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Comparison of particle swarm optimization and generalized geometric programming for the design of a discrete controller

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***Abstract*—The main purpose of this paper is the design of a discrete fixed low order controller with time specifications. This controller is synthetized to reach some step performances such as settling time and overshoot. The determination of the controller parameters leads to resolve a non-convex optimization problem. As the resolution of this problem must generate a global solution, the use of a global optimization method is suggested. A comparative study between Particle Swarm Optimization (PSO), Generalized Geometric Programming (GGP) and the gradient methods with different initializations is proposed. Simulation results are presented to show the efficiency of each proposed method.**

Keywords—fixed low order controller; non-convex optimization; time response; particle swarm optimization; generalized geometric programming

I. INTRODUCTION

The design of a controller that meets specific performances has interested many researchers in different fields. Many of them focused on the PID controller because of its simple structure and robust performance to resolve these problems [1-3]. At present, the PID controller is used for many applications such as, aerospace, renewable energy, medicine, etc. Yet, industrial plants are burdened with characteristics such as high order, time delays and nonlinearities [4]. Accordingly, tuned PID with Particle Swarm Optimization (PSO) has been proposed, to solve the problem of parameter estimation for nonlinear dynamic rational filters [5]. Also, for the highly complex and nonlinear processes, Fuzzy Logic Controllers (FLC) have been developed [6]. Additionally, for higher order systems an algebraic scheme using model order formulation has been proposed to design a PID controller [4]. Unfortunately, one of the major drawbacks of these PID controllers is that it cannot fulfil the accuracy of the desired step performances.

In order to come over these difficulties, authors in [7-8] developed a method for the design of a continuous fixed low order controller using non-convex optimization. Solving the non-convex optimization problem has interested noted researchers in different fields with the object to find the global minimum [9-11].

Stochastic search methods such as Particle Swarm Optimization (PSO) proposed by Eberhart and Kennedy are well known for achieving high efficiency and searching global optimal solution in problem space [12-13]. PSO has been applied to many control systems [14]. In addition, the deterministic method Generalized Geometric Programming (GGP) has made its proof in global optimization that mainly appeared in engineering design, management and chemical process industry [15].

In this paper, we are going to extend works in [7] for the discrete Linear Time Invariant (LTI), Single Input Single Output (SISO) plant in order to develop a controller that reaches the target time specifications. The characteristic polynomial coefficients are defined by the user as shown in [16]. A comparative study between the PSO, the GGP and the gradient methods with different initializations is established to obtain a controller that fits the most with the time specifications.

This paper is organized as follows, in section II, the problem statement is presented. In section III, the PSO method is introduced. The GGP method is developed in section IV. Simulation results are proposed with a comparison between the PSO, the GGP and the Gradient methods in section V. The last section is devoted to conclude this paper.

II. PROBLEM STATEMENT

Let consider the closed loop system in Fig.1

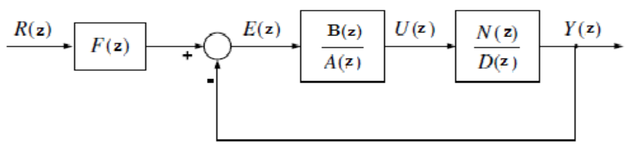

Fig.1. A feedback control system with cascade configuration.

This system is presented by a plant *G(z)* and a fixed low order controller *C(z)*, such as:

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$$G(z)=\frac{N(z)}{D(z)}=\frac{n_m z^m+n_{m-1}z^{m-1}+...+n_0}{d_l z^l+d_{l-1}z^{l-1}+...+d_0}\quad,\ m\le l \tag{1}$$

The fixed low order controller is:

$$C(z)=\frac{B(z)}{A(z)}=\frac{b_r z^r+b_{r-1}z^{r-1}+\ldots+b_0}{z^t+a_{t-1}z^{t-1}+\ldots+a_0} \tag{2}$$

For a low order controller $r\le t\le l-1$.

