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Modeling and simulation the nonlinear dynamics of renewable energy system in Tunisia

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Abstract— **Modeling the nonlinear dynamics of renewable energy systems recently became the most attracted domain for many researchers. Several study have approved that such systems are highly nonlinear dynamics systems and can exhibit usually a cascade of bifurcation phenomenon and in many cases complexes chaotic oscillations. The subject of this paper is to show the role of exploring methods for modeling and simulation the nonlinear dynamics of renewable energy system in Tunisia which can contribute to investigate these basically dynamical behaviors and study their synchronization problems. Under the assumption of endogenous instability, the conditions for existence and characterizing endogenous cycles of the model are obtained. Our research question is to know how sustainable energy transitions could take place in specific economic sector of Tunisia. Using dynamical systems theory, modeling and simulations results are presented and several different interpretations have been developed.**

Keywords— **Chaos, Endogenous cycle, Nonlinear dynamical system, Renewable energy in Tunisia.**

I. INTRODUCTION

Given that according the Schumpeterian conception the innovation is the engine of growth [1]. In particular, the innovation in the renewable energy sector became among the main factors of economic performance. It's grown considerably in recent years and has become the most important factor which has a significant effect on economic growth. Thereby, the innovation as the company's competitiveness factor and the technology development will be the pillars upon which based the foundations of the Tunisian energy policy in the future period. However, currently in Tunisia the emergence of innovation remains a very complex process due to a non-adequacy between several actors, with iterative and nonlinear aspects. For this reason, several aspects of this complex phenomenon seem still elusive.

So, to study its sustainability and its intrinsic relationships, we need recognizing the role of interactions between these involving multiple actors, and understand precisely the nonlinear dynamics of innovation system in its particularity.

Over the years several major theories have been proposed to explain the complex interactions intra-actors of innovation system dynamics.

Paradoxically, most of these approaches, which calls for reinstatement of innovation in economic theory have been criticized and subsequently neglected, since their institutional heterogeneity is ignored.

Empirically, the major difficulty of research in nonlinear dynamics the level of the innovation system is to characterize the different types of interdependence while distinguishing for dynamic interaction between changes caused by intrinsic factors to the phenomenon studied and the influence of external factors.

To support innovation in Tunisia, our methodology is based firstly on a conceptual base for the overall look of the systemic approach to innovation and then the empirical work with simulation based on the complex dynamical system approach, such as the Lorenz process.

This paper is organized as follows: The first section provides some theoretical background on the concept of endogenous cycle of industrial innovation and nonlinear dynamic system associated while recalling the limits of linear modeling. A second section will by Reference to the chaos of econometrics representation of nonlinear dynamics model adequate able to generalize the study phenomenon. The third section will be devoted to a description of the structure of industrial innovation system in Tunisia. The fourth section describes the econometric methodology of endogenous simulation of industrial innovation cycle as well as discussion of the empirical results. Finally, a conclusion is for summarizing the results and discussions.

II. THEORITICAL FRAMEWORK

The dynamics of Innovation system (IS) which is a challenging task is defined as an evolutionary complex system generated by the emergence of scientific and technological structures and processes that influence performance and economic development. However, it has institutional and organizational determinants that included an interactive multi-dimensional framework governed by multiple actors. Now, this suggests not only are responsible for the process of knowledge circulation inside the IS is complicated, but that its policy and management decision is difficult because current tools do not take into consideration the essential characteristics of nonlinear dynamic operation of complex systems such as feedback loops , delays and stocks and flows.

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Therefore, the modeling for sustainable system dynamics requires taking into account the dynamics, structure and performance of the IS. Therefore, the modeling for sustainable system dynamics requires taking into account the complexity, non-linarity of structures and performance of the IS.

The govemance of IS is highlighted by several theoretical perspectives. The main objective is to conduct new approaches that can be used to explain the dynamics, structure and performance of the SI.

In this context, [1] offers a "triple helix" model as nonlinear evolutionary development of innovation. This is composed of three entities: universities, companies and public administrations.

