

# Pachycondyla APIcalis ants (API) Algorithm for Multi-user Detection of SDMA-OFDM System

Saliha Azzeddinne, Mahdjoub Zoubir  
Université de Sidi-Bel-Abbes-Algeria

\* Corresponding author's Email: [sali.azzeddine@yahoo.com](mailto:sali.azzeddine@yahoo.com)

**Abstract**—Space Division Multiple Access (SDMA) based technique as a subclass of Multiple Input Multiple Output (MIMO) systems achieves high spectral efficiency through bandwidth reuse by multiple users. On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) mitigates the impairments of the propagation channel. The combination of these tools has emerged as a most competitive technology for future wireless communication system. Different Multiuser Detection (MUD) schemes have been proposed at the BS receiver to identify users correctly by mitigating the multiuser interference. However, most of the classical MUDs fail to separate the users signals in the over load scenario, where the number of users exceed the number of receiving antennas. In this work, MUD tool based on the Pachycondyla APIcalis ants (API) algorithm is proposed in the multiuser MIMO-OFDM system and its performance is compared to existing MUDs such as MMSE (Minimum mean square) and ML algorithm. The simulation results show that the API-MUD algorithm can achieve higher performance.

**Keywords**—OFDM, SDMA, Multiuser Detection (MUD), Optimisation, Pachycondyla APIcalis ants (API).

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) and Multiple Input Multiple Output (MIMO) are the most competitive technologies to satisfy future requirements of wireless access systems. OFDM is an efficient technique for transmission over frequency selective channels [1]. The diversity can be maximized by transmitting symbols through different antennas at different frequencies using combination of OFDM and multiple antenna systems which results in smaller probability of error because of diversity gain. Instead of achieving diversity gain, we can use multiple antennas for multiple users in space division multiple access (SDMA) [2, 3]. Therefore, SDMA have been combined with OFDM in order to mitigate the technical challenges of channel impairments and enhance the spectral efficiency [4].

In the SDMA uplink, multiple users communicate simultaneously with a multiple antenna Base Station (BS) sharing the same frequency band by exploring their unique user specific-special spatial signature. However, the non-orthogonality nature of the channel impulse responses (CIRs) raises a challenge to the process of multi-user detection (MUD), and different schemes have been proposed at the Base Station (BS) receiver to identify the user signals based on their

CIRs. Maximum likelihood (ML) methods are known as an optimum tool for MUD, but they are associated with high computational complexity that grows exponentially with the increase of the number of simultaneous supported users. Additionally, in the overloaded scenario problem which means that number of users are higher than number of receive antennas, the classic MUD can't detect and separate the users signal, large search space and the number of solutions is increasing [5].

Hence, cost function minimization based Minimum Error Rate (MER) detectors are preferred, which basically minimize the probability of error by iteratively updating receiver's weights using adaptive algorithms.

The linear detectors like Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) detect signals with the aid of a linear combiner. These detectors cannot mitigate the nonlinear degradation caused by the fading channel, because the channel's output phasor constellation often becomes linearly non-separable. Hence, these detectors result high residual error. On the other hand, the nonlinear and computationally intensive Maximum Likelihood (ML) detector is capable of achieving optimal performance through an exhaustive search, which prohibits its usage in practical systems [6, 1]. Considering the tradeoff between complexity and performance, some non-linear MUD techniques like Successive Interference Cancellation (SIC) [7, 8], Parallel Interference Cancellation (PIC) [6], Sphere Decoding (SD) [9–1150] and QR Decomposition (QRD) [12–14] MUDs are introduced. Modifications of SD [15–18] and QRD [19–21] techniques are also proposed in several literatures.