Thus, the closed-loop transfer function is

$$T(z)=\frac{F(z)B(z)N(z)}{A(z)D(z)+B(z)N(z)}=\frac{F(z)B(z)N(z)}{\delta(z)} \tag{3}$$

Then the closed-loop equation is given by:

$$\delta(z)=A(z)N(z)+B(z)N(z) \tag{4}$$

where F(z) is considered as:

$$F(z)\ =\ f_q z^q+\ f_{q-1}z^{q-1}+\ldots+\ f_0 \tag{5}$$

F(z) is introduced in order to fix the closed-loop system's gain.

Hence, the characteristic polynomial *δ (z)* is represented by:

$$\delta(z)=\delta_n z^n+\delta_{n-1}z^{n-1}+\ldots+\delta_0\qquad,\ n=l+t \tag{6}$$

Once the model and its structure are set, the main purpose is to design the controller that matches the desired settling time and overshoot. Accordingly, the target model is defined as:

$$T^*(z)=\frac{F(z)B(z)N(z)}{\delta^*(z)} \tag{7}$$

where *F(z)* is chosen as $T^*(1)=1$.

We determine the desired characteristic polynomial $\delta^*(s)$ [2] [16]. This polynomial allows reaching the required settling time and overshooting. After that, $\delta^*(s)$ is discretized using the zero order holder. Then, we represent the controller parameters *C (z)* with the vector

$$x=\begin{bmatrix}b_0 & b_1 & ... & b_r & a_0 & a_1 & ... & a_{t-1}\end{bmatrix} \tag{8}$$

Let the coefficient vectors of $\delta(z)$ and $\delta^*(z)$ be respectively:

$$\delta=\ \begin{bmatrix}\delta_0\,\delta_1\,...\,\delta_{n-1}\delta_n\end{bmatrix} \tag{9}$$

$$\delta^*=\begin{bmatrix}\delta^*{}_0\,\delta^*{}_1\,...\,\delta^*{}_{n-1}\delta^*{}_n\end{bmatrix} \tag{10}$$

The closed-loop characteristic polynomial can be expressed as

$$\delta\ (z)=A(z)D(z)+B(z)N(z)=Px+q \tag{11}$$

where

$$P=\begin{bmatrix} n_0 & 0 & 0 & d_0 & & 0\\ n_1 & n_1 & 0 & d_1 & \cdots & 0\\ \vdots & \vdots & \vdots & d_2 & & 0\\ n_m & n_{m-1} & n_{m-r} & \vdots & \cdots & \vdots\\ \vdots & \vdots & \vdots & d_l & & d_{l-t+1}\\ 0 & 0 & n_m & \vdots & & \vdots\\ \vdots & \vdots & \vdots & 0 & \cdots & d_l\\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$q=\begin{bmatrix}0 & ... & 0 & d_0 & ... & d_{l-1} & ... & d_l\end{bmatrix}^T$$

$P\in \mathbb{R}^{(n+1)*(r+t+1)}$, $x\in \mathbb{R}^{(r+t+1)}$, $q\in\ \mathbb{R}^{n+1}$

The controller parameters result from the minimization between *δ and δ**. We define the weighted cost function:

$$f_0(x)=\ \left[\delta(z)-\ \delta^*(z)\right]^T W\ \left[\delta(z)-\ \delta^*(z)\right] \tag{12}$$

Where W is the weighted matrix [17].

By using (11) and (12) we obtain:

$$f_0(x)=x^T\left[P^TWP\right]x+2\left[(q-\delta^*)^TWP\right]x$$
$$+\left[(q-\delta^*)^TW(q-\delta^*)\right] \tag{13}$$

Then, $f_0(x)$ is minimized with regards to x as follows: $\min\limits_x\ f_0(x)$

This non-convex problem can be resolved by local approach the gradient method or some several global optimization techniques. Accordingly, the comparison between the PSO, the GGP and the gradient methods is suggested.

III. Particle swarm optimization

PSO is a heuristic population-based optimization technique. It is one of the most used methods because of its robustness in solving problems with nonlinearities [18].

