3rd International Conference on Automation, Control, Engineering and Computer Science (ACECS'16)

Proceedings of Engineering & Technology (PET)
pp. 617-624

Robust Fault Detection Performances for Stochastic Systems based on Adaptive Threshold

Marwa Houiji
Gabès university
National Engineering School of Gabès
MACS, 6029 Tunisia
Email: marwa.houiji.TN@ieee.org

Rim Hamdaoui
Gabes university
National Engineering School of Gabes
MACS, 6029 Tunisia
Email:rim.hamdaoui@gmail.com
Fax: +216 75 39 21 90

Mohamed Aoun
Gabes university
National Engineering School of Gabes
MACS, 6029 Tunisia
Email: mohamed.aoun@gmail.com

***Abstract*—This paper investigates the problem of fault detection and isolation for discrete linear systems subjected to unknown disturbances, actuator and sensor faults. A bank of Robust Two Stage Kalman filters is adapted to estimate both the state and the fault as well as to generate the residuals. Besides, this paper presents the evaluation of the residuals with Bayes test of binary hypothesis test for fault detection to adaptive threshold compared with fixed threshold. This test allow the detection of low magnitude faults as fast as possible with a minimum risk of errors, the increase of detection probability and the reduction of false alarm probability.**

Keywords: Fault Detection,Fault isolation, Stochastic Systems, Adaptive Threshold, detection delay.

I. Introduction

The problem of fault detection and isolation (FDI) for stochastic linear systems with unknown inputs has received considerable attention in intelligent control systems[1], [2]. In [3], [4]the optimal filtering and robust fault diagnosis problem has been studied for stochastic systems with unknown disturbances. An optimal observer is proposed to estimate the state which is designed to be decoupled from unknown disturbances with minimum variance for time varying systems with both noise and unknown disturbances. Recently,unknown input filtering has been extensively studied using the Kalman filtering approach [5] in which the residual is designed to be decoupled to unknown disturbances, modeling errors and noises, whilst it's sensible to faults. In fact Chien Shu Hsieh in [6], has developed a robust filter structure, that can solve the problem of simultaneously estimating the state and the fault in the presence of the unknown disturbances. The procedure of fault detection and isolation can be divided into the following two steps [7], [8]: the first step considers the residuals'generation which is based on a physical model of the system to be monitored. The generation phase consists in calculating the residuals which are consistency indicators between recorded measures and the model behavior. The second step describes the residuals'evaluation(converting the residuals'value symptoms). The detection problem is to establish a rule of decision that can detect the earliest possible passage of an available functioning hypothesis H_0, to an abnormal state, where there are failures, called hypothesis H_1.

However, the problem reduces the system performances of fault diagnosis due to modeling errors and unmeasurable distribution. it is difficult to distinguish between the effects of an actual fault and those caused by uncertainties and disturbances, when perfect de-coupling cannot be achieved. We must make a difference between ”low” residuals which are characteristics of normal state and ”big” residuals that indicates the presence of faults. The implementation of the statistical tests of binary hypotheses in this work makes it possible to analyze the statistical characteristics of the residuals and their sensitivity to the changes of the system [9]. In this contexts, our work consists in proposing a robust decision making with a statistical approach of fault detection of linear stochastic systems with unknown disturbances.

This paper is organized as follows: Section 2 states the system and the fault modeling. Section 3 presents fault diagnosis for stochastic systems using the Robust Two Stage Kalman filter(RTSKF). The fault detection delay is presented in the Section 4. Section 5 demonstrates the influence of using an adaptive threshold in improving the performance of the fault detection. In Section 6, the performances of the proposed method are assessed through a numerical example. Finally, concluding notices are given in section 7.

II. System and Fault Modeling

Consider the linear time-varying discrete stochastic systems:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ed_k + w_k^x \\ y_k &= Cx_k + v_k \end{aligned} \tag{1}$$

where $x_k \in \Re^n$ is the state vector, $y_k \in \Re^m$ is the output vector, $u_k \in \Re^p$ is the known input vector, and $d_k \in \Re^q$ is the unknown disturbances. w_k^x and v_k are uncorrelated white noises sequences of zero-mean and the covariances matrices are $Q_k^x = \varepsilon\left[w_k^x w_k^T\right] \geq 0$, and $R_k^x = \varepsilon\left[v_k v_k^T\right] \geq 0$, where$\varepsilon\left[.\right]$ denotes the expectation operator. The matrices A, B and C are known and have appropriate dimensions. We assume that (A, C) is observable, $m \geq r + q$ and rank(CE) = rank (E). The initial state is correlated with the white noises processes w_k^x and v_k. The initial state x_0 is a gaussian random variable with $\varepsilon\left[x_0\right] = \hat{x}_0$ and $\varepsilon\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right] = P_0^x$.

