5th International Conference on Control Engineering&Information Technology (CEIT-2017)
Proceeding of Engineering and Technology –PET
Vol.33 pp. 23-26

Solutions for mobile localization with hybrid TOA/AOA, GPS and Extended Kalman Filter

Rabaa Ibrahmi[1], Tahar Ezzedine[*2]

[#] *Communications Systems Laboratory,* [#]*University of Tunis El Manar*
National Engineering School of Tunis
[1]ibrahmirabaa@gmail.com
[2]tahar.ezzedine@enit.rnu.tn

***Abstract*—The mobile robots and the mobile localization are nonlinear systems. So, a linear process model is generated out of the non-linear dynamic systems in this paper. This document presents three algorithms; the first is a hybrid TOA/AOA (TS/LS) (Time Of Arrival, Angle Of Arrival)(Taylor Series-Least Square), the second is the Global Positioning System and the finally is the Extended Kalman Filter. The GPS and EKF can trace the objects relatively well, further reducing the positioning error.**

***Keywords*— MS; BTS; hybrid TOA/AOA; GPS; EKF.**

I. INTRODUCTION

Now, the information of the present mobile's position is very important because it gives any service of autonomous vehicle and C-ITS (cooperative Intelligent Transportation System) [1]. Use GPS is the same way to provide information of mobile's position. This document presents in the first a hybrid TOA/AOA (TS- LS) localization algorithm which extends the Taylor Series/Least Square [2] and in the second it presents a vehicular positioning with GPS.

The EKF is a method through which the state propagation equations and the sensor models can be linearized about the current state estimate.

II. HYBRID TOA/AOA LOCALIZATION ALGORITHM

In the first, designate (x_i,y_i) as the position of the ith BS and (x,y) as the position of the MS. The home BS is the first BS. t_i is the Time Of Arrival measurement at the ith BS and θ is the Angle Of Arrival measurement at the home BS. $r_i = ct_i$, where r_i is the range measurement between the ith BS and the MS, c is the speed of light.
Designate the noise free value of {*} as $\{*\}^0$, the range measurement r_i can be modeled as:

$$r_i = r_i^0 + n_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} + n_i, \quad i = 1 \ldots M \quad (1)$$

where n_i is the measurement error which is modeled as a zero-mean Gaussian variable i.e, $n_i \sim N(0, \sigma_n^2)$
Expanding (1), and introducing a new variable R = $x^2 + y^2$ we obtain:

$$r_i^{02} = (r_i - n_i)^2 = (x-x_i)^2 + (y-y_i)^2$$
$$r_i^{02} = r_i^2 + n_i^2 - 2r_i n_i = x^2 + y^2 + x_i^2 + y_i^2 + 2xx_i + 2yy_i$$
$$r_i^2 + n_i^2 - 2n_i r_i = R + K_i - 2xx_i - 2yy_i,$$
$$i = 1,2 \ldots M \quad (2)$$

then

$$r_1^0 \sin n_\theta = (x - x_1)sin\theta - (y - y_1)cos\theta \quad (3)$$

and

$$r_1^0 n_\theta \approx -x_1 sin\theta + y_1 cos\theta + x sin\theta - y cos\theta \quad (4)$$

Let $Z = [x \quad y \quad R]^T$, and rewriting (2), (4) in the matrix form, we have

$$\varphi = h - G_a Z_a^0 \quad (5)$$

Where

$$h = \begin{bmatrix} (r_1^2 - K_1) & / & 2 \\ (r_M^2 - K_M) & / & 2 \\ -x_1 sin\theta & + & y_1 cos\theta \end{bmatrix} \quad (6)$$

$$G_a = \begin{bmatrix} -x_1 & -y_1 & 1/2 \\ -x_M & -y_M & 1/2 \\ -sin\theta & cos\theta & 0 \end{bmatrix} \quad (7)$$

Note from (1) that $r_i = r_1^0 + n_i$:

$$n_i r_i - n_i^2/2 = n_i(r_1^0 + n_i) - n_i^2/2 = n_i r_1^0 + n_i^2 - n_i^2/2$$
$$n_i r_i - n_i^2/2 = n_i r_1^0 + n_i^2/2$$

