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Nonlinear Control of MIMO System Using Feedback Linearization Control Method and PD Controller for Tracking Purpose

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***Abstract*— this paper presents feedback linearization and decoupling algorithm for control of a robot manipulator with six degrees of freedom "EpsonC4" using PD controller for purpose of tracking reference trajectory. the nonlinear multi-input Multi-output MIMO system is transformed into six independent single-input single-output SISO linear systems. An extensive MATLAB simulation program was developed and the obtained results in different simulations show the efficiency of the proposed approach.**

***Keywords*— Feedback linearization ,Nonlinear system, robot arm ,MIMO system, SISO linear systems,PD controller.**

I. INTRODUCTION

In this paper we discuss the notion of feedback linearization of nonlinear systems. The basic idea of feedback linearization is to build a nonlinear control law as a so-called inner loop control which, in the ideal case, exactly linearizes the nonlinear system after a suitable state space change of coordinates[2]. The designer can then design a second stage or outer loop control in the new coordinates to satisfy the traditional control design specifications such as tracking, disturbance rejection, and so forth.

In this case,we applied this technique to derive a control law for each joint of a manipulator robot with six degrees of freedom "EpsonC4" .which the dynamic equations form a complex, nonlinear, and multivariable system.

Using matlab, we have applied the technique of Input-Output feedback linearization of nonlinear systems with PD linear controller for each individual decoupled obteined subsystem to control the angular position,velocty and accelration of each joint of this robot for tracking purposes.

The obtained results in different simulations show the efficiency of the proposed approach.

II. INPUT-OUTPUT FEEDBACK LINEARIZATION METHOD FOR MIMO SYSTEM

In this section we discuss the notion of Input-Output feedback linearization of nonlinear systems, The basic idea of feedback linearization is to obtain a linear relation between the output Y and a new input V.

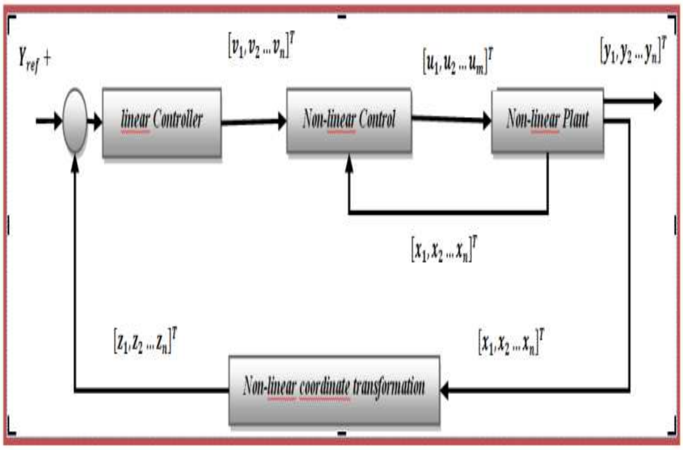

Fig. 1 Structure of feedback linearization controller using exact linearization method

The basic condition for using exact linearization method is nonlinear dynamic MIMO of n -order with p number of inputs and outputs described in the affine form;

$$\begin{cases} \dot{X}(t) = f\big(X(t)\big) + \sum_{i=1}^{p} g_i\big(X(t)\big)U_i(t) \\ Y_i(t) = h_i\big(X(t)\big) \\ i = 1,2, \dots p \end{cases} \quad (1)$$

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Where,
$X = [x_1, x_2 \dots x_n]^T \in R^n$: is the state vector.
$U = [u_1, u_2 \dots u_p]^T \in R^p$: is the control input vector.
$Y = [y_1, y_2 \dots y_p]^T \in R^p$: is the output vector,
$f(X), and\ g_i(X)$: are n-dimentional smooth vector fields.
$h_i(X)$: is smooth nonlinear functions,with i=1,2...n.