The population is assimilated to a swarm of particles updating from iteration to iteration. The particles change their state in the search space until they reach the optimal solution. Each particle moves in the direction to its previously best (pbest) position and the global best (gbest) position in the swarm [18].

In addition, the experiences are accelerated by two factors c_1 and c_2, and two random numbers r_1 and r_2 generated between [0,1] while the movement is multiplied by an inertia factor w varying between $[w_{\min}, w_{\max}]$. With each population, update of the velocity v of each dimension D is adjusted by the combination of particles information to compute the new position of particles [13].

For the population of size N_p and dimension D, each particle's position is $X_i=\left[X_{i,1}\,X_{i,2}\ ...\ X_{i,D}\right]$ and the initial velocity of each particle X_i is $V_i=\left[V_{i,1}\ V_{i,2}\ ...\ V_{i,D}\right]$, where $i=1,..,N_p$ and $j=1,..,D$.

The PSO algorithm is given as follows [13]:

1. Set parameters w_{min}, $w_{\max}$, c_1, c_2, D, r_1, r_2 and *N*max of PSO.
2. Initialize population of particles having positions *X* and velocities *V*.
3. Set iteration *t =1*.
4. Calculate the fitness of the particles $F_i^t=f(X_i^t)$ and find the index of the best particle b.
5. Select $pbest_i^t=X_i^t,\forall i$ and $gbest^t=X_b^t$
6. Update the inertia factor
   $w\ =\ w_{\max}-t(w_{\max}-w_{\min})/N\max$
   where $N\max$ is the maximum number of iterations.
7. Update the velocity and position of particles
   $V_{i,j}^{t+1}=w.V_{i,j}^t+c_1.r_1(pbest_{i,j}^t-X_{i,j}^t)$
   $+c_2.r_2.(gbest_j^t-X_{i,j}^t);\ \ \forall i$ and $\forall j$

$X_{i,j}^{t+1} = X_{i,j}^{t} + V_{i,j}^{t+1};\ \forall i$ and $\forall j$

8. Evaluate the fitness $F_i^{t+1} = f(X_i^{t+1})$ and find the index of the best particle b_1.
9. Update *pbest* of population $\forall i$, if $F_i^{t+1} < F_i^t$ then $pbest_i^{t+1} = X_i^{t+1}$ else $pbest_i^{t+1} = pbest_i^t$
10. Update *gbest* of the population
If $F_b^{t+1} < F_{b_1}^t$ then $gbest^{t+1} = pbest_{b_1}^{t+1}$ and set $b = b_1$ else $gbest^{t+1} = gbest^t$
11. If $t<N$max then $t = t+1$ and repeat from step 6 else go to 12
12. Print the optimum solution *gbest.*

IV. GENERALIZED GEOMETRIC PROGRAMMING:

GGP is a deterministic global optimization method based on variable transformation. This mathematical transformation is required for the convexification of the objective function [10].
The mathematical formulation of a GGP problem is expressed as follows [19]:

$$\min_X Z(X) = \sum_{p=1}^{T_0} c_p z_p \tag{14}$$

Where

$$z_p = x_1^{\alpha_{p1}} x_2^{\alpha_{p2}} \ldots x_n^{\alpha_{pn}},\ p = 1,\ldots,T_0 \tag{15}$$

$$X = (x_1, x_2, \ldots, x_m, x_{m+1}, \ldots, x_n),\ \underline{x}_i \le x_i \le \overline{x}_i \tag{16}$$

$x_i > 0$, for $1 \le i \le m$ and $x_i \le 0$, for $m+1 \le i \le n$, $c_p \in \Re$, $\alpha_{pi} \in \Re$ for $1 \le i \le m$, α_{pi} is integer $m+1 \le i \le n$ and $\underline{x}_i, \overline{x}_i$ are respectively, lower and upper bounds of continuous variables x_i.

Some definitions should be presented before introducing the convexification propositions and property.