In recent years due to a succession of events that occurred after the revolution the analysis of the situation in Tunisia has confirmed that it suffers from a severe recession particularly characterized by a simultaneous fall in activity of merchant services especially in industrial production. During the chaotic transition phase the country is witnessing a period of deep transformation that spawned challenges and opportunities in particular for industry. Many efforts until now have tried to go beyond this impasse by trying to revitalize the industrial sector to stimulate the dynamics of the process of structural transformation of the economy and redirect it to a more proven to be beneficial reorganization Unfortunately these efforts have all failed to revitalize the economy to floating growth path.

The failure first trained a multiplication of assessments and diagnosis efforts of real symptoms of complexity phenomenon involved. In this sense the study initiated by [3] on the national industrial strategy has the 2016 horizon has concluded that the malfunction including periods delicate juncture could come from a dynamic complexity of the system, which makes unpredictable the possible changes. Then it necessarily requires the strategic positioning of our industry, towards a more innovative and creative industry. Or, this is only possible thanks to the renovation of industrial policy in Tunisia to support innovation including the introduction of a new effective strategy.

Such a strategy requires the integration of the concept of innovation as being the interactive process by which knowledge (scientific, technological, organizational, commercial and Financials), manifests into economic growth and social well -Being must consider not only internal changes underway while dealing with the problems and identified weaknesses tired but also must be able to establish the issues and challenges to meet and adapt rational and fast way to changes that occur as a result of the crisis. The fact that in crisis and in a chaotic field rich in turbulences and instability of these structures endogenous the problem actually comes from the complex nature related to nonlinear dynamic character relationships and multiplicity of the elements in interaction complicate their studies and make impossible the mathematical resolution of their global dynamics.

It is the process based on the transfer of knowledge that will be able to encourage creativity in industrial entities and thus directs the companies aware of the necessity to innovate continuously to competitiveness in the context of the knowledge economy or innovation is considered the engine of growth and well-being. Since then, it insists we understand the manner in which the industrial innovation process takes place in Tunisia and grows. By means of the crucial question of chaos theory in this study is therefore to the study of the nonlinear dynamics of industrial innovation system in Tunisia which became a subject of news and exciting research.

1) *The concept of industrial innovation*

First, according to the traditional explanation, the concept of innovation had its genesis in direct relationship with industry [15]. In fact, it's industrialized countries who have found the concept of industrial innovation at the time of the industrial revolution in light of not only seek to explain the problem of the efficiency of industrial companies but also for industrial performance trends of fabric. Besides, from the economic point of view and outside the specialization, the emergence of the concept of innovation has undergone several phases of mutation. But, whatever its application fields, it generally tried to maintain a sense of its basic meaning of invention and innovation to designate more specifically the research and use of new knowledge and methods for purely creative and also Live connection with the search for improvement that exists.

2) *Endogenous cycle and industrial innovation*

A system covers a set of interacting elements. Interactions are playing an essential role in maintaining of the structure of the system or change. When the evolution of the system shows a change of state, the system is so dynamic.

Industrial cycles are characteristic fluctuations in industrial activity [14]. In a crisis, when the dynamic of the industrial system confirms that it is in a condition to imbalance, it loses its stability (dynamic and structural) and it will be known, by fluctuations in its dynamic behavior that will lead to profound structural changes.

Depending on the design of Schumpeter is the recurring fluctuation that makes these movements appears with a certain regularities which give their cyclical notion. According to this assumption, the cycles are seen as endogenous dynamics generated by industrial activity itself. Indeed, the evolution of this process can not result from simple quantitative changes of its factors of production, but rather it comes from qualitative changes in the vicinity of the dynamic equilibrium of the system when it loses its stability [1]. By design, mechanism is realized only through the dominance of endogenous pulse a factor of evolution is innovation.

This then is the origin of dynamic endogenous cyclic structures [15]. It includes both products of creations of the practical implementation of new manufacturing process. It is at the heart not only of development of the production system, but also the endogenous qualitative change in its dynamic cyclic structures.

Furthermore, analysis of ([8], [11]) shows that innovation is dynamic and endogenous phenomenon but also has a complexity feature. In addition, [9] states that there is a relationship of interaction and complementarities between innovation and the complexity induced by the instability of the system. Recently, [1] assumes not only that the instability is an endogenous weakening mechanism which is causing the dynamics of endogenous cyclic structures bifurcation, but also the industrial expansion can be induced by endogenous factors, mainly the impulse of endogenous cycles.