φ is found to be

$$\varphi = \begin{bmatrix} BE + 0.5E.E \\ r_1^0 n_\theta \end{bmatrix} \quad (8)$$

where

$$B = diag\{r_1^0, r_2^0 \ldots r_M^0\}, \quad E = [n_1 \ n_2 \ldots \ n_M]^T \quad (9)$$

and the covariance matrix Ψ is evaluated as

$$\Psi = E(\varphi\varphi^T) = E[B(EE^T)B^T - 0.5B(EE^T)E^T + 0.5E(EE^T)B^T + 1/4E(EE^T)E^T]$$
$$\Psi = E[B(EE^T)B^T + 1/4E(EE^T)E^T]$$
$$\Psi = E[\varphi\varphi^T]$$

$$\Psi = B' \begin{bmatrix} 4\sigma_n^2 I_M & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} B' + \begin{bmatrix} 2\sigma_n^4 + I_M + \sigma_n^4 1_M & 0 \\ 0 & 0 \end{bmatrix} \quad (10)$$

Assuming the independence of x, y and R the Maximum Likelihood (ML) estimator of Z_a is

ISSN 2356-5608

$$Z_a = \arg\min\{(h - G_a Z_a)^T \Psi^{-1} (h - G_a Z_a)\} = (G_a^T \Psi^{-1} G_a)^{-1} G_a^T \Psi^{-1} h \quad (11).$$

III. GPS

A. GPS DEFINITION

GPS (The Global Positioning System) is a satellite-based navigation system that produces information correlated to location of object equipped with GPS receiver [3].

In this section, we define the vehicular positioning with GPS[4] for trajectory tracking of the MS and we improve the localization accuracy.

B. THE ESTIMATION ERRORS WITH GPS

Designated the obtained initial value of (x, y) as $(x\hat{}, y\hat{})$, we obtain:

$$\hat{x} = \hat{x} + \Delta x, \quad \hat{y} = \hat{y} + \Delta y \quad (12)$$

where Δx and Δy are the estimation errors to be determined.

Utilizing (1) into Taylor Series and restraining the first order terms, we obtain:

$$r_i^0 = \hat{r}_i + \frac{\hat{x} - x_i}{\hat{r}_i}\Delta x + \frac{\hat{y} - y_i}{\hat{r}_i}\Delta y \quad (13)$$

Where $\hat{r}_i = \sqrt{(\hat{x} - x_i)^2 + (\hat{y} - y_i)^2}$

Substituting (12) into (4), we obtain:

$$r_1^0 n_\theta = (\hat{x} - x_1)\sin\theta - (\hat{y} - y_1)\cos\theta + \Delta x \sin\theta - \Delta y \cos\theta \quad (14)$$

Defining $Z'_a = [\Delta x \ \ \Delta y]^T$ and expressing (13), (14) in the matrix form, we have:

$$\varphi' = h' - G'_a Z'_a \quad (15)$$

Where

$$\varphi' = [n_1 \ n_2 \ldots n_M \ (r_1 - n_1)n_\theta]^T \ (16)$$

$$h' = [r_1 - \hat{r}_1 \ldots r_M - \hat{r}_M \ \ (\hat{x} - x_1)sin\theta - (\hat{y} - y_1)cos\theta]^T \ (17)$$

$$G'_a = \begin{bmatrix} \frac{\hat{x} - x_1}{\hat{r}_1} & \ldots & \frac{\hat{x} - x_M}{\hat{r}_M} & -sin\theta \\ \frac{(\hat{y} - y_1)}{\hat{r}_1} & \ldots & \frac{(\hat{y} - y_M)}{\hat{r}_M} & cos\theta \end{bmatrix} \quad (18)$$

The covariance matrix $\psi' = E[\varphi' \varphi'^T]$ can be easily evaluated. The WLS estimation of (15) is then given by:

$$Z'_a = (G'^T_a \Psi'^{-1} G'_a)^{-1} G'^T_a \Psi'^{-1} h' \quad (19)$$

and the covariance matrix of Z_a' is:

$$var(Z'_a) = \left(G'^T_a \Psi'^{-1} G'_a\right)^{-1} \quad (20)$$

The location estimate can then be updated using (12)

$$\hat{x} = \hat{x} + \Delta x, \quad \hat{y} = \hat{y} + \Delta y \quad (12)$$

IV. SIMUTATONS RESULTS

We obtain four conventional algorithms (TOA (TS-LS), TOA/AOA (TS-LS), TOA/AOA positioning with single BS and TOA/AOA positioning with multiple utilizing LS)

In the first figure, we compare the performance of the TOA/AOA (TS-LS) with the other algorithms.