ISSN: 2356-5608

The unknown disturbances $d_k \in \Re^q$ can be used to describe additive disturbances and modeling errors such as nonlinear terms in the system dynamics. In the system modeling, faults are described in two different types: 1)additive faults, characterizing actuator or sensor faults, 2)multiplicative faults, designating plant faults.In the sequel, only actuator and sensor faults are considered. For instance, an actuator fault should be represented by

$$B_f = B(I + diag(\xi_k^a)) \tag{2}$$

with $\xi^a = \begin{bmatrix} \xi^{a_1}.. & \xi^{a_i}.. & \xi^{a_p} \end{bmatrix}^T$, B_f is an unknown matrix, the state-space representation of the faulty system requires the definition of an unknown inputf_a, which is equal to zero in the fault-free case ;

$$x_{k+1} = Ax_k + Bu_k + Ed_k + F^a f_k^a + w_k^x \tag{3}$$

where

$$F^a = \begin{bmatrix} F^{a_1} & F^{a_i} & F^{a_p} \end{bmatrix} \quad f_k^a = \begin{bmatrix} f_k^{a_1} & f_k^{a_i} & f_k^{a_p} \end{bmatrix}$$

Likewise, sensor faults characterize a scaling change in the state measurement and are represented by modifying the matrix C as:

$$C_f = (I + diag(\xi_k^s))C \tag{4}$$

with$\xi^s = \begin{bmatrix} \xi^{s_1}.. & \xi^{s_i}.. & \xi^{s_p} \end{bmatrix}^T$, and expression of the faulty system is :

$$y_k = Cx_k + F^s f_k^s + v_k \tag{5}$$

where

$$F^s = \begin{bmatrix} F^{s_1} & F^{s_i} & F^{s_m} \end{bmatrix} \quad f_k^s = \begin{bmatrix} f_k^{a_1} & f_k^{a_i} & f_k^{a_p} \end{bmatrix}$$

The system with an actuator fault is thus modeled by replacing the state equation in(1)as:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ed_k + F^{a_i} f_k^a + w_k^x \\ f_{k+1}^a &= f_k^a + w_k^f \end{aligned} \tag{6}$$

and the one with a sensor fault is modeled by substituting the output equation in(1)as:

$$\begin{aligned} y_k &= Cx_k + F^{s_i} f_k^s + v_k \\ f_{k+1}^s &= f_k^s + w_k^f \end{aligned} \tag{7}$$

III. FAULT DIAGNOSIS

Fault diagnosis divided into the two tasks: 1)fault detection, determining if the system is faulty or not regardless disturbances. 2)fault isolation, deciding which element of the system is faulty. In order to achieve this task, we need to look for fault symptoms. Residual is the most common defect symptom that is used for fault diagnosis. It is composed of the state and the estimation error where the state and the estimation error are generated using robust filtering for the system subjected of unknown disturbances. To solve the simultaneous state and fault estimation problem of linear stochastic discrete-time with unknown disturbances, the natural approach is to augment the fault as a part of the state, and to apply the Kalman filter. The Robust Two-Stage Kalman filter(RTSKF)is developed on two steps. Firstly, a two-stage $U - V$ transformations are made in order to decouple the covariance matrix on the augmented state Kalman Filter (ASKF)so, reduced order Kalman filter called two-stage Kalman filter is obtained. Secondly, by making use of the two-stage Kalman filtering technique and a new proposed unknown inputs filtering technique, a robust two-stage Kalman filter(RTSKF)which is unaffected by the unknown inputs[6].

A. Robust Two Stage Kalman filter

The algorithm of the Robust Two Stage Kalman filter is summarized in the Table I.

B. Residual generation and fault detection

To detect a fault, the residual is synthesized from the difference between the real and the estimated output of the system described by its mathematical model.

$$\begin{aligned} y_{k/k} &= Cx_{k/k} + F_k^{s_i} f_{k/k} \\ r_k &= y_k - y_{k/k} = Ce_k^x + F_k^{s_i} e_k^f + v_k \end{aligned} \tag{8}$$

where $e_k^x = x_k - x_{k/k}$ and $e_k^f = f_k - f_{k/k}$ are the state and the fault estimation errors. Note that these errors have minimum variances.

The residual is examined in terms of the probability of a fault, therefore a logical decision-making process is applied aiming to decide if the fault has occurred and avoided wrong decisions, such as false alarm and non-detection. Different techniques to evaluate residuals is as follows.