However, all these MUDs either fail to detect users in overload or rank deficient scenarios, where the number of users is more than the number of receiving antennas, or suffer from high complexity. Hence, S. Chen et al. proposed Minimum Bit Error Rate (MBER) MUD by minimizing BER directly rather than minimizing mean square error for CDMA system to support in overload condition [22]. Conjugate Gradient (CG) algorithm is used for updating receiver's adaptable weights [23]. But, it requires proper selection of initial solutions and differentiable cost functions. These drawbacks can be eliminated by incorporating metaheuristic optimization methods, as they start the search process from random positions. M. Y. Alias et al. Proposed Genetic

Algorithm (GA) based MBER MUD and implemented it for the SDMA-OFDM system [24, 25]. Subsequently, the MBER MUD algorithm was modified using other well-known optimization technics like Particle Swarm Optimization (PSO) [26, 27] and Differential Evolution Algorithm (DEA) [28]. The MBER MUD technique is basically designed for Binary Shift Keying (BPSK) modulation scheme.

This work investigates the feasibility of employing the API algorithm for solving the MUD problem in the SDMA-OFDM system. The major advantage of the proposed API-MUD is that it provides a good compromise between the performance and computational complexity than MMSE and ML algorithm for the multi user detection problem in SDMA-OFDM system. The rest of this paper is arranged as follows: Section 2 describes the SDMA-OFDM model, whereas in Section 3 the API-MUD Algorithm is illustrated. The simulation results are introduced in Section 4 and conclusions are drawn in Section 5.

## II. UPLINK SDMA-OFDM MODEL

In this section, we consider an uplink OFDM-SDMA system as shown in figure 1. There are  $T$  active users, each equipped with a single transmit antenna. Let the  $t^{\text{th}}$  user communicates with the base station, transmitting BPSK symbols  $\mathbf{a}_t$ , so that each  $\mathbf{a}_t \in \{\pm 1\}$ . A serial stream of input data symbols at each transmit antenna is converted into parallel form and the OFDM signal is obtained by IFFT operation, that can be detected at each receiver antenna by FFT operation. The received  $m^{\text{th}}$  OFDM symbol of  $k^{\text{th}}$  subcarrier at  $r^{\text{th}}$  receiver antenna after FFT operation can be written as:

$$\mathbf{x}_r(k, m) = \sum_{t=1}^T \mathbf{H}_{r,t}(k, m) \mathbf{a}_t(k, m) + \mathbf{n}_r(k, m) \quad (1)$$

Where  $\mathbf{a}_t(k, m)$  is the transmitted symbol from  $t^{\text{th}}$  transmit antenna on  $k^{\text{th}}$  subcarrier and  $m^{\text{th}}$  OFDM symbol,  $\mathbf{n}_r(k, m)$  is the AWGN at  $r^{\text{th}}$  receiver antenna and  $\mathbf{H}_{r,t}(k, m)$  is the channel coefficient in frequency domain between the  $t^{\text{th}}$  transmit antenna and  $r^{\text{th}}$  receiver antenna.  $\mathbf{H}_{r,t}(k, m)$  is obtained by linear combination of dispersive channel taps [1]:

$$\mathbf{H}(k, m) = \sum_{l=0}^{L-1} \mathbf{h}_l(m) e^{-j \frac{2\pi k l}{N}} \quad k = 0, 1, \dots, N-1 \quad (2)$$

Where  $L$  is the length of channel impulse response,  $N$  is the size of FFT/IFFT,  $\mathbf{h}_l$ , ( $l = 0, 1, \dots, L-1$ ) are channel impulse response coefficients modelled as an independent zero mean complex Gaussian random process. We assume that cyclic prefix of at least  $L$  samples is added to each symbol after IFFT operation at the transmitter. Cyclic prefixes are removed at the receiver before the FFT operation. For notational convenience, let us omit the indices and write (1) in matrix form as:

$$\mathbf{X} = \mathbf{H}\mathbf{a} + \mathbf{n} \quad (3)$$

Where  $\mathbf{X}$  is an  $(R \times 1)$  dimensional received signal,  $\mathbf{a}$  is an  $(T \times 1)$  dimensional transmitted signal,  $\mathbf{n}$  is an  $(R \times 1)$

dimensional noise matrix,  $\mathbf{H}$  is an  $(R \times T)$  dimensional channel matrix in frequency domain. The received signal (3) consists of linear overlapping of the transmitted layers. The task is to detect  $T$  transmitted symbols from a set of  $R$  received symbols corrupted by non-ideal communication channel.

