinate are given by:

$$\bar{G}_l = \begin{bmatrix} \bar{A}_{12} \\ \frac{1}{\mu}(\bar{A}_{22} - \bar{A}_{22}^s) \end{bmatrix} \quad and \quad \bar{G}_n = \begin{bmatrix} 0 \\ I_p \end{bmatrix} \quad (10)$$

where $\bar{A}_{22}^s \in \Re^{p\times p}$ is any design matrix with stable eigenvalues and where μ is a positive number. The discontinuous function υ is defined by:

$$\upsilon(t) = \begin{cases} -\rho \left\|\bar{D}_2\right\| \frac{P_2 e_y(t)}{\|P_2 e_y(t)\|} & if \quad e_y(t) \neq 0 \\ 0 & otherwise \end{cases} \quad (11)$$

where $P_2 \in \Re^{p\times p}$ is a Lyapunov matrix and ρ is a positive scalar.

The objective is to design a SMO where the sliding motion is attained, during finite time, the sliding surface which is defined by:

$$S = \{\bar{x}_2, \hat{\bar{x}}_2 \in \Re^p : s_e(t) = \hat{y}(t) - y(t) = \bar{e}_2(t-\tau)\} \quad (12)$$

with $\bar{e}_2(t-\tau) = \hat{\bar{x}}_2(t-\tau) - \bar{x}_2(t-\tau)$.

Defining the state estimation error as:

$$\begin{cases} \bar{e}_1(t) = \hat{\bar{x}}_1(t) - \bar{x}_1(t) \\ \bar{e}_2(t) = \hat{\bar{x}}_2(t) - \bar{x}_2(t) \end{cases}$$

then it is straightforward to show:

$$\dot{\bar{e}}_1(t) = \bar{A}_{11}\bar{e}_1(t) + \bar{A}_{12}\bar{e}_2(t) - \bar{A}_{12}\bar{e}_2(t-\tau) \quad (13)$$

$$\begin{aligned} \dot{\bar{e}}_2(t) = \bar{A}_{21}\bar{e}_1(t) + \bar{A}_{22}\bar{e}_2(t) - \tfrac{1}{\mu}(\bar{A}_{22} - \bar{A}_{22}^s)\bar{e}_2(t-\tau) \\ +\upsilon(t-\tau) - \bar{D}_2 f_i(t) \end{aligned} \quad (14)$$

$$e_y(t) = \bar{e}_2(t-\tau) \quad (15)$$

Theorem: The state estimation error dynamics is quadratically stable, if there exist symmetric positive definite matrices $P_1 \in \Re^{(n-p)\times(n-p)}$, $Q \in \Re^{p\times p}$ and a symmetric matrix $P_2 \in \Re^{p\times p}$ such that the following condition LMI is satisfied:

$$\begin{bmatrix} \bar{A}_{11}^T P_1 + P_1\bar{A}_{11} & P_1\bar{A}_{12} + \bar{A}_{21}^T P_2 & \rightarrow \\ * & \bar{A}_{22}^T P_2 + P_2\bar{A}_{22} + Q & \\ * & * & \end{bmatrix}$$
$$\begin{bmatrix} \leftarrow & -P_1\bar{A}_{12} \\ & \frac{1}{\mu}P_2(\bar{A}_{22} - \bar{A}_{22}^s) \\ & -Q \end{bmatrix} < 0 \quad (16)$$

Proof : To prove the stability and the convergence of the designed SMO, choose a Lyapunov-Krasovskii functional as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) > 0 \quad (17)$$

where

$$V_1(t) = \bar{e}_1^T(t)P_1\bar{e}_1^T(t)$$
$$V_2(t) = \bar{e}_2^T(t)P_2\bar{e}_2(t)$$
$$V_3(t) = \int_{t-\tau}^{t} \bar{e}_2^T(\theta)Q\bar{e}_2(\theta)d\theta$$