Definition 1 [20]**:** A "*monomial*" function is a product of power **t**erms and it can be given by:

$$f(X) = c\prod_{i=1}^{n} x_i^{p_i} \tag{17}$$

where c is a real constant and p_i can be negative or positive power for $1 \le i \le n$.

Definition 2: A "*signomial*" function is constituted of a sum with products of power terms, where each product with power terms is multiplied by a real constant [20]:

$$f(X) = \sum_{j=1}^{T} c_j \prod_{i=1}^{n} x_i^{p_{i,j}} \tag{18}$$

The constants c_j and powers $p_{i,j}$ for $1 \le i \le n$ and $1 \le j \le T$ can be positive or negative.

Definition 3: The function $f(X)$ is called a" *posynomial*", when all constants c_j for $1 \le j \le T$ in a signomial function of equation $f(X) = \sum_{j=1}^{T} c_j \prod_{i=1}^{n} x_i^{pi} = \sum_{j=1}^{T} c_j \exp(\sum_{i=1}^{n} p_i y_i)$ are positive.

Optimization problems that possess only signomial terms are called GGP problems.

The following propositions allow analyzing the convexity of a function.

Proposition 1 [20]: A twice-differential function $f(X) = c\prod_{i=1}^{n} x_i^{pi}$ is convex in $\Re_+^n$ for $c \ge 0$ if $p_i \le 0$.

Proposition 2 [19]: A twice-differential function $f(X) = c\prod_{i=1}^{n} x_i^{pi}$ is convex in $\Re_+^n$ for $c \le 0$ if $p_i \ge 0$ and $(1 - \sum_{i=1}^{n} p_i) \ge 0$.

Property 1 [21]: The function $c\exp(\sum_{i=1}^{n} p_i x_i)$ is convex in $\Re_+^n$ for $c \ge 0$ and $p_i \in \Re$.

Convexification strategy :

The convexification strategy is based on variable transformation that permits to convexify each monomial of the signomial depending on their signs [10].

Positively signed term (c>0):

Consider the monomial function (17) where $p_i > 0$.

New variable y_i are presented according to $x_i = \exp(y_i)$, $i = 1,..,n$.

$$f(X) = c\prod_{i=1}^{n} x_i^{pi} = c\exp(\sum_{i=1}^{n} p_i y_i) \tag{19}$$

According to property 1, the signomial equation is convex relatively to y_i. The transformation is called exponential transformation.

Negatively signed terms (c<0):

Consider the monomial function $f(X) = c\prod_{i=1}^{n} x_i^{pi}$, where $p_i > 0$, and $(1 - \sum_{i=1}^{n} p_i) < 0$, new variable z_i are presented according to $x_i = z_i^{\frac{1}{\beta}}$, $i = 1,2,\ldots,n$ where $\beta = \sum_{i=1}^{n} p_i$.

We obtain the equality: $$f(X) = c\prod_{i=1}^{n} x_i^{pi} = c\prod_{i=1}^{n} z_i^{\frac{P_i}{\beta}} \tag{20}$$

According to proposition 2, the function is convex according to z_i, as the sum of exponent is equal to 1 and they are all positive. We can also convexify $f(x)$ by choosing

$\beta > \sum_{i=1}^{n} p_i$. This transformation is related to power transformation.

V. SIMULATION RESULTS:

In this section, Gradient method, PSO and GGP are applied for the design of a discrete fixed low order controller.

The solutions of these methods will be used to evaluate the efficiency of each optimization technique.