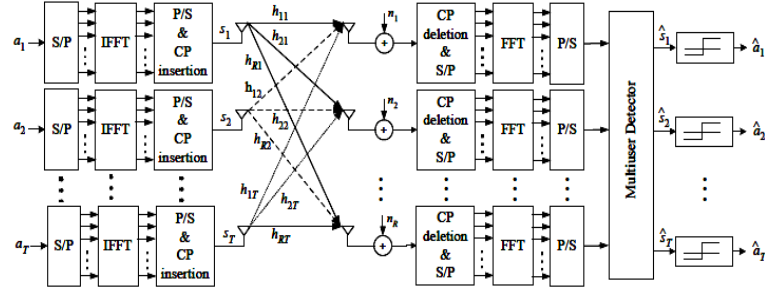


Fig.1 OFDM-SDMA system model [5].

## III. MUD SDMA-OFDM OPTIMIZATION

### A. API algorithm

In this paper, we are interested in a model of the foraging strategy of the *Pachycondyla APicalis Ponerinants* [29,30] and in its application to MUD MIMO-SDMA-OFDM optimization problems.

The powerfulness of API is that it can be easily designed for new search spaces and also because of its simplicity that allows including specialized and dedicated algorithms or sub-heuristics. The API algorithm is based on the natural behavior of *Pachycondyla APicalis* ants.

The main principles of the algorithm can be stated as follows: the nest plays a role of a central point in the considered search space and this point can be moved (either automatically or contextually) as iterations are progressing. Ants are searching in parallel for solutions around the nest in the search space, and they can consider an improvement in the fitness associated to a given hunting site, i.e. a point in the search space, as a prey capture. Ants can also punctually exchange information as with the “tandem running” recruitment (this can be uncorrelated to nest moves) [30].

These ants use relatively simple principles to search their preys, both from global and local view-points. Starting from their nest, they globally cover a given surface by partitioning it into many hunting sites. Each ant performs a local random exploration of its hunting sites and its site choice is sensitive to the success previously met on the sites. These principles can be used to implement a new strategy for the search of a global minimum of a function  $f$  in a search space  $S$ .

The stepwise procedures of the API algorithm are described as following [30]:

- (1) Initialization: set the algorithm parameters.
- (2) Generation of new nest (exploration)
- (3) Exploitation
  - (3.1) Intensification search:
    - For** each ant ,
    - if** has less than hunting sites in its memory,

**then** create a new site in the neighborhood of and exploit this new site;

**ElseIf** the previous site exploitation is successful, then exploits the same site again;

**Else** exploit a probabilistically selected site (among its sites in memory).

**(3.2) Information sharing**

Probabilistically replace a site in the memory of the ant by the best one searched so far in this cycle.

**(3.3)Nest movement:** If the condition for nest movements is satisfied, go to **(4)**; otherwise, go to **(3.1)**.

**(4)Termination test:**

If the test is passed, stop; otherwise, empty the memories of all ants and then go to **(2)**.

The following diagram illustrates the principle of intensified research by each ant.

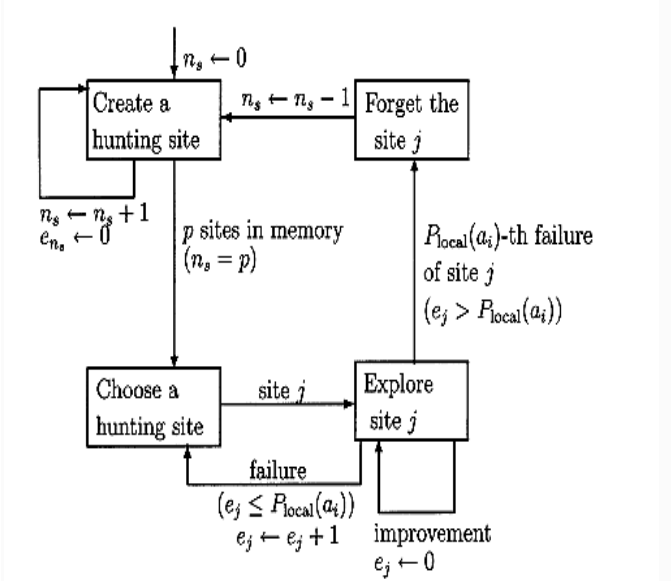


Fig 2. The behavior model of a Pachycondyla APicalis ant [29].

