5th International Conference on Control Engineering&Information Technology (CEIT-2017)
Proceeding of Engineering and Technology –PET
Vol.32 pp. 80-85

Box-Jenkins approach for modelling the global solar radiation time series

Mohammed ali JALLAL[1] / Samira CHABAA[1,2]
[1]Department of physics, Cadi Ayyad University
Faculty of Sciences, Semlalia Marrakesh, Morocco
[1,2] Industrial Engineering Department, National School of Applied Sciences, Ibn Zohr University, Agadir, Morocco
mohammedali.jallal@edu.uca.ac.ma

Abdelouhab ZEROUAL/ Abdessalam EL YASSINI / Saida IBNYAICH
Department of physics, Cadi Ayyad University
Faculty of Sciences, Semlalia
Marrakesh, Morocco

***Abstract*— In this paper, our objective is to provide to the solar power plant managers a simple and an adequate mathematical models for the prediction of global solar radiation in order to reconstruct the global solar radiation data series in the absence or the rarity of real measurement. In this contribution we are interested to apply the Box and Jenkins method for modeling and predicting the global solar radiation using ARMA model. Different models have been developed and compared with each other using statistical criteria (MAE, MSE, RMSE, and MAPE). The obtained results demonstrate that the most adequate model for modeling our time series is the process MA (1) added to a polynomial tendency of order r = 8, with a linear correlation coefficient R = 0.99942 Close to one which describe the approximation of the output and the target.**

***Keywords*— ARMA; Box-Jenkins approach; Databases; Global solar radiation; Identification; Modelization.**

I. Introduction

The sun can satisfy all our needs if we know how to rationally exploit the energy it radiates to the earth. It shines in the sky for almost 4.6 billion years and scientists have calculated that it is half its existence [1]. It is difficult for us to imagine that in the course of a year the sun diffuses into the earth ten thousand times more energy than that consumed by the whole world population [2]. Today it seems trivial not to benefit because we have the necessary technological means. Moreover, it must be considered that is the cleanest and most abundant renewable energy source available, and Morocco has some of the richest solar resources in the world [3]. Contemporary technology can harness this energy for a diversity of uses, as well as producing electricity, providing light or a comfortable interior environment, and heating water for domestic, commercial, or industrial use. But when setting up an installation project, we usually confront the problems related to the rarity or even the absence of the global solar radiation series of measurements. To overcome these difficulties, several models have emerged that consist in reconstructing the global solar radiation series using models developed from meteorological data such as temperature, humidity, duration of sunshine...etc. In the absence of climate data, it is useful to have an idea about the evolution of the radiation. Since the solar radiation received on the ground is composed of a deterministic component and stochastic component, it has been shown that the sequences of the latter can be described and simulated by artificial intelligence techniques [4,5,6], or by statistical type models, that use the previous data to predict the observations [7], statistical regression models AR (Auto-Regressive), MA (moving Averages) and ARMA (Auto Regressive moving Averages). These models have proved their effectiveness in the prediction and modeling of solar processes [8,9,10]. In this paper we used the Box-Jenkins approach for modeling the global solar radiation to generate estimated values in case of absence or the scarceness of data.

II. Theory for Modeling the time series with an ARMA

A. *The Autoregressive Moving Average process*

ARMA(p,q) model is a grouping of AR(p) and MA(q) models it is appropriate for univariate time series modeling. In an AR(p) model the future value of a variable is supposed to be a linear combination of p previous observations and a random error together with a constant term. Mathematically the AR(p) model can be articulated as [11,12]:

$$X_t = \mu + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \ldots + \varphi_p X_{t-p} + \varepsilon_t \quad (1)$$

Such as X_t and ε_t are respectively the real value and the random error at time period t, φ_i (i = 1, 2,..., p) are the model factors and μ is a constant. The numeral constant p is known as the order of the model. Occasionally the constant term is omitted for simplicity.

Usually for estimating AR process parameters using the given time series, the YuleWalker equations [13] are used.