The cellular networks utilize a hexagonal layout that we use it.

Three BSs are deployed at:

1- $(0m, 0m)$
2- $(2000\sqrt{3}m, -2000m)$
3- $(2000\sqrt{3}m, 2000m)$.

The MS is randomly arranged within a $2000\sqrt{3}m \times 4000m$ rectangle comprised the three BSs.

The standard deviation of the Time Of Arrival measurement error σn is disposed to $200m$, and the other parameters utilized in the simulation are defined in Table I, unless otherwise stated.

The performance measure of the algorithm is obtained as the Root Mean Square Error (RMSE) defined in (21), and is computed over 10,000 independent runs.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[(\hat{x}_i - x_1^0)^2 + (\hat{y}_i - y_1^0)^2} \ (21)$$

TABLE 1

PARAMETERS SETTING IN THE SIMULATION

Standard Deviation σ	**System Model**		**Observation Model**		
	ax (m/s^2)	ay (m/s^2)	θ (°)	v (m/s)	a (m/s^2)
Value	0.1	0.1	2	3	1

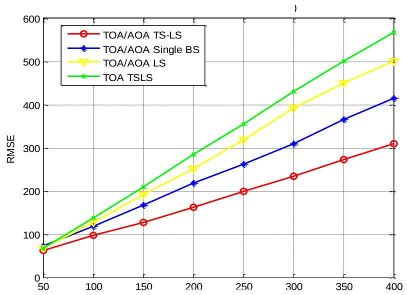

Fig. 1. RMSE based on the measurement error of TOA

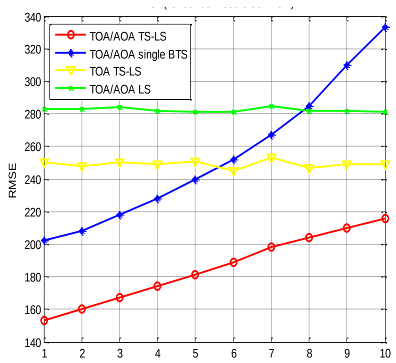

Fig. 2. RMSE as a function of the measurement error of the angle of arrival AOA

V. SCENARIO

For Δx and Δy to be weak, we will use the following method:

- ✓ Install a GPS module on each BTS in the chosen area (Military Zone for example)
- ✓ Or install a GPS module on the BTS
- ✓ There are two methods to know the position of BTS:
  - ❖ By GPS module that gives us the estimated value
  - ❖ The exact value of BTS
- ✓ Install a GPS module on the vehicle
- ➢ How can one calculate the distance $\boldsymbol{r_1}$ with this method?

1) Calculate the estimated position using the GPS / IMU module of BTS Home

$$\hat{x}_k^- = \begin{bmatrix} (V_{Long}^-)_{k-1} + \Delta t.(a_{Long}^-)_k \\ (ACC_x - g.sin\theta_{k-1}^-).cos\theta_{k-1}^- \\ \theta_{k-1}^- + \Delta t.(GYRO_y)_k \end{bmatrix}$$

$$z_k = \begin{bmatrix} (V_{GPS})_k \\ (V_{GPS})_k \\ \left(\tan^{-1}\frac{V_Z}{V_{XY}}\right)_k \end{bmatrix}$$

- $\hat{X}_K^-, \hat{Y}_K^-$ are vehicle's position that are estimated as longitude and latitude.
- Where θ is pitch angle.
- $GYRO_y$ and $GYRO_z$ are y and z-axis angular velocity.
- ACC_x is x-axis acceleration.
- g is acceleration of gravity.
- a_{Long} and V_{Long} are the longitudinal acceleration and velocity.
- V_{GPS} and ψ_{GPS} and X_{GPS} and Y_{GPS} are velocity and yaw and longitude and latitude from NMEA of GPS data.
- V_Z and V_{XY} is velocity calculated by variation of altitude and longitude and latitude, respectively.