By calculating the derivative of (16) which must be negative and to determinate the gains of the observer, the derivative of V along the trajectory of (13) and (14) is:

$$\begin{aligned} &\dot{V}(t) = 2\bar{e}_1^T(t)P_1\dot{\bar{e}}_1(t) + 2\bar{e}_2^T(t)P_2\dot{\bar{e}}_2(t) \\ &+\bar{e}_2^T(t)Q\bar{e}_2(t) - \bar{e}_2^T(t-\tau)Q\bar{e}_2(t-\tau) \\ &= \bar{e}_1^T(t)\left[\bar{A}_{11}^T P_1 + P_1\bar{A}_{11}\right]\bar{e}_1(t) + 2\bar{e}_1^T(t)P_1\bar{A}_{12}\bar{e}_2(t) \\ &-2\bar{e}_1^T(t)P_1\bar{A}_{12}\bar{e}_2(t-\tau) + 2\bar{e}_2^T(t)P_2\bar{A}_{21}\bar{e}_1(t) \\ &+\bar{e}_2^T(t)\left[\bar{A}_{22}^T P_2 + P_2\bar{A}_{22}\right]\bar{e}_2(t) \\ &-\tfrac{2}{\mu}\bar{e}_2^T(t)P_2(\bar{A}_{22} - \bar{A}_{22}^s)\bar{e}_2(t-\tau) \\ &+2\bar{e}_2^T(t)P_2\upsilon(t-\tau) - 2\bar{e}_2^T(t)P_2\bar{D}_2 f_i(t) \\ &+\bar{e}_2^T(t)Q\bar{e}_2(t) - \bar{e}_2^T(t-\tau)Q\bar{e}_2(t-\tau) \end{aligned} \quad (18)$$

Using the fact that $\|f_i(t)\| \leq \beta$ and (11), one obtains :

$$\begin{aligned} &2\bar{e}_2^T(t)P_2\upsilon(t-\tau) - 2\bar{e}_2^T(t)P_2 D_2 f_i(t) \\ &= -2P_2\bar{e}_2^T(t)\left[\rho\frac{P_2\bar{e}_2(t-\tau)}{\|P_2\bar{e}_2(t-\tau)\|} + \bar{D}_2 f_i(t)\right] \\ &\leq -2\left\|P_2\bar{e}_2^T\right\|\left[\rho\frac{\|P_2\bar{e}_2(t-\tau)\|}{\|P_2\bar{e}_2(t-\tau)\|} + \beta\left\|\bar{D}_2\right\|\right] \\ &\leq -2\left\|P_2\bar{e}_2^T\right\|\left[\rho + \beta\left\|\bar{D}_2\right\|\right] \end{aligned} \quad (19)$$

Substituting (19) into (18) yields

$$\begin{aligned} &\dot{V}(t) \leq \bar{e}_1^T(t)\left[\bar{A}_{11}^T P_1 + P_1\bar{A}_{11}\right]\bar{e}_1(t) + 2\bar{e}_1^T(t)P_1\bar{A}_{12}\bar{e}_2(t) \\ &-2\bar{e}_1^T(t)P_1\bar{A}_{12}\bar{e}_2(t-\tau) + 2\bar{e}_2^T(t)P_2\bar{A}_{21}\bar{e}_1(t) \\ &+\bar{e}_2^T(t)\left[\bar{A}_{22}^T P_2 + P_2\bar{A}_{22} + Q\right]\bar{e}_2(t) \\ &-\tfrac{2}{\mu}\bar{e}_2^T(t)P_2(\bar{A}_{22} - \bar{A}_{22}^s)\bar{e}_2(t-\tau) \\ &-\bar{e}_2^T(t-\tau)P_2 Q\bar{e}_2(t-\tau) - 2\left\|P_2\bar{e}_2^T\right\|\left[\rho + \beta\left\|\bar{D}_2\right\|\right] \end{aligned} \quad (20)$$

defining the vector $\xi(t) = \begin{bmatrix} \bar{e}_1(t) \\ \bar{e}_2(t) \\ \bar{e}_2(t-\tau) \end{bmatrix}$, the inequality (20) becomes :