$n_s \leq p$  represents the number of sites memorized by the ant.  $e_j$  represents the number of unsuccessful explorations successively performed by the ant on site  $j$ .

**B. API algorithm for MUD MIMO-SDMA-OFDM optimization**

Naturally, the problem formulation and the corresponding objective function as well as the optimisation procedure used has a crucial impact on the attainable performance as well as complexity of the MUD. ML optimum detector considered as a reference to other detection methods [1] and the ML method minimizes the Euclidean distance between the received signal vectors and the product of all possible transmitted signal vectors with the given channel. The ML detection method determines the estimate of the transmitted signal vector  $x$  as:

$$\hat{s}_{ML} = \arg \left\{ \min_{\check{s} \in M^L} \|x - H\check{s}\|^2 \right\} \quad (4)$$

In the context of the SDMA-OFDM system employinig R receiver antenna elements, the decision metric required for the  $r^{th}$  receiver antenna-specific objective function [25] can be derived from (3), yielding :

$$\Omega_r(\check{s}) = |x_r - H_r\check{s}|^2 \quad (5)$$

Where  $x_r$  is the received symbol at the input of the  $r^{th}$  receiver at a specific OFDM subcarrier, while  $H_r$  is the  $r^{th}$  row of the channel transfer function matrix  $H$ . Therefore, the decision rule for the optimum MUD associated with the  $r^{th}$  antenna is to choose the specific T symbol vector  $\check{s}$ , which minimizes the objective function given in (5). Thus, the estimated transmitted symbol vector of the T users based on the knowledge of the received signal at the  $r^{th}$  receiver antenna and a specific subcarrier is given by:

$$\check{s}_{API} = \arg \{ \min_{\check{s}} [\Omega_r(\check{s})] \} \quad (6)$$

Since there is a total of R number of receiver antennas, the combined objective function can be formulated as [5]:

$$\Omega(\check{s}) = \sum_{r=1}^R \Omega_r(\check{s}) \quad (7)$$

Hence, the decision rule of the API MUD is to find the specific estimated transmitted T symbol vector  $\check{s}_{API}$  that minimizes  $\Omega(\check{s})$  for every OFDM subcarrier considered.

The strategy adopted here, consists in overall modifying population of initial solutions repeatedly to lead to a satisfactory final solution in a reasonable time.

For this purpose, the API method uses movements to pass from a solution to another inside a research space. It is divided into two essential iterative phases: a diversification phase for the promising solution detection by applying Orand stochastic operator, followed by an intensification phase to intensify research in the zone of this solution by applying Oexplo stochastic operator, for finding the best phase vector.

Each API ant presents a feasible solution of symbol vector  $\check{s}_{API}$ . It is much better than the fitness value is minimal as seen in Eq6. Table 1 shows the parameters description of API algorithm.

Table 1. API Parameters Description

Ant number	F
------------	---

Locale patience (iteration number in the intensification process for each ant without improving the current solution).	P
Number of hunting site for each ant.	$H_u$
Operator for generating ant population in diversification process.	$O_{rand}$
Operator for generating neighbor solutions in intensification process for each ant.	$O_{explo}$
Stopping criterion.	itr
Initial nest (Output of the ML MUD for MIMO-OFDM system);	$\tilde{\mathbf{S}}_{ML}$

### C. Ant population generation

A population of ants represents a part of the total search space composed of all the feasible solutions of MUD problem. This operation consists to create randomly a matrix of feasible solution of size  $\mathbf{F} \times \mathbf{H}_u$ . The population will be partitioned into  $\mathbf{F}$  subpopulation. Each subpopulation will be designed for an ant which presents its hunting sites. The population is a class of neighbor solutions generated by applying **Orand** operator which consists of two random swapping in initial solution (symbol vector  $\tilde{\mathbf{S}}_{ML}$ ).