1) Thresholding: Evaluation consists in defining a threshold to detect the presence of faults. The main difficulty of detection lies in the calculation of the threshold residue. A high threshold is likely to cause non detection. On the contrary, a low threshold will possibly cause false alarms[7].
2) Statistical decision: For this assessment, the well-known examples of these statistical test techniques are as follows :
   - Generalized Likelihood Ratio (GLR) test introduced by Willsky and Jones [10] performs statistical tests on the innovations sequence of a Kalman filter state estimator.
   - Bayes test: the function of decision which is the likelihood ratio test of conditional probability densities noted by $\lambda(r)$ will be compared with a threshold η, which is determined by the minimization of an optimality criterion [9]. We can detect the fault using the following detection rule

$$S(r_k) = \begin{cases} 0 & if \quad r_k < \eta\} \\ 1 & otherwise \end{cases} \tag{9}$$

C. Fault isolation

Fault isolation requires the generation of a residual that must be sensitive to faults able to distinguish between different types of faults. Thus, a bank of two-stage kalman filters is set up according to the system model with actuator fault and with

TABLE I

THE ALGORITHM OF THE ROBUST TWO STAGE KALMAN FILTER

Correction of state estimation

$$x_{k+1/k+1} = \bar{x}_{k+1/k+1} + \beta_{k+1/k+1} f_{k+1/k+1}$$
$$P^x_{k+1/k+1} = \bar{P}^x_{k+1/k+1} + \beta_{k+1/k+1} P^f_{k+1/k+1} \beta^T_{k+1/k+1}$$

$with$

$$\bar{x}_{0/0} = x_0 - \beta_{0/0} f_0, \beta_{0/0} = P^{xf}_0 (P^f_0)^{-1}$$
$$\bar{P}^x_{0/0} = P^x_0 - \beta_{0/0} P^f_0 \beta^T_{0/0},$$

State Subfilter

$$\bar{x}_{k+1/k+1} = \bar{x}_{k+1/k} + \bar{L}^x_{k+1} \bar{\gamma}_{k+1}$$
$$\bar{P}^x_{k+1/k+1} = (I - \bar{K}^x_{k+1} C) \bar{P}^x_{k+1/k} + \eta^x_{k+1} \Pi^x_{k+1} G^x_{k+1} \Pi^{x^T}_{k+1} \eta^{x^T}_{k+1}$$

with

$$\bar{L}^x_{k+1} = \bar{K}^x_{k+1} + \eta^x_{k+1} \Pi^x_{k+1}$$
$$\eta^x_{k+1} = E - \bar{K}^x_{k+1} CE$$
$$\Pi^x_{k+1} = \left[(CE)^T G^{x^{-1}}_{k+1} CE \right]^{-1} (CE)^T G^{x^{-1}}_{k+1}$$

where

$$\bar{\gamma}_{k+1} = y_{k+1} - C \bar{x}_{k+1/k}$$
$$\bar{x}_{k+1/k} = A \bar{x}_{k/k} + B u_k + \alpha_k f_{k/k} - \beta_{k+1/k} f_{k/k}$$
$$\bar{K}^x_{k+1} = \bar{P}^x_{k+1/k} C^T G^{x^{-1}}_{k+1}$$
$$\bar{P}^x_{k+1/k} = A \bar{P}^x_{k/k} A^T w^x + \alpha_k P^f_{k/k} \alpha^T_k - \beta_{k+1/k} P^f_{k+1/k} \beta^T_{k+1/k}$$
$$G^x_{k+1} = C \bar{P}^x_{k+1/k} C^T + v$$

Fault Subfilter

$$f_{k+1/k+1} = f_{k/k} + L^f_{k+1} \gamma^f_{k+1}$$
$$P^f_{k+1/k+1} = (I - K^f_{k+1} H_{k+1/k}) P^f_{k+1/k} + \eta^f_{k+1} \Pi^f_{k+1} G^f_{k+1} \Pi^{f^T}_{k+1} \eta^{f^T}_{k+1}$$

with

$$L^f_{k+1} = K^f_{k+1} + \eta^f_{k+1} \Pi^f_{k+1}$$
$$\eta^f_{k+1} = K^f_{k+1} CE$$
$$\Pi^f_{k+1} = \left[(CE)^T G^{f^{-1}}_{k+1} CE \right]^{-1} (CE)^T G^{f^{-1}}_{k+1}$$

where

$$\gamma^f_{k+1} = \bar{\gamma}_{k+1} - H_{k+1/k} f_{k/k}$$
$$K^f_{k+1} = P^f_{k+1/k} H^T_{k+1/k} G^{f^{-1}}_{k+1}$$
$$G^f_{k+1} = H_{k+1/k} \bar{P}^x_{k+1/k} H^T_{k+1/k} + G^x_{k+1}$$