Just as an AR (p) model regress against previous values of the series, an MA (q) model uses past errors as the descriptive variables. The MA (q) model is given by [11,12,13]:

$$X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \quad (2)$$

Here μ is the mean of the series, θ_j (j = 1, 2,…, q) are the model parameters and q is the model order. The random errors are assumed to be a white noise process [12,13], i.e. a sequence of independent and identically distributed (i.i.d)

ISSN 2356-5608

random variables with zero mean and a constant variance σ^2. Mostly, the random errors are supposed to follow the typical normal distribution. Thus theoretically a moving average model is a linear regression of the current observation of the time series against the random errors of one or more earlier observations. Appropriate an MA model to a time series is more complicated than appropriate an AR model because in the former one the random errors terms are not fore-seeable. Autoregressive (AR) and moving average (MA) models can be efficiently joined together to form a general and suitable class of time series models, known as the ARMA models. Mathematically an ARMA(p,q) model is represented as [11,12,13]:

$$X_t = \mu + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \qquad (3)$$

Here the p,q orders model refer to p autoregressive and q moving average terms. Frequently ARMA models are deployed using the lag operator [12,13] notation. The lag or backshift operator is defined as $LX_t = X_{t-1}$. Polynomials of lag operator or lag polynomials are used to represent ARMA models as shadows [12]:

AR (p) model: $\varepsilon_t = \varphi(L)X_t$.

MA (q) model: $X_t = \theta(L)\varepsilon_t$.

ARMA (p,q) model: $\varphi(L)X_t = \theta(L)\varepsilon_t$.

Here $\varphi(L) = 1 - \sum_{i=1}^{p} \varphi_i L^i$ and $\theta(L) = 1 + \sum_{j=1}^{q} \theta_j L^j$.

It is exposed in [13] that an significant property of AR(p) model is invertibility, i.e. an AR(p) model can permanently be written in terms of an MA(∞) model. Although for an MA(q) model to be invertible, all the roots of the equation θ (L) = 0 must lie external the unit circle. This condition is identified as the invertibility condition for an MA process.

B. Unit root and stationarity tests

The stationarity tests make it possible to check whether a series is stationary or not. The main tests used in the literature are the Augmented Dickey-Fuller and Philipps-Perron tests for which the null hypothesis is the non-stationarity of the series studied [14].

C. Selection criteria : AIC and BIC

It is more expensive in computations to deduce the order p and q for an ARMA(p,q) process, since a two-variable function must now be minimized. The AIC and BIC criteria for an ARMA(p,q) process take the following form [15]:

$$\text{and} \quad \begin{cases} AIC = 2k - 2\ln(\hat{L}) \\ BIC = ln(n)k - 2\ln(\hat{L}) \end{cases} \qquad (4)$$

$\hat{L}$: The maximized value of the likelihood function of the model M, i.e. $\hat{L} = p(X/\hat{\theta}, M)$, where $\hat{\theta}$ are the parameter values that maximize the likelihood function.

X : The observed data.

n : The number of data points in X, the number of observations, or equivalently, the sample size.

k : The number of free parameters to be estimated.

In order to minimize these functions, one method consists in making two iterative loops on p and q to test all the pairs couples (p,q) up to certain terminals p < P and q < Q. Inside these loops, the estimators $\hat{\varphi}$, $\hat{\theta}$ using, for example, the least squares or the maximum likelihood, are calculated first and then the AIC and BIC criteria are calculated for these different couples (p,q) values And we find the minimum of these criteria.

Therefore we have the values $\hat{p}$ and $\hat{q}$ which minimize the AIC or the BIC. This makes it possible to calculate the efficient estimators of the parameters of the ARMA($\hat{p}$,$\hat{q}$) model using the maximum likelihood method.

D. Validation tests:Ljung-Box

The Q(k) statistic of Ljung-Box allows to test the hypothesis of serial independence of a series (or that the series is white noise). More specifically, this statistic tests if the k autocorrelation coefficients are equal to zero. It is based on the sum of the autocorrelations of the series and is distributed according to a chi-squared law with k degrees of freedom. We can be judged the good quality of the estimate values the residue analysis, therefore study the ACF and PACF functions of the residue series $\hat{\varepsilon}_t$, but often this study is summarized by carrying out the test [16]:

H_0 : $\hat{\varepsilon}_t$ Is a white noise.