$$\dot{V}(t) \leq \xi^T(t) \begin{bmatrix} \bar{A}_{11}^T P_1 + P_1 \bar{A}_{11} & P_1 \bar{A}_{12} + \bar{A}_{21}^T P_2 & \rightarrow \\ * & \bar{A}_{22}^T P_2 + P_2 \bar{A}_{22} + Q & \rightarrow \\ * & * & \end{bmatrix}$$

$$\leftarrow \begin{matrix} -P_1 \bar{A}_{12} \\ \frac{1}{\mu} P_2 (\bar{A}_{22} - \bar{A}_{22}^s) \\ -Q \end{matrix} \Bigg] \xi(t) - 2 \left\| P_2 \bar{e}_2^T \right\| \left[\rho + \beta \left\| \bar{D}_2 \right\| \right] \quad (21)$$

if $\begin{bmatrix} \bar{A}_{11}^T P_1 + P_1 \bar{A}_{11} & P_1 \bar{A}_{12} + \bar{A}_{21}^T P_2 & \rightarrow \\ * & \bar{A}_{22}^T P_2 + P_2 \bar{A}_{22} + Q & \rightarrow \\ * & * & \end{bmatrix}$

$\leftarrow \begin{matrix} -P_1 \bar{A}_{12} \\ \frac{1}{\mu} P_2 (\bar{A}_{22} - \bar{A}_{22}^s) \\ -Q \end{matrix} \Bigg] < 0$, so $\dot{V(t)} < 0$. Therefore the state estimation error dynamics is quadratically stable.

III. FAULT RECONSTRUCTION

A. *Actuator fault reconstruction*

The sliding motion is holding on to the sliding surface, for a sufficiently small number of μ, the equations (13) and (14) become:

$$\dot{\bar{e}}_1(t) = \bar{A}_{11} \bar{e}_1(t) \quad (22)$$

$$\begin{aligned} 0 \approx \bar{A}_{21} \bar{e}_1(t) - \tfrac{1}{\mu} (\bar{A}_{22} - \bar{A}_{22}^s) \bar{e}_2(t-\tau) + \upsilon(t-\tau) \\ -\bar{D}_2 f_i(t) \end{aligned} \quad (23)$$

The matrix $\bar{A}_{11}$ is stable, therefore $\bar{e}_1 \longrightarrow 0$, so the equation (23) becomes:

$$0 \approx -\frac{1}{\mu} (\bar{A}_{22} - \bar{A}_{22}^s) \bar{e}_2(t-\tau) + \upsilon_{eq}(t-\tau) - \bar{D}_2 f_i(t) \quad (24)$$

where $\upsilon_{eq}(t)$ the equivalent output injection defined by [27] :

$$\upsilon_{eq}(t) = -\rho \left\| \bar{D}_2 \right\| \frac{P_2 e_y(t)}{\| P_2 e_y(t) \| + \delta} \quad (25)$$

with δ is a smoothing small positive scalar.

Since $rank(\bar{D}_2) = q$, it follows that:

$$\hat{f}_i(t) = (\bar{D}_2^T \bar{D}_2)^{-1} \bar{D}_2^T (\upsilon_{eq}(t) - \frac{1}{\mu} (\bar{A}_{22} - \bar{A}_{22}^s) \bar{e}_2(t-\tau)) \quad (26)$$

B. *Sensor faults reconstruction*

Assuming that the actuator fault is zero $f_i(t) = 0$, and the sensor fault $f_o(t) \neq 0$, where :

$$y(t) = Cx(t-\tau) + f_o(t) \Rightarrow e_y(t) = \bar{e}_2(t-\tau) - f_o(t) \quad (27)$$

then the dynamic error estimation is :

$$\dot{\bar{e}}_1(t) = \bar{A}_{11} \bar{e}_1(t) + \bar{A}_{12} \bar{e}_2(t) - \bar{A}_{12} \bar{e}_2(t-\tau) + \bar{A}_{12} f_o(t) \quad (28)$$