Coupling Equations

$$H_{k+1/k} = F^{s_i} + C \beta_{k+1/k}, F^{s_i} = 0 \text{ for model (6)}$$
$$\beta_{k+1/k+1} = \beta_{k+1/k} - \bar{L}^x_{k+1} H_{k+1/k}$$
$$\alpha_k = A \beta_{k/k} + F^{a_i}, F^{a_i} = 0 \text{ for model (7)}$$
$$\beta_{k+1/k} = \alpha_k P^f_{k/k} P^{f^{-1}}_{k+1/k}$$

sensor fault. The residual generated from the bank of two-stage kalman filters in case of an actuator or sensor fault summarize as follows.

1)The residual of the ith filter in case of the jth actuator fault $(i = a_1, ..., a_p, s_1......s_m, j = a_1, ..., a_p)$

$$r^{a_i}_k = C e^x_k + v_k \tag{10}$$

where

$$\begin{aligned} e^x_{k+1} &= (I - L_{k+1} C) F^{a_j} f_k - (I - L_{k+1} C) F^{a_i} f_{k/k} \\ &+ (I - L_{k+1} C) A e^x_k + (I - L_{k+1} C) w^x_k \\ &- L_{k+1} v_{k+1} \\ e^f_{k+1} &= e^f_k - L^f_{k+1} C F^{a_j} f_k + L^f_{k+1} C F^{a_i} f_{k/k} \\ &- L^f_{k+1} C A e^x_k + w^f_k - L^f_{k+1} (C w^x_k + v_{k+1}) \end{aligned} \tag{11}$$

with

$$L_{k+1} = \bar{L}^x_{k+1} + \beta_{k+1/k+1} L^f_{k+1} \tag{12}$$

$$r^{s_i}_k = C e^x_k - F^{s_i} f_{k/k} + v_k \tag{13}$$

where

$$\begin{aligned} e^x_{k+1} &= (I - L_{k+1} C) F^{a_j} f_k + L_{k+1} F^{s_i} f_{k/k} \\ &+ (I - L_{k+1} C) A e^x_k + (I - L_{k+1} C) w^x_k \\ &- L_{k+1} v_{k+1} \\ e^f_{k+1} &= e^f_k - L^f_{k+1} C F^{a_j} f_k + L_{k+1} F^{s_i} f_{k/k} \\ &- L^f_{k+1} C A e^x_k + w^f_k - L^f_{k+1} (C w^x_k + v_{k+1}) \end{aligned} \tag{14}$$

2) The residual of ith filter in case of the jth sensor fault

$$(i = a_1, ..., a_p, s_1......s_m, j = s_1, ..., s_p)$$

$$r^{a_i}_k = C e^x_k + F^{s_j} f_k + v_k \tag{15}$$

where

$$\begin{aligned} e^x_{k+1} &= -L_{k+1} F^{s_j} f_k - (I - L_{k+1} C) F^{a_i} f_{k/k} \\ &+ (I - L_{k+1} C) A e^x_k + (I - L_{k+1} C) w^x_k \\ &- L_{k+1} (F^{s_j} w^f_k + v_{k+1}) \\ e^f_{k+1} &= e^f_k - L^f_{k+1} F^{s_j} f_k + L^f_{k+1} C F^{a_i} f_{k/k} \\ &- L^f_{k+1} C A e^x_k + (I - L^f_{k+1} F^{s_j}) w^f_k \\ &- L^f_{k+1} (C w^x_k + v_{k+1}) \end{aligned} \tag{16}$$

$$r^{s_i}_k = C e^x_k + F^{s_j} f_k - F^{s_i} f_{k/k} + v_k \tag{17}$$

where

$$\begin{aligned} e^x_{k+1} &= -L_{k+1} F^{s_j} f + L_{k+1} F^{s_i} f_{k/k} \\ &+ (I - L_{k+1} C) A e^x_k + (I - L_{k+1} C) w^x_k \\ &- L_{k+1} (F^{s_j} w^f_k + v_{k+1}) \\ e^f_{k+1} &= e^f_k - L^f_{k+1} F^{s_j} f_k + L^f_{k+1} F^{s_i} f_{k/k} \\ &- L^f_{k+1} C A e^x_k + (I - L^f_{k+1} F^{s_j}) w^f_k \\ &- L^f_{k+1} (C w^x_k + v_{k+1}) \end{aligned} \tag{18}$$