This confirms the foundation of the endogenous cycle theory where recurrence dynamic will be capable of generating endogenous cyclical fluctuations which are intrinsically linked to the operation of the system itself, without excited by another source of exogenous ([1], [14]). In this sense, on the basis of "innovation clusters" and the principle of "creative destruction" [16] postulated that innovation is a cyclical phenomenon and intrinsic dynamism of the industrial system, it is at the origin of determining endogenous cycles, that is why we speak of endogenous innovation cycle.

3) *Structure of industrial innovation system*

The industrial innovation system described structures and scientific and technological processes that influence and drive the emergence of economic development. This approach is a conceptual framework recently used by researchers and policymakers to identify workable structures that dominate the industrial innovation system. But in reality there are a variety of possible models of industrial innovation system with a multitude of structures with different achievable representations. This makes it difficult to select the most appropriate, able to guide policy makers towards good exploration and implementation of appropriate policies. The bad accuracy of the actual mode of operation of industrial innovation system negatively influences the identification of its structures. This is at the origin of failure in developing more effective policies to revive the industry.

Industrial innovation can affect both products as services, processes, management tools and organizational structures. Therefore, it is recognized that to operate in an innovative environment, it is necessary that looked for relationships are established between the market, the government, academia, human capital and industrial enterprise. In this research, we adopt the conceptualization of the innovation system as that of [19]. Because it provided the appropriate generic framework to meet all the detailed characteristics previously.

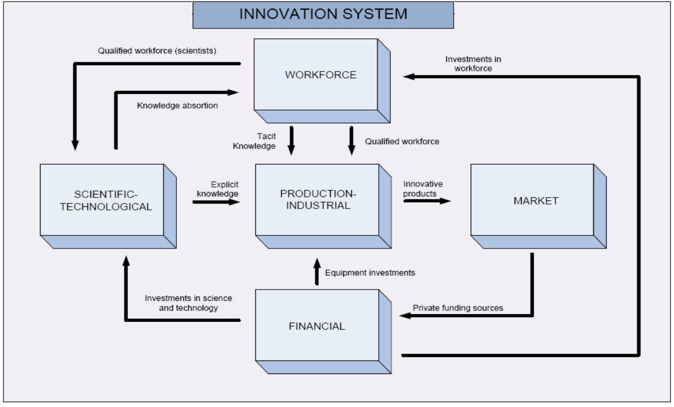

Fig. 1 Chart generic of the industrial innovation system (Source: [19], p.10).

The complexity of the industrial innovation system is intended as a new way of conceiving its nonlinear dynamics which requires new ways of thinking, that is to say, a new paradigm. Indeed, because of the presence of a large number of actors in the reality of an industrial innovation system, it is equipped, according to this generic chart of Figure (1) of complex structures. These structures are intrinsically endogenous to industrial innovation system and determine the complex interactions and especially the non-linear relationships between the actors and institutions that compose it [19].

Therefore, given the recurrent nature of cyclical fluctuations, fluctuations in the dynamics of industrial innovation system are considered endogenous and self-sustained. [12] This ensures not only an explanation of the cycles from imbalances but also to have an important idea about the nature of favors specification to be used in the study. This is the deterministic dynamic framework because it is the only one capable of generating itself from fluctuations outside exogenous shocks. Ultimately, in order to select the most appropriate econometric modeling to study the dynamics of industrial innovation system, according to this approach this requires to have a model that follows the statement features (dynamic, complex, non-linear, deterministic generates imbalances with self-sustained endogenous fluctuations). So this model sought cannot be a model that refers to the chaos of econometrics.

A dynamic system behaves chaotically, when there is at least a significant part of the dynamic that meets both the following two properties:

- Described by deterministic equations;
- Apparently irregular, but non-random;
- Sensitivity to initial conditions;
- A high recurrence.

In addition, the existence of a chaotic behavior of a process can be detected by the following:

- It is a nonlinear dynamic process;
- It is a deterministic process, bounded and aperiodic which will keep its dimension when placed in a higher embedding dimension;
- It is a process that has strange attractors;
- It is a very sensitive process to initial conditions.
- It is deterministic, but not predictable.