$$\begin{aligned} \dot{\bar{e}}_y(t) = \bar{A}_{21} \bar{e}_1(t) + \bar{A}_{22} \bar{e}_y(t) - \bar{A}_{22} f_o(t) \\ -\tfrac{1}{\mu} (\bar{A}_{22} - \bar{A}_{22}^s) \bar{e}_y(t-\tau) + \upsilon(t) - \dot{f}_o(t) \end{aligned} \quad (29)$$

while the matrix $\bar{A}_{11}$ is stable, so $\dot{\bar{e}}_1 \mapsto 0$, then from the equation (28) can be drawn $\bar{e}_1(t) \approx -\bar{A}_{11}^{-1} \bar{A}_{12} f_o(t)$.

Therefore the estimated sensor fault is given by:

$$\begin{aligned} \hat{f}_o(t) = (\bar{A}_{21} \bar{A}_{11}^{-1} \bar{A}_{12} + \bar{A}_{22})^{-1} (\upsilon_{eq}(t) \\ -\tfrac{1}{\mu} (\bar{A}_{22} - \bar{A}_{22}^s) \bar{e}_y(t-\tau)) \end{aligned} \quad (30)$$

IV. SIMULATION EXAMPLE

To illustrate the theoretical concepts which has been developed in this paper, consider an inverted pendulum system where the equation of motion is:

$$(M+m)\ddot{x} + F_x \dot{x} + ml(\ddot{\theta} \cos\theta - \dot{\theta}^2 \sin\theta) = u \quad (31)$$

$$J\ddot{\theta} + F_\theta - mlg \sin\theta + ml\ddot{x} \cos\theta = 0 \quad (32)$$

where the particular values of the system parameters are given in Table 1

$M(kg)$	$m(kg)$	$J(kgm^2)$	$l(m)$	$F_x(kg/s)$	$F_\theta(kgm^2)$	$g(m/s^2)$
3.2	0.535	0.062	0.365	6.2	0.009	9.807

Table1. Parameters of inverted pendulum system.

A linearized matrix model of equation motion has been made about the equilibrium point at the origin is given by:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1.9333 & -1.9872 & 0.0091 \\ 0 & 36.9771 & 6.2589 & -0.1738 \end{bmatrix},$$

$$B = D = \begin{bmatrix} 0 \\ 0 \\ 0.3205 \\ -1.0095 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Taking as states the angular position of the pendulum θ, angular velocity $\dot{\theta}$, the position d and velocity of the cart $\dot{d}$, then $x(t) = \begin{bmatrix} \theta & \dot{\theta} & d & \dot{d} \end{bmatrix}^T$.

A linear state feedback controller $u = [-7.8265 - 83.7077 - 15.6042 - 12.9578]x$ has been introduced to stabilize the system.

The new matrix dynamics after stabilization is :

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2.5084 & 24.8950 & 3.0139 & 4.1621 \\ -7.9009 & -47.5258 & -9.4935 & -13.2547 \end{bmatrix}$$

After the change of coordinates, we obtain the following matrices:

$$\bar{A} = \begin{bmatrix} -2 & 0 & 27.1779 & 6.2992 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1.8549 & -3.1498 \\ 4.1621 & 2.5084 & 32.6151 & -10.0957 \end{bmatrix},$$

$$\bar{B} = \bar{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.3205 \end{bmatrix}, \bar{C} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $\bar{A}_{22}^{s} = diag\{-11; -12; -13\}$.