The resultant vectors,$S^a(r_k)$ for residuals from the filters with F^{a_i} and $S^s(r_k)$ for residuals from the filters with F^{s_i},are produced as

$$\begin{aligned} S^a(r_k) &= \left[\begin{array}{ccc} S^{a_1}(r_k)... & S^{a_i}(r_k)... & S^{a_p}(r_k) \end{array} \right]^T \\ S^s(r_k) &= \left[\begin{array}{ccc} S^{s_1}(r_k)... & S^{s_i}(r_k)... & S^{s_p}(r_k) \end{array} \right]^T \end{aligned} \tag{19}$$

Then for fault isolation, $S^a(r_k)$ and $S^s(r_k)$ are compared to the fault signatures $S^a(ref, f_i)$ and $S^s(ref, f_i)$ which are the column vectors if the fault signature matrices defined in the Table II. Note that the $'0'$ element of the fault signature matrices are designed based on the underlying principle that only the filter associated with the fault occurred can estimate the state and the fault correctly. Now, if $S^a(r_k)$ coincides with a column of the actuator fault signature matrix in the Table

II, the corresponding fault indicator $I(f_{a_i})$ or$I(f_s)$ is set to ”one”. If $I(f_{a_i}) = 1$, the ith actuator is declared to faulty. If $I(f_s) = 1$, we guess a sensor fault. Further, by checking if $S^s(r_k)$ is same as which column of sensor fault signature matrix in theTable II, the corresponding fault indicator $I(f_{s_i})$ set to 1, and the ith sensor is declared to be faulty [6].

TABLE II
FAULT SIGNATURE MATRICES

$S^a(r)$	$S^a(ref, nofault)$	$S^a(ref, f_1)$	$S^a(ref, f_i)$	$S^a(ref, f_p)$
$S^a1(r)$	0	0	1	1
$S^ai(r)$	0	1	0	1
$S^ap(r)$	0	1	1	0

$S^s(r)$	$S^s(ref, nofault)$	$S^s(ref, f_1)$	$S^s(ref, f_i)$	$S^s(ref, f_m)$
$S^s1(r)$	0	0	1	1
$S^si(r)$	0	1	0	1
$S^sp(r)$	0	1	1	0

IV. FAULT DETECTION DELAY

The main goal of fault detection is to detect the fault when it occurs, by generating an alarm. However, we must attach a great importance to the time taken before generating an alarm which depends on the decision method. This step aims to analyze the residuals in order to detect the fault.

A. Fault detection threshold and detection time

The residual r_k is compared with a threshold r_{th} as follow:

$$\begin{cases} r_k < r_{th}, & \text{fault free system} \\ r_k \geq r_{th}, & \text{faulty system} \end{cases} \quad (20)$$

The choice of thresholds vector affects directly the detection time of the anomaly. This time is the difference between the time of occurrence t_f and the detection instant t_d, the time taken for detection is denoted $T_d = t_d - t_f$ [11].

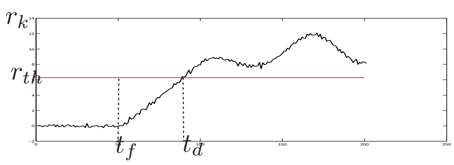

Fig. 1. Effect of the threshold on the detection time

In an ideal case, a fault must be detected immediately after its occurrence. However, because of the position of threshold alarm there is always a detection delay. In order to show the effect of the threshold value variation on the detection time, we consider the probability tools to estimate the variable T_d for different threshold values [12]. We assume that the residual signal r_k is an independent and identically distributed random variable (IID). The probability of exceeding the threshold detection r_{th} by the residue at the occurrence time of the fault is given by P_2 , which is in fact the probability of immediate detection.

The probability of not exceeding the threshold residual signal at the time of the fault occurrence with h delay time is P_1, which is the probability of the time detection delay of ih. The probability of detection delay is expressed by:

$$\begin{aligned} P(T_d = 0h) &= P(r(t_f) > r_{th}) = P_2 \\ P(T_d = 1h) &= P(r(t_f) < r_{th}, r(t_f + h) > r_{th}) \\ &= P_2 P_1 \\ &\vdots \\ P(T_d = ih) &= \\ P(r(t_f + ih - h) &< r_{th}, r(t_f + ih) > r_{th}) \\ &= P_2^i P_1 \end{aligned} \quad (21)$$

Asr_k is IID, then $P_2 = 1 - P_1$, the average value of the expected detection time T_d is expressed by equation (22)where$E$ denotes the expected value:

$$\begin{aligned} &\bar{T}_d = E(T_d) = h \sum_{i=0}^{\infty} iP(T_d = ih) \\ &= h \sum_{i=0}^{\infty} iP_1 P_2^i \\ &= hP_1 P_2 (\sum_{i=0}^{\infty} P_2^i)' \\ &= hP_2 P_1 (\frac{1}{1-P_2})' \\ &= \frac{P_2}{P_1} h \end{aligned} \quad (22)$$