4) *Limits of the linear approach of modeling the innovation system*

Often innovation is generated so as (express or implied) with the linear approach. It describes the process along the path of innovation as a causal sequence. However, the conceptualization of linear sulfur types the following limits:

- It gives a representation of the different paths of innovation activity intent, without being able to identify or its degree of complexity or dynamic properties associated with the innovation process;
- It considers science as technology oriented (natural sciences and life) and R & D closely related to the manufacture, causing his lack especially to the social

and behavioral sciences. Accordingly, the omission of emotional components (or transitory) Innovation can cause many failures;

- The complex interactions between new technological capacity and emerging markets are an essential part of the innovation process, but they are omitted in connection with the linear approach.

All these limits neglect the usefulness of the linear approach. That is why we are facing nonlinear systems approach including complex systems with chaos, which is a framework for privilege this research. In what follows, we present the Lorenz model [13] to simulate the nonlinear dynamic behavior of industrial innovation system using several differential equations; we identify the qualitative behavior and visualize the strange attractor associated to recover by simulating the theoretical results meet the many issues raised by the forward-looking analysis of the endogenous cycle of industrial innovation system in Tunisia.

III. THE SYSTEM DYNAMICS METHODOLOGY

1) *The standard modeling process*

The standard methodology of modeling nonlinear process using the system dynamics theory is iterative and consists in following a described procedure [21], based on the five following steps:

- Step 1: Specify the problem articulation (formal the general of interest, describe the key of variables and identify the relevant time statement);
- Step 2: Define the dynamic hypothesis (a sophisticated assumption which can explain the dynamic behavior, formal all the possible of causal feedback, that can endogenously explains the reference models);
- Step 3: Choose the appropriate structure of the system (identify all differential equations model based on the dynamic hypothesis);
- Step 4: Using a sequence of hypothesis tests for model validation;
- Step 5: Policy formulation and evaluation (after modeling and evaluate the appropriate model of system dynamic using diagnostic tests, our objective is to explain the design policies for the system advance).

These steps are briefly detailed as following:

2) *Representation of the Rössler system*

In 1963, Lorenz proposed a simple deterministic dynamical system model for prediction the atmospheric convection. After that, in order to model the diffusion of dynamical equilibrium in chemical reactions, Rössler discovered in 1976 a three dimensional chaotic system known as Rössler attractor, which was simulated by a three dimension set of nonlinear ordinary differential equations, given by:

$$\frac{dx_t}{dt} = -y_t - z_t$$

$$\frac{dy_t}{dt} = x_t + a\,y_t$$

$$\frac{dz_t}{dt} = b + z_t(x_t - c)$$

Where t is the time, a, b and c are three positive real parameters set and x_t, y_t and z_t denote the dynamic variables specifying the system status at any time. Also it has only a single quadratic nonlinearity, which to be easier to analyze quantitatively.

This model with seven terms and three parameters is a three dimensional representation of a nonlinear dynamic system that will be capable of generating a minimal[1] autonomous chaotic flows by bifurcation through a period doubling cascade under certain conditions. Applying this model, [12] discovered a major indication of the chaotic behavior characterized by boundary, aperiodicity and sensitivity to initial conditions. That is to say, a small change in the initial solutions can lead to a radically different evolution. The tracing of the dynamics of the Rössler system simulation for the constant coefficients a = 0.1, b = 0.1 and c = 14, gives the following chaotic behavior[2] characterized by the system explicit expression:

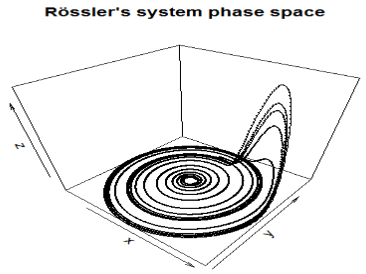

Fig. 2 Simulation of the Rössler attractor in a three-dimensional phase space (x, y, z) for a = 0.1, b = 0.1 and c = 14.