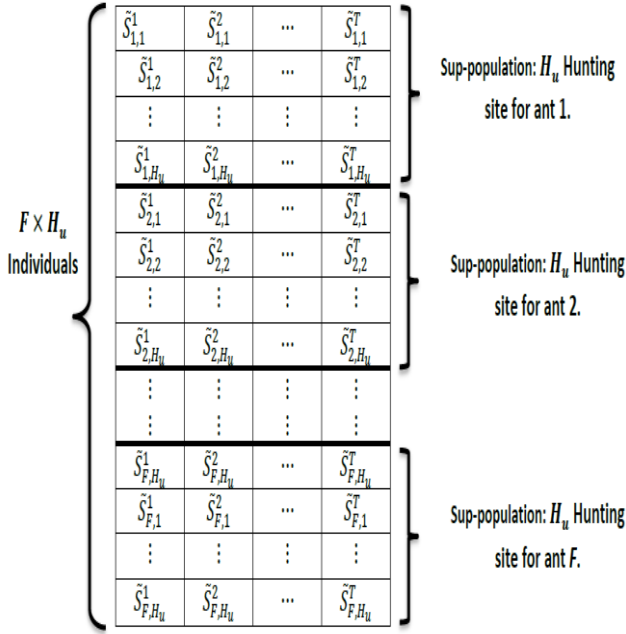


Fig 3. Ant population generation.

The columns number of this matrix is the T (number of users)

### D. Foraging behavior

In parallel, for each ant, we adopt the following intensification process for local site.

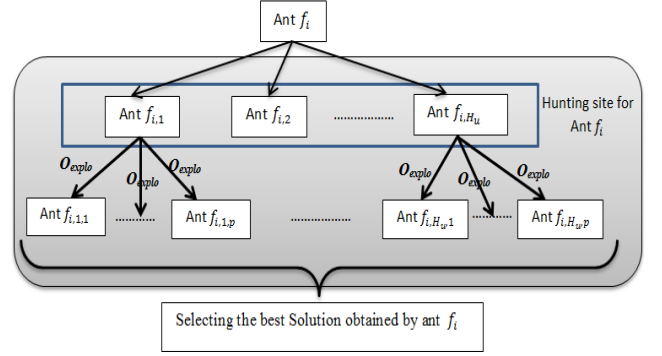


Fig 4. Flowchart of the foraging process for ant  $f_i$ .

With  $f_{i,j,p}$  is a search space node, it is a vector symbol  $\tilde{\mathbf{S}}$  generated by applying  $O_{explo}$  operator;  $i$  is the ant index;  $j$  is the node index,  $p$  is the neighbor index around the node  $j$ . The evaluation and classification of solutions is performed according to the value of fitness function previously defined in Eq.6. For each solution (selected hunting site), a class of neighbor solutions (candidate list) is generated by applying  $O_{explo}$ . It is a perturbation obtained by changing randomly a value of one symbol in the selected solution. This process is repeated for P. It is a parameter of local patience for each ant. After this process, we perform recruitment (select a best site copying between all ants: best vector symbol  $\tilde{\mathbf{S}}_{obtained}$ ) and remove from the ants memories all sites which have been explored unsuccessfully more than P. If more than itr iterations have been performed then reset the memories of all ants.

## IV. SIMULATION RESULTS

In this section, the performance of SDMA-OFDM system under Rayleigh fading channel is evaluated by computer simulations. SDMA-OFDM system is implemented under Matlab environment. Perfect channel estimation is assumed to be done at the BS. The curves show the average BER as a function of average signal-to-noise energy ratio per bit (Eb/N0). The MUD methods used are MMSE, optimal ML, and API tool. The output of the ML method is applied as an initial population for API optimization. The simulation parameters are summarized in table 2.

Table 2. Parameters initialisation

Modulation	BPSK
FFT/IFFT size	128
Length of cyclic prefix	16
Population size (F)	10
number of iterations (itr)	20
Hunting site (Hu)	5
Number of users (T)	Varying
receiver antennas (R)	varying
Local patience (P)	10

In figure 5, performance of API, ML and MMSE based detectors is compared in an OFDM-SDMA system, for four

users equipped with four receiver antennas respectively. It can be seen that the performances of API outperforms the MMSE detector. The reason is quite obvious; API finds the optimal weight vector to minimize the BER while MMSE minimizes the mean square error not the BER.