Definition1:

Let f: $\mathcal{R}^n \to \mathcal{R}^n$ be a smooth vector field on $\mathcal{R}^n$ and let h : $\mathcal{R}^n \to \mathcal{R}^n$ be a scalar function. The Lie Derivative of h, with respect to f, denoted $L_f h$, is defined as,

$$L_f h = \frac{\partial h}{\partial x} f(x) = \sum_{i=1}^{p} \frac{\partial h}{\partial x_i} f_i(x)$$

The Lie derivative is simply the directional derivative of h in the direction of f(x), equivalently
the inner product of the gradient of h and f. We denote by $L_f{}^2 h$ the Lie Derivative
of $L_f h$ with respect to f:

$$L_f{}^2 h = L_f\,(L_f h)$$

In general we define:

$$L_f{}^k h = L_f(L_f{}^{k-1} h) \quad \text{for } k = 1,\dots,p$$

with $L_f{}^0 h = h$

Definition2:

The function $\Phi: \mathcal{R}^n \to \mathcal{R}^n$ defined in a region he $\Omega \subset \mathcal{R}^n$ is called difeomorphisme if it checks the following conditions:
-A diffeomorphism is simply a differentiable function whose inverse exists and is also differentiable.
-We shall assume both the function and its inverse to be infinitely differentiable. Such functions are customarily referred to as ℂ∞ diffeomorphisms
The diffeomorphisrne is used to transform one nonlinear system in another nonlinear system by making a change of variables of the form:

$$z = \Phi(x)$$

Where Φ(x) *represents n variables*

$$\Phi(x) = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_p \end{bmatrix} = \begin{bmatrix} [h_1 \quad L_f h_1 \quad \dots \quad L_f{}^{r_1 - 1} h_1]^T \\ \vdots \\ [h_p \quad L_f h_p \quad \dots \quad L_f{}^{r_p - 1} h_p]^T \end{bmatrix}, x = [x_1, x_2 \dots x_n]^T$$

The problem is to find a linear relation between the input and the output by deriving the Output until at least one input appears using the expression:

$$y_j{}^{(r_j)} = L_f{}^{r_j} h_j(x) + \sum_{i=1}^{p} L_{g_i}(L_f{}^{(r_{j-1})} h_j(x)) u_i \quad (2)$$
$$i, j = 1,2, \dots p$$

Where: $L_f{}^i h_j$ and $L_g{}^i h_j$:Are the i^{th} Lie derivatives of $h_j(x)$ respectively in the direction of f and g.

$$L_f h_j(x) = \frac{\partial h_j}{\partial x} f(x), \ L_g h_j(x) = \frac{\partial h_j}{\partial x} g_i(x)$$

r_j: is the relative degree corresponding to the y_j output, it's the number of necessary derivatives so that at least one of the inputs appear in the expression.
If expression $L_g h_j(x) = 0$,for all i, then the inputs have not appeared in the derivation and it's necessary to continu the derivation of the output y_j.
The system (1) has the relative degree (r) if it satisfies:

$$\begin{cases} L_{g_i} L_f{}^k h_j = 0 & 0 < k < r_{j-1}, 0 \le i \le n, 0 \le j \le n \\ L_{g_i} L_f{}^k h_j \ne 0 & k = r_{j-1} \end{cases} \quad (3)$$

The total relative degree (r) is defined as the sum of all the relative degrees obtained using (2) and must be less than or equal to the order of the system:

$$r = \sum_{j=1}^{n} r_j \le n \quad (4)$$

To find the expression of the nonlinear control law U that allows to make the relationship linear between the input and the output, the expression (2) is rewritten in matrix form:

$$[y_1{}^{r_1} \dots y_p{}^{r_p}]^T = \propto(x) + \beta(x).U \quad (5)$$
$$V = [v_1\, v_2 \dots v_p]^T = [y_1{}^{r_1} \dots y_p{}^{r_p}]^T \quad (6)$$

Where:

$$\propto(x) = \begin{bmatrix} L_f{}^{r_1} h_1(x) \\ . \\ . \\ . \\ L_f{}^{r_p} h_p(x) \end{bmatrix} \quad (7)$$

$$\beta(x) = \begin{bmatrix} L_{g_1}(L_f{}^{(r_1-1)} h_1(x)) & L_{g_2}(L_f{}^{(r_1-1)} h_1(x)) & \dots & L_{g_p}(L_f{}^{(r_1-1)} h_1(x)) \\ L_{g_1}(L_f{}^{(r_2-1)} h_2(x)) & L_{g_2}(L_f{}^{(r_2-1)} h_2(x)) & \dots & L_{g_p}(L_f{}^{(r_2-1)} h_2(x)) \\ . & . & \dots & . \\ . & . & & . \\ L_{g_1}(L_f{}^{(r_p-1)} h_p(x)) & L_{g_2}(L_f{}^{(r_p-1)} h_p(x)) & \dots & L_{g_p}(L_f{}^{(r_p-1)} h_p(x)) \end{bmatrix}$$