A. Example:

We consider the following continuous system:

$$G(s) = \frac{s+3}{s^5 + 16s^4 + 72s^3 + 224s^2 + 81.6s + 13.4} \quad (21)$$

By using a zero order holder we obtain the discrete system:

$$G(z) = \frac{0.02699z^4 + 0.02163z^3 - 0.0005379z^2 - 1.647.10^{-06} z - 2.128.10^{-09}}{z^5 - 1.208z^4 + 0.4225z^3 + 0.0001046z^2 + 3.503.10^{-05} z - 5.162.10^{-16}} \quad (22)$$

The objective is to design a 3rd order controller with the following specifications:

- Overshoot $\leq 1\%$
- 2% settling time $\leq 11s$

We proceed with the design as follows:

Step 1: We use the CRA method to obtain the target model (7) [7]. For that, the following parameters are chosen: $\tau = 5.25$ and $\alpha_1 = 2.4$. By using the method presented in [7], we obtain the target continuous polynomial:

$$\delta^*(s) = s^8 + 18.63s^7 + 139.2s^6 + 562.4s^5 + 1339s^4 + 1921s^3 + 1624s^2 + 742.4s + 141.4 \quad (23)$$

The application of a zero order holder to the continuous polynomial $\delta^*(s)$ gives rise to the discrete polynomial defined by $\delta^*(z)$, whose coefficients are shown in Table I.

Step 2: The 3rd order controller is

$$C(z) = \frac{b_2 z^2 + b_1 z + b_0}{z^3 + a_2 z^2 + a_1 z + a_0} \quad (24)$$

We set the matrix P and the vector q, then we define the weighted matrix coefficients

$$w_{ij} = \begin{cases} 0.3 \text{ for } i, j = 0,..,3 \text{ and } i = j \\ 0.025 \text{ for } i, j = 4,..,7 \text{ and } i = j \\ 0 \text{ for } i \neq j \end{cases}$$

Case 1 Gradient method:

Using the gradient method in the resolution of (13) gives rise to different solutions depending on the initialization. In fact, the choice of two different starting points: $x_1 = [0\ 0\ 0\ 0\ 0\ 0]$ and $x_2 = [-5\ 1\ -3\ 2\ -1\ 5]$, leads to two different controllers $C_1(z)$ and $C_2(z)$:

$$C_1(z) = \frac{-0.73z^2 - 0.1071z - 0.1279}{z^3 - 0.302z^2 + 0.1028z} \quad (25)$$

$$C_2(z) = \frac{-2.4862z^2 + 2.630z - 1.1013}{z^3 - 0.3020z^2 + 0.168z + 0.0535} \quad (26)$$

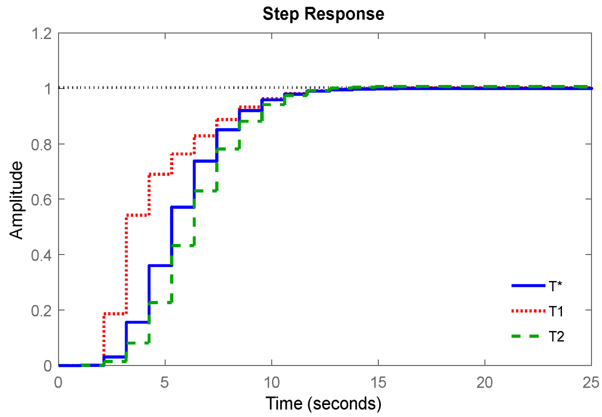

Fig. 2. Step response of the closed-loop system for different controllers

From Fig. 2 it is shown that, we obtained a closed-loop system response with a settling time about 9.11s and without overshoot by considering the first controller $C_1(z)$. While, the second controller $C_2(z)$ gave rise to a settling time about 8.9s without overshoot. Accordingly, as we are looking for a global solution this method is going to be discarded because of its local character.

Case 2 comparing PSO and GGP:

To resolve (13) with the PSO we set:

- Inertia weight: $w_{\min} = 0.4$, $w_{max} = 0.9$, $r_1, r_2 \in rand[0,1]$ and Nmax=1000.
- Acceleration factors: $c_1 = c_2 = 2$
- Population size $N_p = 100$ with the dimension $D = 1$
- Initial velocity : 10% of the initial position X_0

Hence, we obtain the 3rd order controller:

$$C_{PSO}(z) = \frac{-0.0456z^2 - 0.1382z - 0.1176}{z^3 - 0.302z^2 + 0.1021z - 0.0005} \quad (27)$$

The resolution of (13) with the GGP leads to the controller:

$$C_{GGP}(z) = \frac{-0.1207z^2 - 0.0489z - 0.1478}{z^3 - 0.3011z^2 + 0.1041z + 0.0011} \quad (28)$$

The characteristic coefficients of both closed-loop systems are compared with the target polynomial $\delta^*(z)$ in Table I. It is shown that the coefficients are approximatively the same.