H_1 : $\hat{\varepsilon}_t$ Is not a white noise.

$$Q(k) = n(n+2)\sum_{j=1}^{k} \frac{\hat{\rho}^2(j)}{(n-j)} \qquad (5)$$

Under H_0, $Q(k) \sim \chi^2(k)$, So if $Q(k)$ is small (the critical probability or the p-value is large i.e. greater than the threshold) the residuals are a white noise.

E. Mesurement of prediction quality

In the field of statistics, the precision prediction is the degree of proximity between the advertised (predicted) quantity and the actual (observed) quantity [17,18].

The statistical tools used to evaluate the performance of deterministic predictions are widespread and used. Recall their formulation:

- Mean Absolute Error :

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|X_i - \hat{X}_i\right| \qquad (6)$$

- Mean Square Error :

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(X_i - \hat{X}_i\right)^2 \qquad (7)$$

- Root mean square error :

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(X_i - \hat{X}_i\right)^2} \qquad (8)$$

- Mean Absolute Percentage Error :

$$MAPE = \frac{100}{N}\sum_{i=1}^{N}\left|X_i - \hat{X}_i\right| \qquad (9)$$

III. Description, processing and visualization of data

A. Description and visualization of data

The data base is constructed from measurements of global solar radiation on horizontal surface (X_t , t ∈ ℤ) at the weather station Agdal Marrakech, the data series considered in this study concerns the global solar radiation measured each half hour from sun set to sun rise during 2014, with (t =1,..., 17520) but in our study we have converted the observation series in the form of a daily average with (t = 1, ..., 365) (Fig. 1).

Before analyzing this series, we start by looking at its evolution in relation to time, sought: abrupt changes, atypical values and to look for an Eventual trend, Eventual seasonality (periodicity).

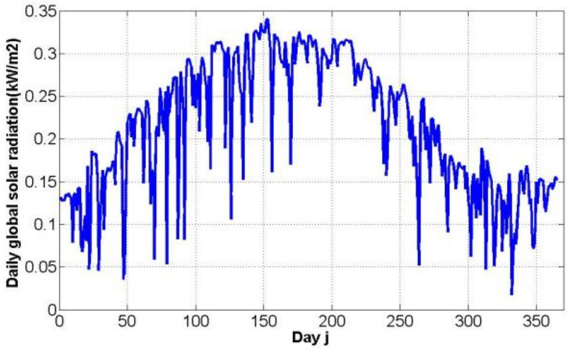

Fig. 1. Daily global solar radiation in Agdal in 2014.

This graph presents the daily evolution of the global solar radiation at Agdal during the year 2014, we note that the observations are not seasonal and evolve according to a polynomial trend as a function of time.

IV. Box and Jenkins Methodology

The various elements presented below, allow us to introduce and discuss the method of Box and Jenkins (1970) which will be very useful for modeling our time series [19].

The modeling steps are as follows:

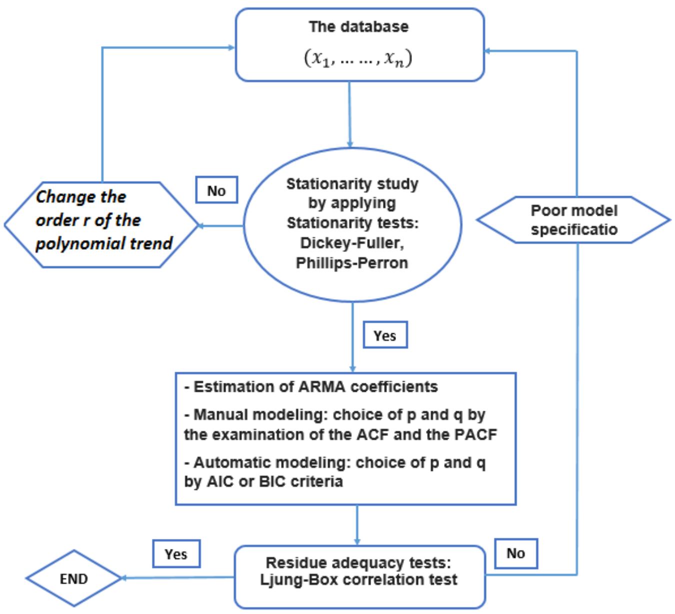

Fig. 2. Algorithm the Box-jenkins for modeling the time series by an ARMA model.