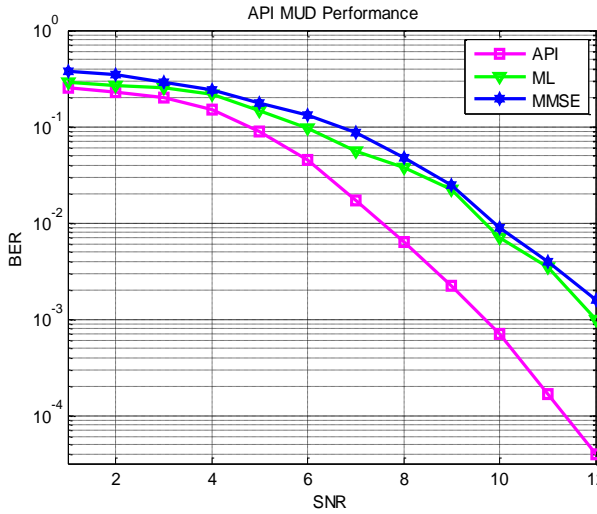


Fig.5 BER performance of two users in an OFDM-SDMA system equipped with two receiver antennas (T=4, R=4) employing API, ML and MMSE algorithm

The Bit Error Rate (BER) of the system in the overloaded system is showing in Fig. 6. Six users (T = 6) are transmitting data and two receive antennas (R = 2) at the BS receiver.

We see that MMSE only supports users, equal in number to the receiver antennas. The addition of a user decreases the order of diversity by one and the diversity becomes equal to one when the number of users becomes equal to the number of receiver antennas. The MMSE MUD becomes incapable of differentiating users as the number of users exceeds the number of receiver antennas. On the other hand, API-MUD supports more user than the MMSE and ML MUD.

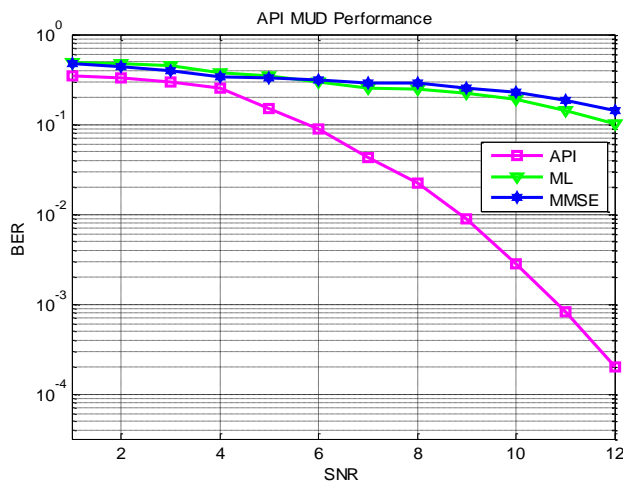


Fig.6 BER performance of two users in an OFDM-SDMA system equipped with two receiver antennas (T=6, R=2) employing API, ML and MMSE algorithm

Figure 7 shows the performance of API for various numbers of ants, where the continuing rising of population size of ants will

develop the performance to approach ML detector output. The system include six users (T = 6) and four receive antennas (R = 4) at the BS. Maximum number of Ants is fixed to F = 20, where the population sizes used are F = 20, F = 10 and F = 5.

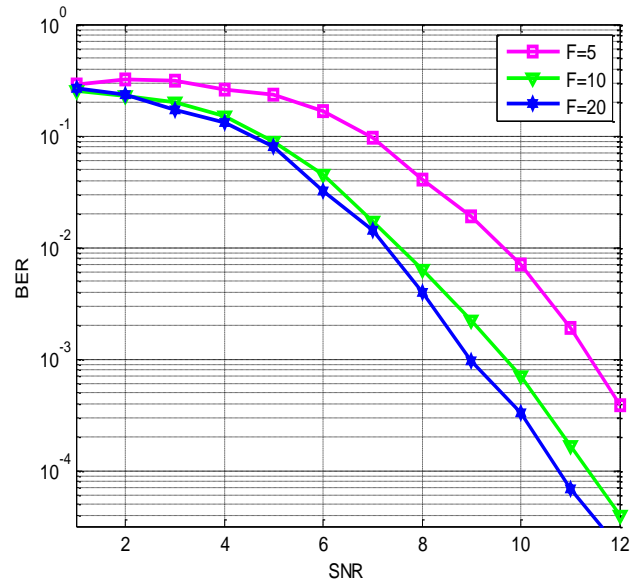


Figure.7 API performance for different population sizes (F).