As shown in Fig. 3, the closed-loop system response with the PSO and the GGP methods, achieved respectively 10.7s without overshoot and 10.9s without overshoot. Accordingly, both methods achieve the desired requirements.

Fig. 4 shows the control signals obtained with the two methods.

We should also note that, the PSO necessitates an execution time of 42.48s to converge into the solution, while the GGP takes only 0.96 seconds.

Our objective now is to synthetize a controller with the following specifications:

- Overshoot $\leq 1\%$
- 2% settling time $\leq 6s$

Let the designed controller be:

$$C(z)=\frac{b_2z^2+b_1z+b_0}{z^3+a_2z^2+a_1z+a_0} \quad (29)$$

The resolution of (13) with the PSO method and the GGP leads respectively to the controllers:

$$C_{PSO}(z)=\frac{-0.4529^2-0.0213z-0.2104}{z^3-0.0130z^2+0.0508\,z+0.0108} \quad (30)$$

$$C_{GGP}(z)=\frac{-0.2533z^2-0.2587z\ -0.1299}{z^3-0.0130z^2+0.0454z\ -0.0063} \quad (31)$$

The coefficients of the closed-loop systems and the characteristic polynomial are given in Table II.

Fig.5 shows that the closed-loop systems achieved the desired time specifications with a settling time about 5.57*s* for the PSO method and 5.67*s* for the GGP method. We also notice that the control signals depicted in Fig.6 provide a small variation between the PSO method and the GGP method. Thus, both of the optimization methods allowed designing closed-loop systems close to the target model.

TABLE I. COEFFICIENTS OF THE TARGET AND CHARCTERISTIC POLYNOMIALS

INDEX (i)	δ_i^*	$\delta_{i_{PSO}}$	$\delta_{i_{GGP}}$
0	$5.556.10^{-09}$	$2.503.10^{-10}$	$3.146.10^{-10}$
1	$-9.581.10^{-07}$	$1.765.10^{-07}$	$2.83.10^{-07}$
2	$8.339.10^{-05}$	$6.701.10^{-05}$	$8.338.10^{-05}$
3	- 0.002696	- 0.00268	- 0.002696
4	0.03766	0.03761	0.03766
5	-0.2561	- 0.256	- 0.2557
6	0.8882	0.8882	0.8871
7	-1.51	-1.51	-1.509
8	1	1	1

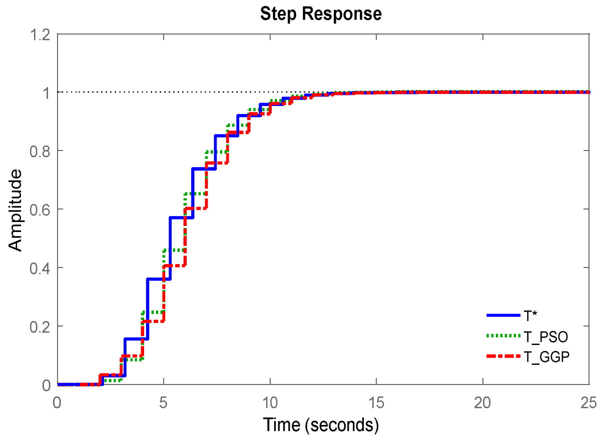

Fig. 3. Step response of the closed-loop system with PSO and GGP for the first example