V. Modeling global solar radiation at Agdal using ARMA process

In this part we are interested to apply the modeling algorithm proposed by Box and Jenkins to model the global solar radiation.

A. Modeling the trend component

We estimate the polynomial trend with a well-determined of the order r which ensures the stationarity of the time series.

Figure 3 shows a slow decay of the auto-correlation function ACF, we can say that the time series X_t is not stationary.

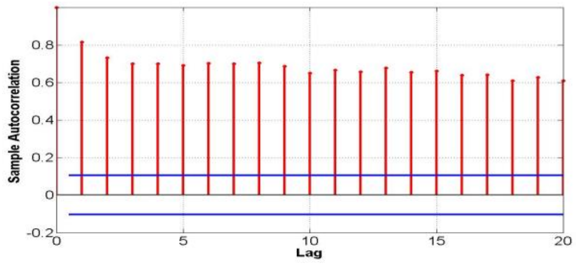

Fig. 3. Sample autocorrelation function the time series X_t.

Such that the P-value of the Phillips Perron and Augmented Dickey-fuller test which corresponds to each autocorrelation coefficient is greater than 5%, thus allowing the null hypothesis of non-stationarity of the series X_t to be accepted.

The tables below summarize the decisions made by the Phillips Perron and Augmented Dickey-fuller tests:

Where:

H = 0: Admit the null hypothesis of non-stationarity X_t.

H = 1: Reject the null hypothesis of non-stationarity X_t.

- Augmented Dickey-fuller (ADF) test:

TABLE I. Decisions made by the application of the augmented Dickey Fuller test on the time series.

ADF_test	**Lag**					
	1	***2***	***3***	***4***	***5***	***6***
P-Value	0.12	0.22	0.31	0.36	0.42	0.45
H	0	0	0	0	0	0

- Phillips Perron (Pp) test:

TABLE II. Decisions made by the application of the Phillips Perron test on the time series.

Pp_test	**Lag**					
	1	***2***	***3***	***4***	***5***	***6***
P-Value	0.08	0.12	0.17	0.20	0.23	0.25
H	0	0	0	0	0	0

For these results we estimate a polynomial tendency with a well–determined order which makes the time series stationary after removal of the trend component.

The estimated polynomial trend is defined as follows:

$$Tr = p_1x^8 + p_2x^7 + p_3x^6 + p_4x^5 + p_5x^4 \quad (10)$$

$$+p_6x^3 + p_7x^2 + p_8x + p_9$$

p_1= 3.41 10^{-19}, p_2= -5.57 10^{-16}, p_3= 3.74 10^{-13}
p_4= -1.32 10^{-10}, p_5= 2.65 10^{-8}, p_6= -3.03 10^{-6},
p_7 = 1.8 10^{-4}, p_8 = -3.3 10^{-3}, p_9 = 0.13.

The following figure shows the series of global solar radiation, the trend and the series obtained after the removal of the trend:

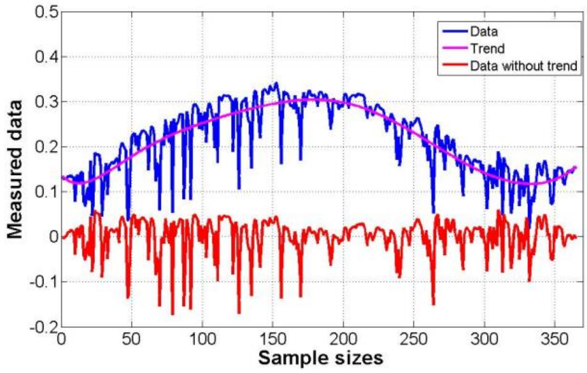

Fig. 4. Global solar radiation, the polynomial trend and the global solar radiation without trend.