## V. CONCLUSION

In this paper, API algorithm is proposed for MUD in OFDM-SDMA communication system. Unlike conventional techniques such as the MMSE-MUD, this tool is capable of supporting an overloaded scenario and provides significant improvement in performance, in which the number of users surpasses the number of receiver antennas. Controlling the operational parameters, the API-MUD algorithm provides a means of adaptively obtaining the necessary performance under complexity limitations. Increasing the population size of ants, for example, allows the gradual enhancement of the results to approach the ML detector output.

We conclude that API-MUD is a promising approach for high data rate OFDM-SDMA communication which can support a large number of users, greater than the number of receiver antennas.

One interesting feature of the API tool is that the operational time can be kept fixed while enhancing the performance making use of the algorithm parallelizability.

## REFERENCES

- [1] S. Verdú, "Multiuser Detection", Camb. Univ. Press, Cambridge, U.K, 1998.
- [2] M. Biguesh and A. B. Gerhman, "Training based MIMO channel estimation: A study of estimation tradeoffs and optimal training signals", IEEE Tran. on Signal Process., vol. 54, pp. 884-893, Mar. 2006.
- [3] X. Dai, "Pilot - aided OFDM/SDMA channel estimation with unknown timing offset", IEE Proc. Commun., vol. 153, pp. 352 - 398, Jun. 2006.
- [4] Vandenameele, P., L. Van Der, and M. G. E. Engels, "A combined OFDM/SDMA approach,"IEEE Journal on Selected Areas in Communications, pp 2312-2321, November 2000.