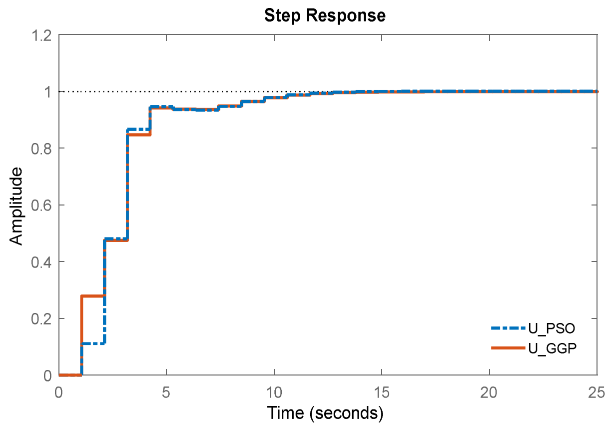

Fig.4. Control signals of PSO and GGP closed-loop systems for the first example

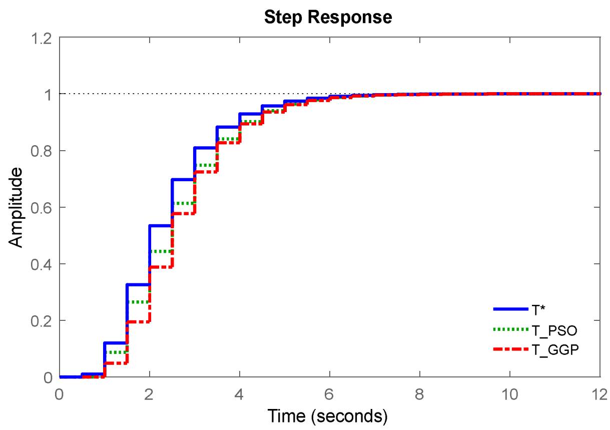

Fig.5. Step response of the closed-loop system for the second example

TABLE II. COEFFICIENTS OF THE TARGET AND CHARCTERISTIC POLYNOMIALS FOR THE SECOND EXAMPLE

INDEX (i)	δ_i^*	$\delta_{i_{PSO}}$	$\delta_{i_{GGP}}$
0	$1.068.10^{-28}$	$4.477.10^{-10}$	$2.764.10^{-10}$
1	$- 3.036.10^{-16}$	$7.249.10^{-07}$	$4.352.10^{-07}$
2	$1.972.10^{-09}$	0.0001161	$7.255.10^{-05}$
3	$- 1.401.10^{-05}$	$2.911.10^{-05}$	$- 4.122.10^{-06}$
4	0.002609	0.002554	0.002639
5	-0.06639	- 0.06633	-0.06639
6	0.4768	0.4768	0.4768
7	- 1.221	- 1.221	- 1.221
8	1	1	1

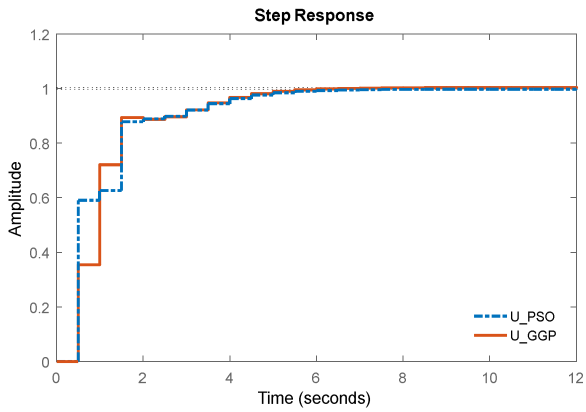

Fig.6. Control signals of PSO and GGP closed-loop systems for the second example

VI. CONCLUSION AND PROSPECT:

A comparative study between the PSO and GGP methods for the design of a discrete fixed low order controller have been presented in this work. The main purpose of these controllers is to meet some time specifications such us settling time and overshoot.

Even though, the PSO is a stochastic method, which has the advantage of solving non-convex optimization with a simple algorithm, it has the disadvantage to be limited by the search space and the population size. Therefore, it may not converge to the global optimum. It is also a time-consuming technique, which can affect the control of real time applications. While the GGP method reaches the global solution and then the desired specifications in a short time. In prospect, we will extend this work to the design of a robust fixed low order controller for uncertain parameters systems.

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