(8)

If β (x) is not singular, then it is possible to define the input transformation "*the nonlinear control law* " which has this form:

$$U = \beta(x)^{-1}.(-\propto(x) + V) \quad (9)$$
$$V = [v_1\, v_2 \dots v_p]^T$$
$$U = [u_1\, u_2 \dots u_p]^T$$

where V is the new input vector.
$\beta(x)$:Is the px p decoupling matrix of the system.

A. Non-linear coordinate transformation:

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_p \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_p \end{bmatrix} = \begin{bmatrix} [h_1 \quad L_f h_1 \quad \dots \quad L_f{}^{r_1-1} h_1]^T \\ \vdots \\ [h_p \quad L_f h_p \quad \dots \quad L_f{}^{r_p-1} h_p]^T \end{bmatrix} \quad (10)$$

By applying the linearizing law to the system,we can transforme the nonlinear system into linear form:

$$\begin{cases} \dot{Z} = Az + BV \\ Y = CZ \end{cases} \quad (11)$$

With,

$$A = \begin{bmatrix} A_{r_1} & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & A_{r_p} \end{bmatrix}, B = \begin{bmatrix} B_{r_1} & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & B_{r_p} \end{bmatrix}, C = \begin{bmatrix} C_{r_1} & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & C_{r_p} \end{bmatrix}$$

And,

$$A_{r_i} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix} \in \mathcal{R}^{r_i \times r_i};\ B_{r_i} = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} \in \mathcal{R}^{r_i};$$

$$C_{r_i} = [1 \quad 0 \quad \dots \quad 0] \in \mathcal{R}^{r_i}$$

B. Design of the new control vector V:

The vector v is designed according to the control objectives, For the tracking problem Considered, it must satisfy:

$$v_j = y_{d_j}{}^{r_j} + K_{r_{j-1}}\left(y_{d_j}{}^{r_{j-1}} - y_j{}^{r_{j-1}}\right) + \dots + K_1\left(y_{d_j} - y_j\right); \quad (12)$$
$$with\ 1 \le j \le p$$

Where the vectors $\left\{y_{d_j}, y_{d_j}{}^2, \dots, y_{d_j}{}^{r_{j-1}}, y_{d_j}{}^{r_j}\right\}$ Denote the imposed reference trajectories for the different outputs. If the K_i are chosen so that the polynomial $s^{r_j} + K_{r_{j-1}}s^{r_{j-1}} + \dots + K_2 s + K_1 = 0$ are chosen Hurwitz(has roots with negative real parts). Then it can be shown that the error $e_j(t) = y_{d_j}(t) - y_j(t)$, satisfied $\lim_{t\to\infty} e_j(t) = 0$.

III. INPUT-OUTPUT FEEDBACK LINEARIZATION METHOD APPLIED TO A ROBOT MANIPULATOR WITH SIX DEGREES OF FREEDOM.

We show that for a nonlinear and coupled system can be transformed to a linear and coupled output system,in this section we will applicate the above algorithme to a robot manipulator with six degrees of freedom "EPSON C4" ,for more mathematical details see [1].

In reality, the dynamic equations of a robot manipulator form a complex, nonlinear, and multivariable system. Let us first reformulate the manipulator dynamic Model in a form more suitable for the discussion to follow.