The probabilities P_1 and $P_2 = 1 - P_1$ can be determined from the probability density function of the decision signal after occurrence of the fault and the threshold value is fixed as shown in figure 2

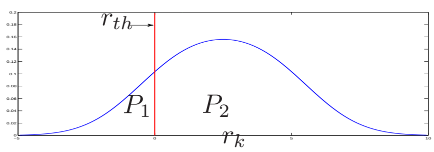

Fig. 2. The Probability density of the residue.

Here fault-free data has a Gaussian distribution with mean 0, variance 1 and faulty data is also Gaussian distributed with mean 2 and variance 2.

Determining the estimation of detection time $E(T_d)$ is directly related to probabilities P_1 and P_2, which are determined by the choice of the threshold detection.Table (III) presents $E(T_d)$ calculated for different threshold values and for $h = 1sec$.

TABLE III
EFFECT OF THRESHOLD CHANGE ON EXPECTED DETECTION DELAY

Threshold r_{th}	Expected detection delay $T_d(s)$
0	0.19
1.5	0.67
2.25	3
4	5.3
5	13.97

According to the table (III), for threshold values $r_{th} \leq 4$ the estimation of the detection time is in the order of 5.3, but it increases considerably for values of $r_{th} \geq 5$.

Based on analysis of probability, we note that:there is always a detection delay which can be considered more or less important depending on the type of fault, type of system dynamics, whether fast or slow. Since the time represents one of the basic indices of the safety process. Detection delay must be limited. However, this limitation should not cause false alarms. In addition, optimizing the detection time due to the choice of a detection threshold ensuring the compromise between the rate of false alarms and the rate of non detection [12], [13].

B. *Optimization of the detection delay*

Detection at optimal time is a detection having optimal performances, a minimum rate of false alarms with an acceptable sensitivity to the faults. The choice of a constant threshold value is limited by the presence of unknown input. The threshold resulted from modeling uncertainties and disturbances can be misinterpreted as a response to sensor and actuator faults thus, set off a false alarm. In fact, perfect decoupling cannot be achieved, a performance index which measures the sensitivity to faults and the insensitivity to uncertainties must be defined and optimised. The implementation of the statistical tests of binary hypothesis makes it possible to analyse the statistical characteristics of these residuals and their sensitivity to change of system behaviour. In fact, the introduction of the technique of decision-making, shows that it is possible to minimise the detection delay and false alarms.

V. Robust Fault Detection based on Bayes Tests with Adaptive Threshold

One of the techniques used to design an adaptive threshold is the method of the Gaussian Kernel(GK) which allows the processing of data directly and provides an estimation of a priori probabilities. More precisely, the measurement data are used directly to calculate weights assigned to the probability densities functions relating to the hypothesis H_0 and H_1. Each kernel has three parameters that can be adjusted during training, the mean μ, the standard deviation σ and a parameter of weight correction w_1 which has a function equivalent to that of a probability. Weights are corrected in a recursive way according to the available observations.

A. *Gaussian kernels algorithm for a priori probability estimation*

Let consider the data μ_0, σ and μ_1, σ of the hypotheses H_0 and H_1, respectively. We consider a stop criterion ς which is the corresponding error to the difference between the estimated value and the true value. The algorithm is given by the following stages.

1) Set initial conditions, by randomly selecting a value for $p(H_1)$ within the range $0 < p(H_1) < 1$ which corresponds to the weight $w_{1,0}$ probability of having the hypotheses H_1 Set the initial value α_0 for $k = 0 \ldots$
2) Calculate the weighted output for each of the two kernels

$$P_i(y_k) = w_{i,k} p_i(y_k/\mu_i, \sigma_i) \quad i = 1, 2$$

3) Calculate the output over the sum of the weighted kernel outputs:

$$P(y_k) = \sum_{i=1}^{2} w_{i,k} p_i(y_k/\mu_i, \sigma_i)$$

4) Update the weights

$$w_{i,k+1} = w_{i,k} + \alpha_k \left[\frac{w_{i,k} P_i(y_k)}{P(y_k)} - w_{i,k} \right]$$