The autocorrelation and the partial autocorrelation functions of the time-series X_t after removal of the trend are presented in figure 5:

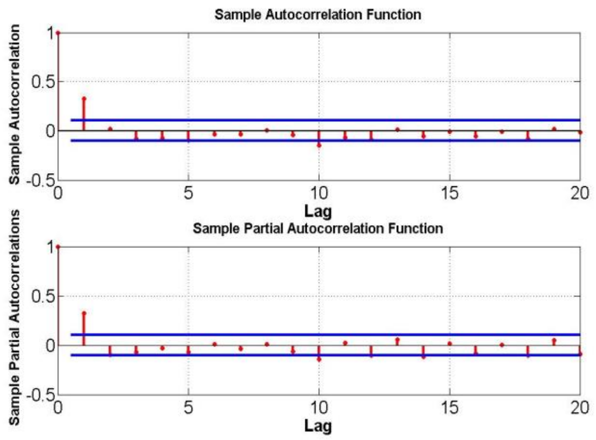

Fig. 5. Autocorrelation and partial autocorrelation functions of the time series X_t without trend.

The figure 5 shows a rapid decay of the autocorrelation and partial autocorrelation function, for this reason, we conclude that the time series X_t after removal of the trend becomes stationary. Such that the P-value of the Phillips Perron and Augmented Dickey-fuller test, which corresponds to each autocorrelation coefficient, is less than 5%, which makes it possible to admit the stationarity of the time-series X_t.

To describe more the obtained results we summarize the decisions made by the Phillips Perron and Augmented Dickey-fuller tests in the following tables:

- Augmented Dickey-fuller test:

TABLE III. DECISIONS MADE BY THE APPLICATION OF THE AUGMENTED DICKEY FULLER TEST ON THE TIME SERIES WITHOUT TREND.

ADF_test	**Lag**					
	1	***2***	***3***	***4***	***5***	***6***
P-Value	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
H	1	1	1	1	1	1

- Phillips Perron test:

TABLE IV. DECISIONS MADE BY THE APPLICATION OF THE PHILLIPS PERRON TEST ON THE TIME SEIES WITHOUT TREND.

Pp_test	**Lag**					
	1	***2***	***3***	***4***	***5***	***6***
P-Value	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
H	1	1	1	1	1	1

B. Choice of the polynomial trend order

We notice that there are several orders which are able to transform the observation time-series X_t to stationary process. The problem is in residual series analysis that will be solved by studying the difference between the observation series and the evolution predicted series (the residual time-series) in the following subsection E.

The only trend order which gives a difference between the measured and predicted series (described in the ensuing subsection E) in the form of a white noise is r = 8 and all the other orders give a residual series containing information (not a white noise). We check this by applying Ljung-Box correlation test, which gives information on the behavior of the residual if it is a white noise or not.

The following table summarizes the prediction quality as a function of the order r of the polynomial trend.

Where:

H = 0: assume that the residual series forms a white noise.

H = 1: reject the hypothesis that the residual series forms a white noise.

TABLE V. LJUNG BOX TEST AS A FUNCTION OF THE TREND ORDER.

Trend order	**Ljung Box Test**	**Lag**				
		1	***2***	***3***	***4***	***5***
7	P_Value	0	2 10^{-8}	6 10^{-8}	9 10^{-8}	4 10^{-7}
	H	1	1	1	1	1
8	P_Value	0.25	0.40	0.23	0.18	0.28
	H	0	0	0	0	0
9	P_Value	0	10^{-12}	10^{-12}	3 10^{-12}	9 10^{-12}
	H	1	1	1	1	1

From this results, we note that the order r = 8 gives a good results than the other orders. Furthermore our optimal trend is a polynomial tendency of order 8.

C. Determination of the optimum couple (p, q) values

We see that P_max = 1 and Q_max = 1 are observed in figure 5 which represent the number of a significant auto-correlation (ACF) and partial auto correlations (PACF) coefficients of the series without trend component.

With the autocorrelogram of (ACF) we can determine q and the partial autocorrelogram of (PACF) allows us to determine p.

We try all the combinations of ARMA (p, q), P∈ {0,1} and Q∈ {0,1} by displaying the results to choose the best suited model.