First, Let us return to the Euler-Lagrange equation of motion, we obtained the following equation.

$$\Gamma = A(q)\ddot{q} + C(q,\dot{q})\dot{q} + Q(q) \qquad (13)$$

Then, since the inertia matrix A is invertible for q $\in \mathcal{R}^n$ we may solve for the acceleration $\ddot{q}$ of the manipulator as.

$$\ddot{q} = f(q,\dot{q},\Gamma) \qquad (14)$$
$$\ddot{q} = -A(q)^{-1}[C(q,\dot{q})\dot{q} + Q(q) - \Gamma] \quad (15)$$

With,

$q = [q_1\ q_2\ q_3\ q_4\ q_5\ q_6]^T$: The angular position vector [6 x 1];
$\dot{q} = [\dot{q}_1\ \dot{q}_2\ \dot{q}_3\ \dot{q}_4\ \dot{q}_5\ \dot{q}_6]^T$: *The* angular velocity vector [6x 1];
$\ddot{q} = [\ddot{q}_1\ \ddot{q}_2\ \ddot{q}_3\ \ddot{q}_4\ \ddot{q}_5\ \ddot{q}_6]^T$: *The* angular acceleration vector [6x 1];
$\Gamma = [\Gamma_1\ \Gamma_2\ \Gamma_3\ \Gamma_4\ \Gamma_5\ \Gamma_6]^T$: *The* input torques vector [6x 1];

For an n-link rigid manipulator the feedback linearizing control is identical to the inverse dynamics control .To see this, consider the rigid equations of motion (15),we define state variables in state space as:

$$x_1 = q_1, x_2 = \dot{q}_1, x_3 = q_2, x_4 = \dot{q}_2, x_5 = q_3, x_6 = \dot{q}_3,$$
$$x_7 = q_4, x_8 = \dot{q}_4, x_9 = q_5, x_{10} = \dot{q}_5, x_{11} = q_6, x_{12} = \dot{q}_6,$$

And derivation of states we obtain,

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = \ddot{q}_1 = -A(x_1)^{-1}[C(x_1,x_2)x_2 + Q(x_1) - \Gamma_1] \\ \dot{x_3} = x_4 \\ \dot{x_4} = \ddot{q}_2 = -A(x_3)^{-1}[C(x_3,x_4)x_4 + Q(x_3) - \Gamma_2] \\ \dot{x_5} = x_6 \\ \dot{x_6} = \ddot{q}_3 = -A(x_5)^{-1}[C(x_5,x_6)x_6 + Q(x_5) - \Gamma_3] \\ \dot{x_7} = x_8 \\ \dot{x_8} = \ddot{q}_4 = -A(x_7)^{-1}[C(x_7,x_8)x_8 + Q(x_7) - \Gamma_4] \\ \dot{x_9} = x_{10} \\ \dot{x_{10}} = \ddot{q}_5 = -A(x_9)^{-1}[C(x_9,x_{10})x_{10} + Q(x_9) - \Gamma_5] \\ \dot{x_{11}} = x_{12} \\ \dot{x_{12}} = \ddot{q}_6 = -A(x_{11})^{-1}[C(x_{11},x_{12})x_{12} + Q(x_{11}) - \Gamma_6] \end{cases} \quad (16)$$

Then ,the affine form of nonlinear model of the robot manipulator with six degrees of freedom "EPSON C4" given by the following system:

$$\begin{cases} \dot{X}(t) = f(X(t)) + \sum_{i=1}^{p} g_i(X(t))U_i(t) \\ Y_i(t) = h_i(X(t)) \\ i = 1,2,\dots 6 \end{cases} \quad (17)$$

Where,

$$f(x) = \begin{bmatrix} x_2 \\ -A(x_1)^{-1}[C(x_1,x_2)x_2 + Q(x_1)] \\ x_4 \\ -A(x_3)^{-1}[C(x_3,x_4)x_4 + Q(x_3)] \\ x_6 \\ -A(x_5)^{-1}[C(x_5,x_6)x_6 + Q(x_5)] \\ x_8 \\ -A(x_7)^{-1}[C(x_7,x_8)x_8 + Q(x_7)] \\ x_{10} \\ -A(x_9)^{-1}[C(x_9,x_{10})x_{10} + Q(x_9)] \\ x_{12} \\ -A(x_{11})^{-1}[C(x_{11},x_{12})x_{12} + Q(x_{11})] \end{bmatrix}; g(x) = \begin{bmatrix} 0 \\ A(x_1)^{-1} \\ 0 \\ A(x_3)^{-1} \\ 0 \\ A(x_5)^{-1} \\ 0 \\ A(x_7)^{-1} \\ 0 \\ A(x_9)^{-1} \\ 0 \\ A(x_{11})^{-1} \end{bmatrix} \quad (18)$$