5) Adjust adaptive gain α:

$$\alpha_{k+1} = \frac{1}{k+1}$$

6) $k = k + 1$ go to step (2) while

$$|(w_{i,k+1} - w_{i,k})/w_{i,k}| > \varepsilon$$

B. *Faults Detection with Bayes Test*

Surveys on design algorithms for failure detection are given in the works [14], [15]. The rule of decision-making of Bayes is written in the following form:

$$\Lambda(r) = \frac{P(r/H_1)}{P(r/H_0)} \begin{array}{c} H_1 \\ > \\ < \\ H_0 \end{array} \frac{P_0(C_{10} - C_{00})}{P_1(C_{01} - C_{11})} = \eta \qquad (23)$$

$\Lambda(y)$: ratio of conditional probabilities densities;
η: threshold which depends on a priori probabilities P_i and C_{ij}. In many practical cases, it is often selected: $C_{00} = C_{01} = 0$ and $C_{10} = C_{11}$, the expression of η depends only on laws a priori P_i, the expression of the threshold of the decision is written as:

$$\eta = \frac{P_0}{1 - P_0} \qquad (24)$$

the progression of adaptive threshold which is given by the algorithm in paragraph (V.A) by the following equation:

$$\eta_k = \frac{(1 - W_{1,k+1})}{W_{1,k+1}} \qquad (25)$$

where $W_{1,k+1}$ corresponds to the estimate P_1 to have hypothesis H_1 at time$(k+1)$.

C. *Bayes Test performance*

To optimize the decision-making by the Bayes test, it is necessary to minimize at the same time the rate of false alarm and the rate of non-detection. Then, it is very delicate to regulate the two probabilities independently. The probability of false alarm, P_{fa} and missed detection P_{nd} can be defined by

$$P_{fa} = \int_{\gamma}^{+\infty} p(r/H_0) \quad dr \qquad (26)$$

$$P_{nd} = \int_{-\infty}^{\gamma} p(r/H_0) \quad dr \qquad (27)$$

VI. ILLUSTRATIVE EXAMPLE

The effectiveness of the present FDI strategy is illustrated through computer simulations for the linearised discrete-time model of a simplified longitudinal flight control system [16]:

$$x_{k+1} = (A + \Delta A)x_k + (B_k + \Delta B)u_k + F^a f_k^a + w_k$$
$$y_k = Cx_k + F^s f_k^s + v_k \tag{28}$$

where the state variables are: pitch angle δ_z , pitch rate w_z and normal velocity η_y, the control input u_k is the elevator control signal. F^a and F^s are the matrices distributions of the actuator fault f_k^a and sensor fault f_k^s. The terms $E^x d_k$ represents the parameter perturbations in matrices A and B:

$$E^x d_k = \Delta A x_k + \Delta B u_k \tag{29}$$

where $\Delta_k \Delta_k^T \leq I$
The system parameter matrices are :

$$A = \begin{bmatrix} 0.9944 & -0.1203 & -0.4302 \\ 0.0017 & 0.9902 & -0.0747 \\ 0 & 0.8187 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4252 \\ -0.0082 \\ 0.1813 \end{bmatrix}$$

$$C = I_{3\times3}$$

$$x = \begin{bmatrix} \eta_y & w_z & \delta_z \end{bmatrix}$$

The covariance matrices for process and measurement noise sequences are $W^x = diag\left\{0.01^2, 0.01^2, 0.1^2\right\}$ and $V = 0.1^2 I_{3\times3}$.

As an actuator fault, we consider a loss in the actuator effectiveness, abruptly (step wise) $f_k^a = -\rho u_k^n$, $0 < \rho < 1$, with the influence pattern,$F^a = B$. Likewise, a sensor fault is modeled by abrupt changes, $f_k^s = \Delta x_k$, for the output measurement with $F^{s_i} = C^i$. The unknown inputs is given by:

$$E_k^x d_k = E_k^x \left\{ \begin{bmatrix} \Delta a_{11} & \Delta a_{12} & \Delta a_{13} \\ \Delta a_{21} & \Delta a_{22} & \Delta a_{23} \end{bmatrix} x_k + \begin{bmatrix} \Delta b_1 \\ \Delta b_2 \end{bmatrix} u_k \right\} \tag{30}$$

where Δa_{ij} and Δb_{ij} $(i = 1, 2.j = 1, 2, 3)$ are perturbations in aerodynamic and control coefficients.

In this example, the aerodynamic coefficients are perturbed by $\pm 50\%$, i.e $\Delta a_{ij} = -0.5 a_{ij}$ and $\Delta b_{ij} = -0.5 b_{ij}$. In addition, we set $u_k = 1, x_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T, P_0 = 0.1^2 eye(3)$

For a better analysis of the sensitivity of residuals compared to the faults and disturbances, we take a low magnitude of faults. Then, we must apply the Bayes test with adaptive threshold, to show its power and its robustness in the procedure of detection of faults.