- [5] Ming Jiang, Lajos Hanzo Multiuser MIMO-OFDM for Next-Generation Wireless Systems. Proceedings of the IEEE | Vol. 95, No. 7, July 2007.
- [6] L. Hanzo, M. Munster, B. J. Choi, T. Keller, "OFDM and MC-CDMA for Broadband Multi-User Communications, WLANs and Broadcasting", IEEE Press/Wiley, West Sussex, 2003.
- [7] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas", Wireless Personal Commun., vol. 6, pp. 311–335, Mar. 1998.
- [8] P.W. Wolniansky, G.J.Foschini, G.D. Golden and R.A. Valenzuela, "V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel," Inter. Symp. on Sig., Sys., and Electron., pp. 295–300, 1998.
- [9] M. Damen, K. A. Meraim and J. C. Belfiore, "Generalized sphere decoder for asymmetrical space-time communication architecture," IEEE Electron. Lett., vol. 36, pp. 166–167, Jan. 2000.
- [10] B. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," IEEE Trans. On Commun., vol. 51, pp. 389–399, Mar. 2003.
- [11] B. Hassibi and H. Vikalo, "On the sphere decoding algorithm: I expected complexity", IEEE Trans. On Signal Process., vol. 53, pp. 2806–2818, Aug. 2005.
- [12] K. J. Kim, J. Yue, R. A. Iltis and J. D. Gibson, "A QRD-M/Kalman filter-based detection and channel estimation algorithm for MIMO-OFDM systems," IEEE Trans. Wireless Commun., vol. 4, pp. 710–721, Mar. 2005.
- [13] M. Arar and A. Yongacoglu, "Efficient detection algorithm for  $2N \times 2N$  MIMO systems using alamouti code and QR decomposition", IEEE Commun. Lett., vol. 10, pp. 819–821, Dec. 2006.
- [14] J. Cha, J. Ha and J. Kang, "Low complexity iterative QRD-M detection algorithm for V-BLAST systems", IEEE Electron. Lett., vol. 43, pp. 1374–1376, Nov. 2007.
- [15] A. Burg, M. Borgmann, M. Wenk, M. Zellweger, W. Fichtner and H. Bolckei, "VLSI implementation of MIMO detection using the sphere decoding algorithm", IEEE J. of Solid State Circ., vol. 40, pp. 1566–1577, Jul. 2005.
- [16] M. El-Khamy, H. Vikalo, B. Hassibi, and R. J. McEliece, "Performance of Sphere Decoding of Block Codes," IEEE Trans. on Commun., vol. 57, pp. 2940–2950, Oct. 2009.
- [17] J. S. Kim, S. H. Moon, and I. Lee, "A New Reduced Complexity ML Detection Scheme for MIMO Systems," IEEE Trans. on Commun., vol. 58, pp. 1302–1310, Apr. 2010.
- [18] J. Jalden and P. Elia, "Sphere decoding complexity exponent for decoding full rate codes over the quasi static MIMO channel", IEEE Trans. on Inf. Theory, vol. 58, pp. 5785–5803, Sep. 2012.
- [19] H. Jian, X. Yao and Y. Shi, "A simplified QRD-M signal detection algorithm for MIMO-OFDM systems", J. of Electron., vol. 27, pp. 88–93, Jan. 2010.
- [20] K. J. Kim, M. O. Pun and R. A. Iltis, "QRD based precoded MIMO-OFDM system with reduced feedback", IEEE Trans. on Commun., vol. 58, pp. 394–398, Feb. 2010.
- [21] X. Wu, and J. S. Thompson, "A simplified unbiased MMSE metric based QRDM decoder for MIMO systems", 6th Inter. Conf. on Wirel. Commun., Netw. And Mobi. Compu., pp. 1–4, 2010.
- [22] S. Chen, A. K. Samingan, B. Malgrew and L. Hanzo, "Adaptive minimum BER linear multiuser detection for DC-CDMA signals in multipath channels", IEEE Trans. on Signal Process., vol. 49, pp. 1240–1247, Jun. 2001.
- [23] B. Widrow and S. D. Stearns, "Adaptive Signal Processing", Pears. Edu. Pub., New Delhi, India, 2009.
- [24] M. Y. Alias, A. K. Samingan, S. Chen and L. Hanzo, "Multiple Antenna Aided OFDM Employing Minimum Bit Error Rate Multiuser Detection," IEEE Electron. Lett., vol. 39, pp. 1769–1770, Nov. 2003.
- [25] M. Y. Alias, S. Chen and L. Hanzo, "Multiple-Antenna-Aided OFDM Employing Genetic-Algorithm-Assisted Minimum Bit Error Rate Multiuser Detection," IEEE Trans. on Veh. Technol., vol. 54, pp. 1713–1721, Sep. 2005.
- [26] S. Chen, A. Livingstone and L. Hanzo, "Minimum Bit-Error Rate Design for Space-Time Equalization-Based Multiuser Detection", IEEE Trans. on Commun., vol. 54, pp. 824–832, May 2006.
- [27] H. Rehman, I. Zaka, M. Naeem, S. I. Shah and Jamil Ahmad, "Minimum Bit Error Rate Multiuser Detection for OFDM-SDMA Using Particle Swarm Optimization", Springer Lect. Notes in Comp. Sci., vol. 4681, pp. 1247–1256, 2007.
- [28] J. I. Ababneh, T. F. Aldalgamouni, A. A. Alqudah, "Minimum Bit Error Rate Multiuser Detection of SDMA-OFDM System Using Differential Evolutionary Algorithm", Proc. of IEEE 6th Inter. Conf. On Wireless and Mob. Comp., Net. and Commun., pp. 273–279, 2010.
- [29] S. Aupetit, N. Monmarché et M. Slimane. "Apprentissage des modèles de markov cachés par l'algorithme API". Fourmis Artificielles, des bases de l'optimisation aux applications industrielles, volume 1, Traité IC2. Hermès-Lavoisier, 2009.
- [30] N. Monmarche, G. Venturini, M. Slimane. "On how Pachycondyla apicalis ants suggest a new search algorithm". Future Generation Computer Systems 16, 937- 946, 2000.