A. Observer design

The obtained SMO gains are:

$$G_l = \begin{bmatrix} 11 & 0 & 1 \\ 0 & 13.8549 & -3.1498 \\ 2.5084 & 32.6151 & 2.9043 \\ -7.9009 & -49.8532 & -8.6911 \end{bmatrix},$$

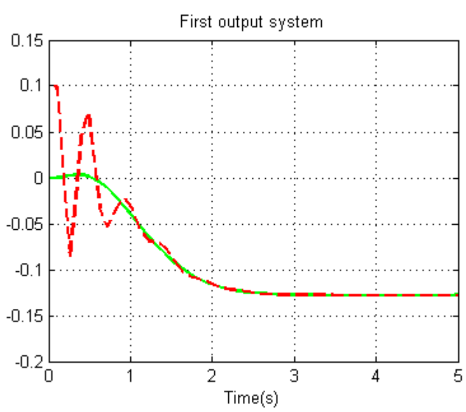
$$\mathrm{G}_n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1.8549 & -3.1498 \end{bmatrix},$$

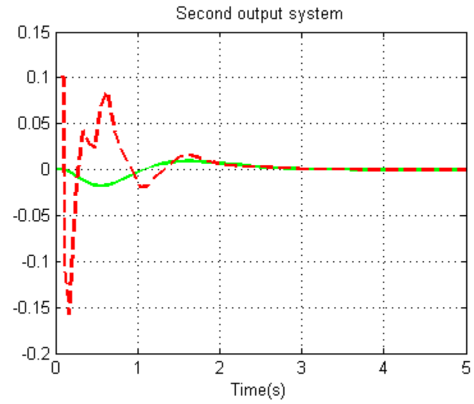
$$\mathrm{P}_1 = 0.0172,\ \mathrm{P}_2 = \begin{bmatrix} 0.0131 & 0.0046 & -0.0023 \\ 0.0046 & 0.0450 & -0.0142 \\ -0.0023 & -0.0142 & 0.0086 \end{bmatrix},$$

$$\mathrm{Q} = \begin{bmatrix} 0.6496 & 0.0024 & -0.0019 \\ 0.0024 & 0.6875 & 0.0036 \\ -0.0019 & 0.0036 & 0.6708 \end{bmatrix} \text{ and } \mu = 0.05.$$

B. Observer simulation

In the corresponding simulation, the constants associated with ν have been chosen to be $\rho = 10$ and $\delta = 0.005$. Furthermore, the system was assumed to have an initial condition $x(0) = [0.1; -0.1; 0.2; 0.1]$ and the observer was assumed to have zero initial condition.





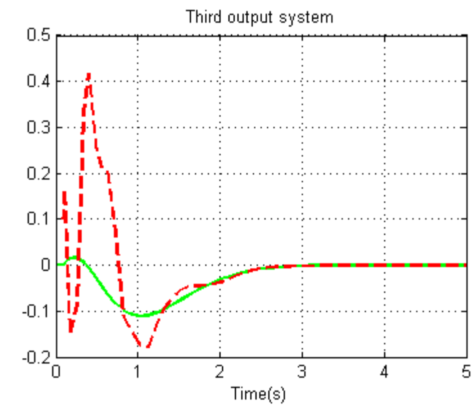


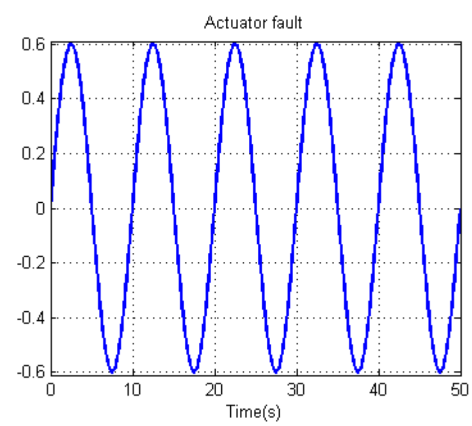
Fig. 1. The system outputs (dash line) and the observer's estimates (solid line).

Figure 1 shows a good estimation of the actual outputs with a constant delay $\tau = 0.1s$.