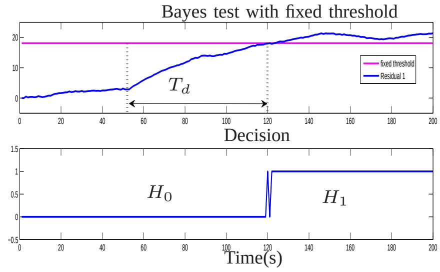

Fig. 3. Bayes test with fixed threshold of the residual1

Figure (3) shows that the Bayes test with fixed threshold does not detected the change of operation at the desired moment $(t_{fa} = 50s, t_{fs} = 90s)$. The decision function indicates that the detection delay is given by $T_d = 70s$, hence, in terms of power the test is very weak. On the other hand, figure (4) which presents the Bayes test with adaptive threshold shows that the fault is detected suitably and the threshold adapts with the evolution of the residual signal. The way it increases the power of the test for the decision-making between the two hypotheses H_0 and H_1. The detection delay for a Bayes test with adaptive threshold is $T_d = 28s$

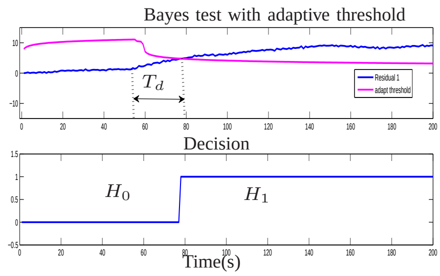

Fig. 4. Bayes test with adaptive threshold of the residual1

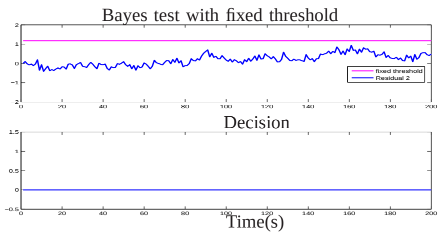

Fig. 5. Bayes test with fixed threshold of the residual2

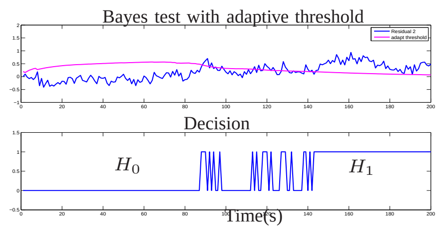

Fig. 6. Bayes test with adaptive threshold of the residual 2

In figure (5) the Bayes test with fixed threshold, for the second residual, does not detect the fault of low magnitude. It is clear that the detection threshold is above the signal. Then, no decision was made between the two hypotheses H_0 and H_1. In figure (6) we notice that the Bayes test with adaptive threshold starts to detect the change of operation by the adaptation of the threshold but with a delay $T_d = 70s$ this results in a number of commutations raised in the graph of decision making.

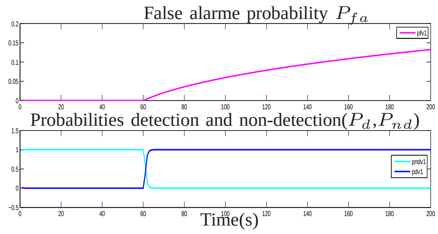

Fig. 7. False alarme and non-detection probabilities progression in Bayes test with adaptive threshold of the residual 1

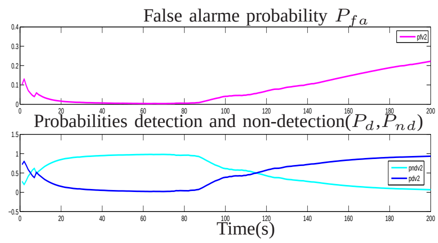

Fig. 8. False alarme and non-detection probabilities progression in Bayes test with adaptive threshold of the residual2

Results of simulation in the figures (7,8), show the performance of Bayes test with adaptive threshold. The use of adaptive threshold increases the probability of detection during the period of presence of faults. On the other hand, we notice that the probability of non-detection is null $P_{nd} = 0$

VII. Conclusion

In this paper, we developed the robust fault detection and isolation of linear stochastic systems subjected to unknown disturbances, actuator and sensor. A bank of Robust To Stage Kalman filter is adapted to estimate the state and the fault as well as to generate the residual sensitive to faults and insensitive to uncertainties. Besides, we implemented the robust decision theory by the adaptive threshold for change detection in a residual can illustrate the faults appearance. This work shown that the improvement of performances can be presented, we decrease the detection time and false alarm probability and we increase the detection probability. This technique is based on the estimate of the a priori probabilities by a non parametric method using Gaussian kernels.

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