The information criteria AIC (Akaike Information Criterium) and BIC (Bayesien Information Criterium) are used to determine the couple (p, q) of the optimal model and the best suited model to our time-series.

The BIC and AIC values of all possible combinations of ARMA(p, q) are presented in table VI:

TABLE VI. BIC AND AIC VALUES OF ALL POSSIBLE COMBINATIONS OF ARIMA (P, Q).

Criteria	ARMA(p,q)			
	ARMA(0,0)	***ARMA(0,1)***	***ARMA(1,0)***	***ARMA(1,1)***
BIC	$-1.282\ 10^3$	$-1.318\ 10^3$	$-1.317\ 10^3$	$-1.313\ 10^3$
AIC	$-1.294\ 10^3$	$-1.3338\ 10^3$	$-1.332\ 10^3$	$-1.3331\ 10^3$

The best value given by the BIC and AIC information criteria is the smallest value, in our case we have:

BIC = $-1.318\ 10^3$ and AIC = $-1.3338\ 10^3$.

For this reason, the optimal process for modeling our time series is corresponding to the process ARMA(0,1) or MA(1) Because that which minimizes the criteria of information.

D. Estimation of the coefficients of the obtained model

After the recognition of the pair (p, q) of processes ARMA(p,q) which is MA(1), the coefficient is estimated using the likelihood method.

The developed model which models the series without trend component Xd_t is defined as follows:

$$Xd_t = -9.68\ 10^{-17} + \varepsilon_t - 8.25\ 10^{-3}\ \varepsilon_{t-1}$$

Where $\varepsilon_t \sim BB(0, \sigma^2)$, with $\sigma^2 = 1$.

Variance $Var[Xd_t] = 2\ 10^{-7}$.

The evolution of the series without trend component and its fitting by this model is as follows:

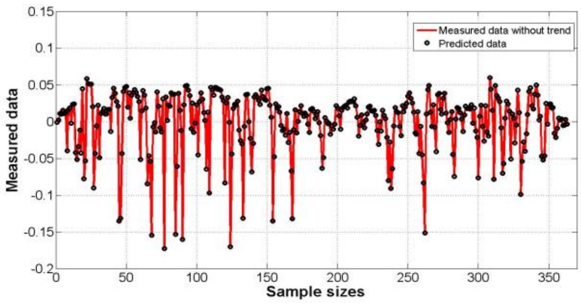

Fig. 6. The series without trend and theirs predicted values by the model MA (1).

The graph shows that there is no difference between the two time-series which permets to conclude that the model fit well the series observation without trend component.

Now we add the trend component to the obtained model MA (1) to define the global model that describes our original time-series X_t of global solar radiation.

The global model becomes:

$$X_t = Xd_t + \text{Tr} \quad \text{Where } \varepsilon_t \sim BB(0, \sigma^2), \text{ with } \sigma^2 = 1.$$

The global solar radiation series X_t and its fit series using the global model are presented in the following figure:

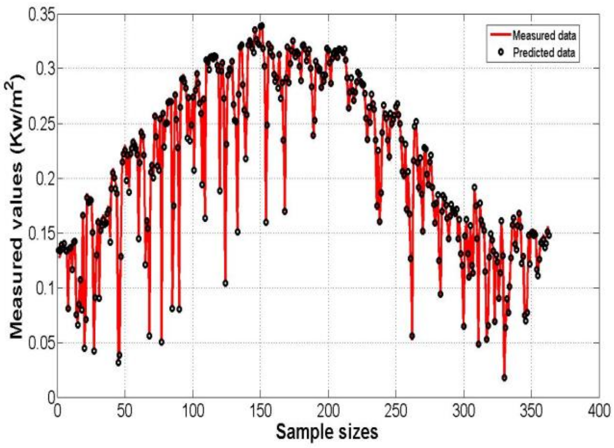

Fig. 7. Global solar radiation and the predicted values by the global model obtained.

From the figure, we notice a good agreement of the measured time series and the predicted time series with the developed model.

E. Study of residuals:

To validate the model, we must to study the residual time-series. The following figure, shows the patterns of the residuals values calculated as the difference between measured values and those predicted by the global model.