And,

$$X = [x_1, x_2, x_3, x_4, x_5, x_6 x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T;$$
$$\dot{X} = [\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4, \dot{x}_5, \dot{x}_6 \dot{x}_7, \dot{x}_8, \dot{x}_9, \dot{x}_{10}, \dot{x}_{11}, \dot{x}_{12}]^T;$$
$$U = [u_1, u_2, u_3, u_4, u_5, u_6]^T = [\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6]^T;$$

The transformed variables $y_1, y_2, \dots\ y_6$ are themselves physically meaningful. We see that:

$$links\ positions \begin{cases} y_1 = h_1(x) = x_1 = q_1; \\ y_2 = h_2(x) = x_3 = q_2; \\ y_3 = h_3(x) = x_5 = q_3; \\ y_4 = h_4(x) = x_7 = q_4; \\ y_5 = h_5(x) = x_9 = q_5; \\ y_6 = h_6(x) = x_{11} = q_6; \end{cases} \quad (19)$$

$$links\ velocities \begin{cases} \dot{y}_1 = \dot{x}_1 = x_2 = \dot{q}_1; \\ \dot{y}_2 = \dot{x}_3 = x_4 = \dot{q}_2; \\ \dot{y}_3 = \dot{x}_5 = x_6 = \dot{q}_3; \\ \dot{y}_4 = \dot{x}_7 = x_8 = \dot{q}_4; \\ \dot{y}_5 = \dot{x}_9 = x_{10} = \dot{q}_5; \\ \dot{y}_6 = \dot{x}_{11} = x_{12} = \dot{q}_6; \end{cases} \quad (20)$$

$$\text{links accelerations}\begin{cases}\ddot{y}_1 = \dot{x}_2 = \ddot{q}_1 = v_1;\\ \ddot{y}_2 = \dot{x}_4 = \ddot{q}_2 = v_2;\\ \ddot{y}_3 = \dot{x}_6 = \ddot{q}_3 = v_3;\\ \ddot{y}_4 = \dot{x}_8 = \ddot{q}_4 = v_4;\\ \ddot{y}_5 = \dot{x}_{10} = \ddot{q}_5 = v_5;\\ \ddot{y}_6 = \dot{x}_{12} = \ddot{q}_6 = v_6;\end{cases} \qquad (21)$$

By using the nonlinear feedback and diffeomorphic transformation given above,the six degrees of freedom "EPSON C4" nonlinear dynamic system (13) with six degree of freedom trajectory ouput is converted into the following Brunovesky canonical form and simultaneously output decoupled .

$$\begin{cases}\dot{Z} = Az + BV\\ Y = CZ\end{cases} \qquad (22)$$

Where,

$$Z = \begin{bmatrix} z_1\\ z_2\\ \vdots\\ z_{12}\end{bmatrix} = \begin{bmatrix} x_1\\ x_2\\ \vdots\\ x_{12}\end{bmatrix}; \dot{Z} = \begin{bmatrix} \dot{z}_1\\ \dot{z}_2\\ \vdots\\ \dot{z}_{12}\end{bmatrix} = \begin{bmatrix} \dot{x}_1\\ \dot{x}_2\\ \vdots\\ \dot{x}_{12}\end{bmatrix}; V = \begin{bmatrix} v_1\\ v_2\\ \vdots\\ v_6\end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & & \cdots & & 0 \\ 0 & 0 & & & & \\ & \ddots & & & & \\ & & \ddots & \cdots & & \\ \vdots & & \cdots & \begin{matrix}0 & 1\\ 0 & 0\end{matrix} & & \vdots \\ & & & \ddots & & \\ & & & & \ddots & \vdots \\ \vdots & & & & \ddots & \vdots \\ \vdots & & \cdots & & \ddots & \vdots \\ 0 & & & \cdots & 0 & 1 \\ & & & & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 & \cdots & & 0 \\ 1 & & & \\ & \ddots & \cdots & \\ \vdots & \cdots & \begin{matrix}0\\ 1\end{matrix} & \vdots \\ \vdots & & & \ddots \quad \vdots \\ \vdots & \cdots & \cdots & 0 \\ 0 & & 0 & 1 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 0 & & \cdots & & 0 \\ 0 & & & & & \\ & & \ddots & \cdots & & \\ \vdots & & \cdots & 1 \quad 0 & & \vdots \\ & & & \ddots & & \\ \vdots & & & & \ddots & \vdots \\ \vdots & & \cdots & & \ddots & \vdots \\ 0 & & & 0 & 1 & 0 \end{bmatrix}$$