C. Fault reconstruction

1) Actuator fault reconstruction

An actuator fault is injected in the input of the system. The figure Fig.2 below shows the robust reconstruction of this fault:



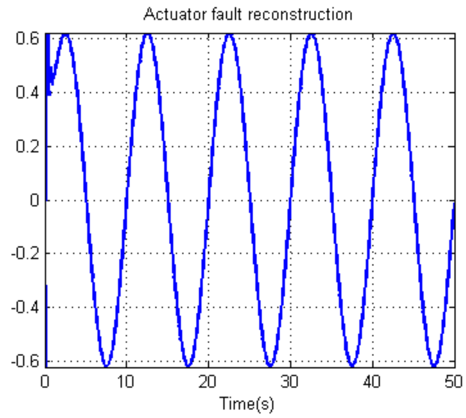


Fig. 2. The actuator fault and its reconstruction.

The figure 2 presents an injected actuator fault and its reconstruction. Therefore,it can be seen in spite the presence of the time delay in the output system and the initial condition the proposed method is able to give a good estimation of this actuator fault.

2) Sensor fault reconstruction

In figures 3 and 4 we present the sensors faults reconstruction using the proposed SMO.

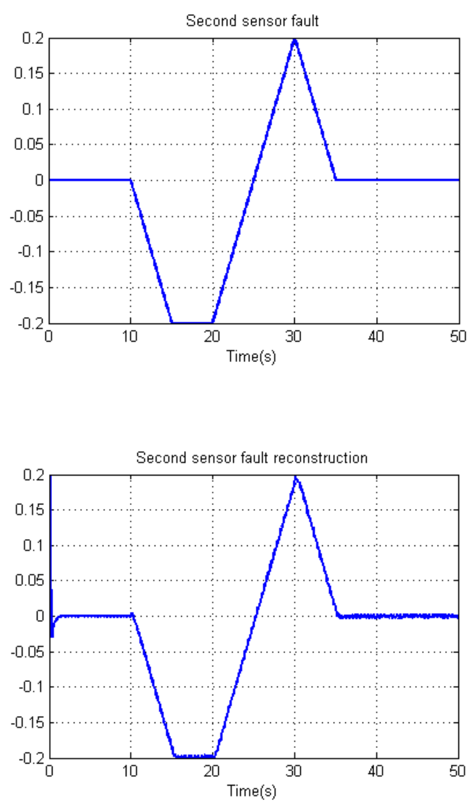


Fig. 3. The fault signal on the second sensor and its reconstruction.

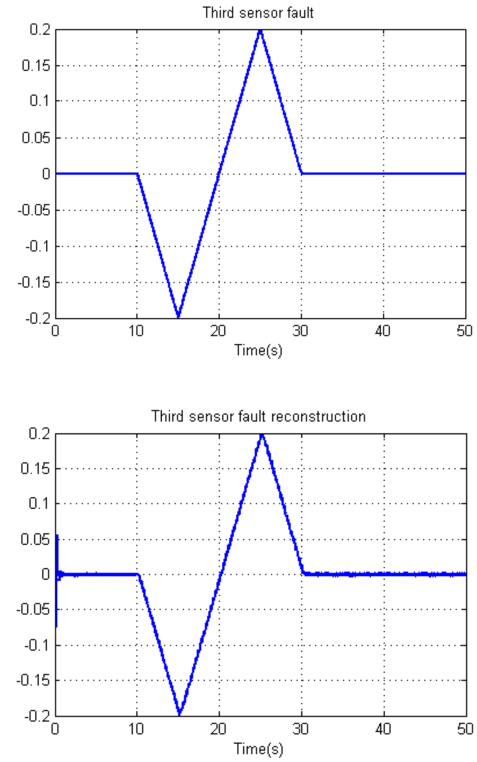


Fig. 4. The fault signal on the third sensor and its reconstruction.

The simulation results show the efficiency of the proposed methods of SMO design and sensor faults reconstruction for the proposed output time-delay system. An appropriate choice of the constants ρ and δ provides a good reconstruction.

V. Conclusion

In this paper a design method of sliding mode observer for a class of linear output delay systems is developed. Using a Lyapunov-Krasovskii functional and the LMI technique, the SMO's gains are derived. The proposed SMO has been used to obtain actuator and sensor faults reconstruction despite the presence of an output tim-delay. Finally the simulation results is used to prove the efficiency of the proposed approaches in fault reconstruction area.

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