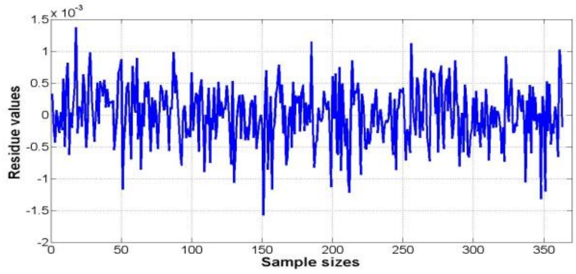

Fig. 8. Evolution of the residual time-series.

We plot the ACF and the PACF to verify that we don't have a significant autocorrelation and a partial autocorrelation coefficients and also to assert the stationarity of the residuals. The Ljung-Box statistic is applied to know if the residuals values describe a white noise or not. The following figure shows the autocorrelograms of ACF and PACF of the residuals series.

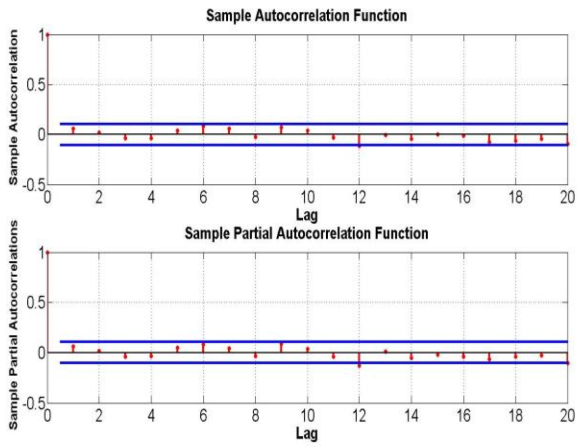

Fig. 9. Autocorrelograms of ACF and PACF of residual time series.

The decisions taken by applying the Ljung-Box statistics test are grouped in table VII:

TABLE VII. DECISIONS TAKEN BY THE APPLICATION OF THE LJUNG-BOX STATISTICS TEST.

Ljung-Box test	Lag					
	1	*2*	*3*	*4*	*5*	*6*
P-Value	0.25	0.40	0.23	0.18	0.28	0.31
H	0	0	0	0	0	0

From the graphs of the autocorrelation function ACF and the partial autocorrelation function PACF of the residuals series; we can affirm that the residuals are stationary because all the values of the ACF and PACF are contained in the blue band. This would mean that at the 5% threshold the residual time series is stationary. In addition all the p-values of the Ljung-Box statistics are greater than 5%; which allows us to conclude that the residual time series is a white noise of variance $\sigma_{\varepsilon}^{2} = 2.01\ 10^{-7}$; hence the validation of this model is achieved.

F. *Measurement of Prediction Quality*

The following table summarizes the values given by the comparison indicators between the measured time-series and predicted time series.

TABLE VIII. QUALITY OF MEASUREMENT.

Comparison indicators	Indicators			
	MSE	***RMSE***	***MAE***	***MAPE***
Value	$7.17\ 10^{-6}$	$2.67\ 10^{-3}$	$2.35\ 10^{-3}$	0.23 %

The comparison indicators values show that this model is efficient.

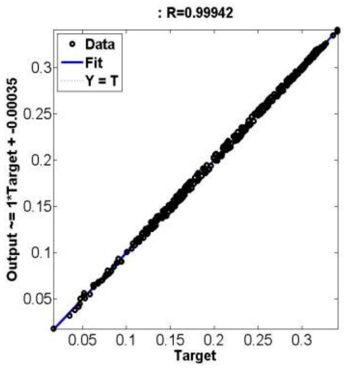

Fig. 10. Linear correlation coefficient to describe the approximation of the predicted values and the observed values.

The following figure shows the concentrations of the time series of measured and predicted values. The linear correlation coefficient describing the approximation of the output (predicted value) and the target (observed value) is R = 0.99942. The value of this coefficient shows that this model is efficient and can be used for modeling purpose with a good accuracy.