The above matrices A,B and C are of dimension respectively : $12 \times 12, 12 \times 6$ *and* 6×12.

We note that linear system () consists of six (i=1,2...,6) independent subsystems of the following form;

$$\begin{cases}\dot{z}_i = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} z_i + \begin{bmatrix}0\\ 1\end{bmatrix} v_i\\ Y_i = \begin{bmatrix}1 & 0\end{bmatrix} z_i\end{cases} \qquad (23)$$

Where, $\quad z_i = \begin{bmatrix} z_{2i-1}\\ z_{2i}\end{bmatrix}$

IV. SIMULATION RESULTS:

The simulations are performed with a six-link manipulator having the parameters see [1].We present the simulation results depicting the output position,velocity,acceleration, and inpout transformation of each joint i=1...6,for the desired trajectories given by

$$y_{d_i}(t) = \sin\left(\frac{\pi}{5}t\right); \qquad (24)$$

$$\dot{y}_{d_i}(t) = 0.6283 \times \cos\left(\frac{\pi}{5}t\right); \qquad (25)$$

$$\ddot{y}_{d_i}(t) = -0.3948 \times \sin\left(\frac{\pi}{5}t\right); \qquad (26)$$

To stabilize the feedback linearized and output decoupled system ,we add a linear PD controller to each linear subsystem these figures illustrate some of the simulation results.