2) Compare this position to the exact position of BTS

$$\left.\begin{matrix} (x,y)_{(BTS)} \\ (x,y)_{(GPS)} \end{matrix}\right\} = (\Delta x , \Delta y)$$

3) Find the error Δx and Δy:

$$(\Delta x , \Delta y) = (x,y)_{(BTS)} - (x,y)_{(GPS)}$$

4) Calculate the estimated vehicle position

$$\hat{x}_k^- = \begin{bmatrix} (V_{Long}^-)_{k-1} + \Delta t.(a_{Long}^-)_k \\ (ACC_x - g.sin\theta_{k-1}^-).cos\theta_{k-1}^- \\ \theta_{k-1}^- + \Delta t.(GYRO_y)_k \end{bmatrix}$$

$$z_k = \begin{bmatrix} (V_{GPS})_k \\ (V_{GPS})_k \\ \left(\tan^{-1}\frac{V_Z}{V_{XY}}\right)_k \end{bmatrix}$$

5) Correct the position of the vehicle with the error found (Δx et Δy)

VI. TRACKING ALGORITHM

A. EKF

In this section, we illustrate the EKF [5] nonlinear filtering for trajectory tracking of the MS and to more ameliorate the localization accuracy:

The basic frame for the EKF necessitates the estimation of the state of a discrete-time nonlinear dynamic system,

$$\begin{cases} x_k = \dot{f}(x_k, u_k) \\ y_x = h(x_k) \end{cases}$$

where $\boldsymbol{x_k}$ is the unobserved state of the system and $\mathbf{Y_k}$ is the only observed signal. The *process* noise $\boldsymbol{u_k}$ drives the dynamic system.

The idea is to approximate the nonlinear functions by linearization:

Equations of linear approximation:

$$\text{Model of prediction: } \phi \approx \frac{\partial f(x,u)}{\partial x}$$

$$\text{Model of measures: } H_k \approx \frac{\partial h(x)}{\partial x}$$

The equations are now linearized.

B. Numerical Result

We analyze the performance of the algorithm for tracking utilizing GPS and EKF in this section, we obtain:

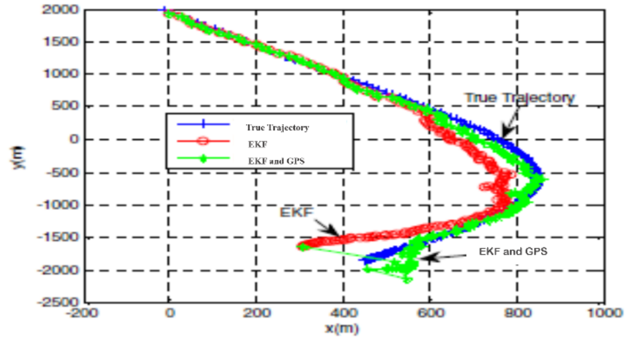

Fig. 3. Sample source trajectory and tracking using the EKF and GPS

The GPS and EKF is generated to give closer tracking of the route than EKF that it truly takes variance of the noises and the statistical mean.

VII. Conclusion

This document present a hybrid TOA/AOA (Time of Arrival/Angle of Arrival). The algorithm use the Taylor Series and Least Square (TS-LS) method. In addition, the GPS and EKF can track the objects, further reducing the positioning error. Simulation results that illustrate the proposed TOA/AOA (TS-LS) can make better performance than conventional schemes in localization accuracy.

References

[1] J. Kim, S. Lee, "A vehicular positioning with GPS/IMU using adaptive control of filter noise covariance" ICT Express 2, 2016, pp. 41-46.

[2] A. K. Wong, "An AGPS-Based Elderly Tracking System," 1st Internantional Conference on Ubiquitous and Future Networks, IEEE ICUFN, June 2009.

[3] S.T Abbas, Z. Ahmed and A. Inam "Study on localization of moving objects Using Wireless Sensor Networks", ICTC, IEEE, 2015.

[4] C. O. Andrei and A. Kukko, "Precise carrier phase-based point positioning of boad-mounted tessestrial remote sensing platform" IEEE, 2014

[5] H. Qasem and L. Reindl, "Unscented and extended kalman estimators for nonlinear indoor tracking using distance measurements," in Positioning, Navigation and Communication, 2007. WPNC '07. 4th Workshop on, 2007, pp. 177-181.