VI. CONCLUSION

In this paper, we used statistical methods to model solar radiation using ARMA processes. To rich our goal, we applied the Box-Jenkins strategy. The simulations carried out in this work are concentrated on the study of the prediction performance, for this we used some statistical criteria namely: MSE, RMSE, MAE, MAPE to compare the predicted values and the observed values of global solar radiation time-series. Finally, we asserted that the most adequate model for modeling our time series is a process MA(1) added to a polynomial tendency of order r = 8.

REFERENCES

[1] A.Bonanno, H.Schlattl, L.Paternò(2008); "The age of the Sun and the relativistic corrections in the EOS". Astronomy and Astrophysics. 390 (3): 1115–1118.doi:10.1051/0004-6361:20020749, 2002.

[2] Michael Grätzel, Photoelectrochemical cells Nature 414, 338-344, doi:10.1038/35104607, 2001.

[3] AfDB helps fund $1.44bn Moroccan solar project.

[4] N. Maleki, E. Maleki; "Modeling of Cathode Pt /C Electrocatalyst Degradation and Performance of a PEMFC using Artificial Neural Network". 2016 International Conference on Engineering & MIS (ICEMIS), doi: 10.1145/2832987.2833000

[5] A. Belayadi, L. Ait-Gougam, F. Mekideche-Chafa; "Automatic Pattern Recognition with Wavelet Neural Network". 2016 International Conference on Engineering & MIS (ICEMIS), doi: 10.1145/2832987.2833011.

[6] S. Kouah, I. Kitouni, D. E. Saidouni; "Towards Agent Oriented Methodology: A Fuzzy based Formal Refinement Approach for Multi-agent". 2016 International Conference on Engineering & MIS (ICEMIS), doi: 10.1145/2832987.2833060.

[7] S. Aljawarneh, V. Radhakrishna, P.V. Kumar, V. Janaki; " A similarity measure for temporal pattern discovery in time series data generated by IoT"; 2016 International Conference on Engineering & MIS (ICEMIS), doi: 10.1109/ICEMIS.2016.7745355.

[8] S. Ferrari, M. Lazzaroni, V. Piuri, L. Cristaldi, M. Faifer; "Statistical Models approach for Solar Radiation Prediction", Instrumentation and Measurement Technology Conference, 2013 IEEE International.

[9] S. Safi, A. Zeroual; "MA system identification using higher order cumulants: application to modeling solar radiation", Journal of Statistical Computation and Simulation, Vol.72 (7) , 2002, pp. 533-548.

[10] Jianxin Li, Kaoru Miyashita, Teruyuki Kato, and Shinichi M iyazaki; "GPS time series modeling by autoregressive moving average method: Application to the crustal deformation in central Japan", Earth Planets Space, 52, 155–162, 2000.

[11] J. Lee; "Univariate time series modeling and forecasting (Box-Jenkins Method)", Econ 413, lecture 4.

[12] John H. Cochrane; "Time Series for Macroeconomics and Finance", Graduate School of Business, University of Chicago, spring 1997.

[13] K.W. Hipel, A.I. McLeod; "Time Series Modelling of Water Resources and Environmental Systems", Amsterdam, Elsevier 1994.

[14] Zivot, Eric, Wang, Jiahui; "Modeling Financial Time Series with S-PLUS", ISBN 978-0-387-32348-0, pp:111-138,2006.

[15] Andrew R. Liddle; "Information criteria for astrophysical model selection", Monthly Notices of the Royal Astronomical Society, 377: L74–L78,2007.

[16] G. M. Ljung; G. E. P. Box; "On a Measure of a Lack of Fit in Time Series Models". Biometrika.65(2):297303.doi:10.1093 /biomet/65.2.297, 1978.

[17] Samira CHABAA; " identification des systèmes non linéaires en utilisant les techniques d'intelligences artificielles et les bases de fonctions de Laguerre pour la modélisation des données du trafic dans les réseaux internet ", THESE Présentée à la Faculté des sciences Semlalia-Marrakech,Page: 32-33, 2011.

[18] Rob J. Hyndman and Anne B. Koehler; "Another look at measures of forecast accuracy", International Journal of Forecasting ,2006, 22(4), 679-688.

[19] Box, George; Jenkins, Gwilym . Time SeriesAnalysis:Forecasting and Control.SanFrancisco:Holden-Day,1970.