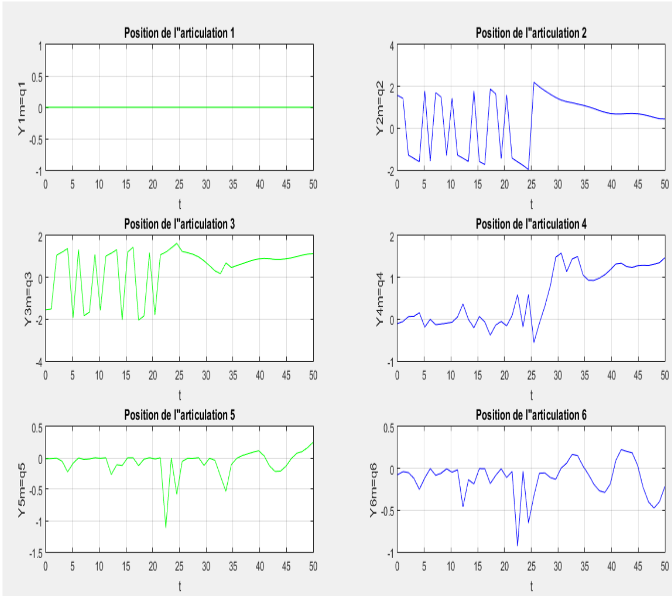

Fig. 2 Position plot $y_i(t)$for the six Joint of the robot

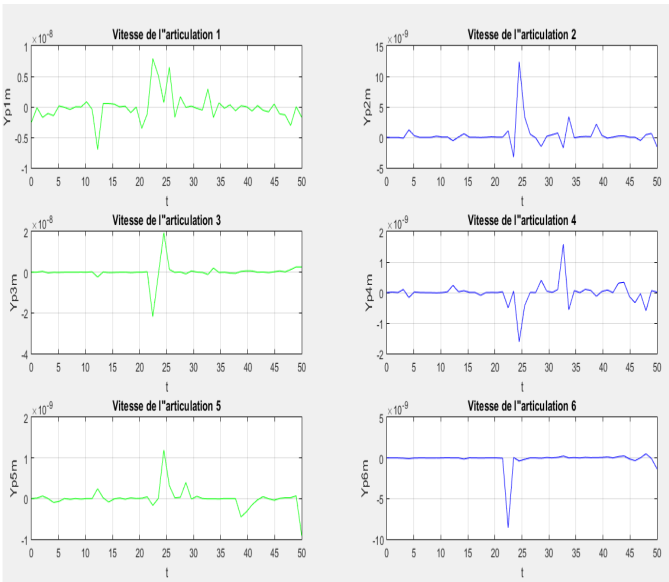

Fig. 3 2 Velocity plot $\dot{y}_i(t)$ for the six Joint of the robot

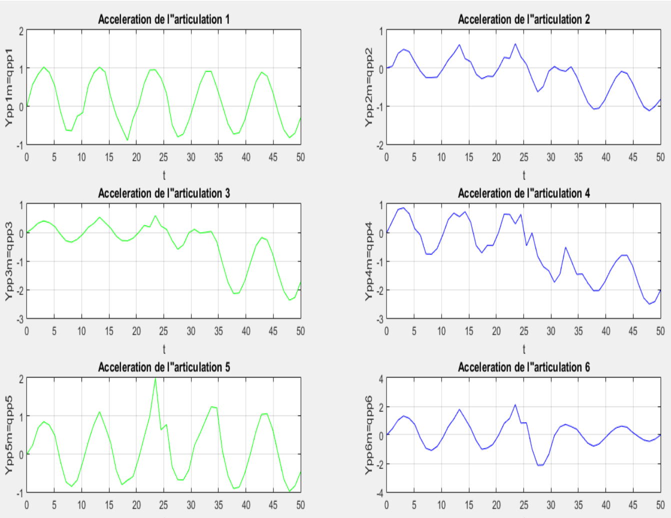

Fig. 4 Acceleration plot $\ddot{y}_i(t)$ for the six Joint of the robot

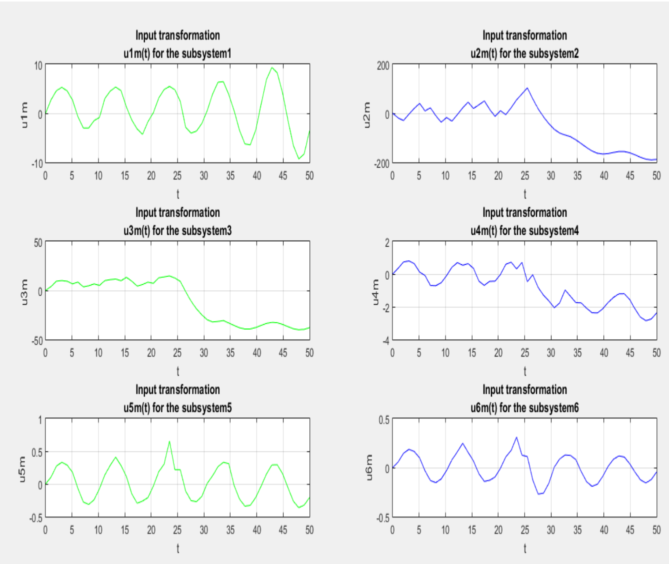

Fig. 5 Inpout transformation plot for the six subystem.

V. CONCLUSION

The design of this approach applied to a robot manipulator with six degrees of fredom discussed in this article has tow major steps:

First,Convert the non linear dynamics model of the "EPSON C4" robot into a feedback linearized and simultaneously output decoupled system by using the above nonlinear feedback and diffeomorphic transformation.

Second,Stablize the feedback linearized and simultaneously output decoupled system by designing a linear correction PD for each individual decoupled subsystem to control the angular position,velocty and accelration of each joint of this robot for tracking purposes.

An extensive MATLAB simulation program was developed and the obtained results in different simulations show the efficiency of the proposed approach which allows us in the futur works to provide more rigorous analysis of the performance of control systems, and also allows us to nonlinear control laws that guarantee stability , tracking of arbitrary trajectories,regulation , disturbance rejection, and so forth.

VI. REFERENCES

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