The phase portrait of the strange attractor shown in Figure (2) shows an orbit within the Rössler attractor follows a spiral behavior close to the x, y plane around an unstable fixe point. Another fixed point of the orbit of the system can influence the trajectory when the graph spirals begins to oscillate out enough, in particular if a rise and twist rhythm in the z-dimension. Thus, these repetitive movements are particularly erratic. Each orbit consists of a series of oscillating point within a fixed range of values. So, the Rössler attractor is a periodic attractor with infinite period. It gives some similarities to the Lorenz attractor, but is simpler and has only one manifold.

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[1] As indicated below this is for three reasons: Its phase space is based on a low dimension (three), its nonlinearity is minimal generated by a single quadratic term and can generate a chaotic attractor with a single lobe.

[2] For another case, the chaotic attractor of Rössler dynamics system can be obtained with the constant coefficients a=0.2, b=0.2 and c=5.7

Theoretically, the strange attractor of Rössler allowed describing chaotic behavior with the fractal properties of the attractor for certain control parameter values. This situation of asymmetry is highlighting where the damped oscillations can be characterized by pulses whose amplitudes are different for ascending and descending phases of generated endogenous fluctuations of the nonlinear dynamical system. Some dynamical and topological properties of the Rössler system can be deduced by either the linear methods such as diagonalization analysis or by the nonlinear methods such as Poincaré section and the bifurcation diagrams. For a complete study of the high complexity of the Rössler system dynamics we need some numerical explorations which can clarify this analysis.

The Takens delay embedding theorem (1981), indicates that during the identification of the dynamics system, it is not necessary to have a completed information data about the chaotic attractor of the system on condition that the initial state space data study is available. In addition, Takens (1981), showed that a chaotic dynamical system can incorporate all topological information needed to reconstruct the smooth attractor in the new phase space from a sequence of observations of initial state of dynamical system while maintaining preserves the properties without any loss of information. The idea of reconstruction phase space proposed by Takens (1981), is to gives the condition[3] which guarantee the reconstruction of chaotic attractor in a Euclidean space of m dimensions using samples delayed the data variable. This procedure is explained as follows:

Thus, let's suppose that we measured only the x component of the Rössler system. According to the Taken's embedding theorem we can assume that the topological properties of the phase space of system are determined from the reconstruction of vectors based on the time delayed versions of that of the x noted as $[x_t, x_{t+\tau}, \ldots, x_{t+m\tau}]$ where τ and m are respectively the delay-parameter and the embedding dimension. First, it must be noted that the delay parameter τ may be estimated using both techniques: the autocorrelation function shown in figure (3) or the function of the average mutual information of the signal shown in Figure (4). While, the embedding dimension m can be computed by using the well-known Cao's algorithm as illustrated in Figure (5).

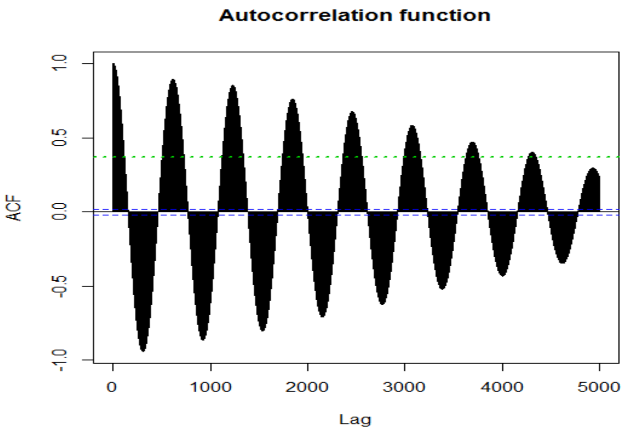

Fig. 3 Estimation the delay parameter using the autocorrelation function of the Rössler attractor.

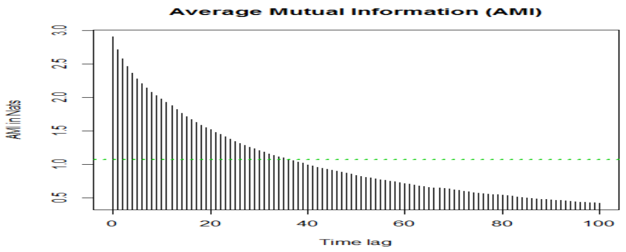

Fig. 4 Estimation the delay parameter using the average mutual information function of the Rössler attractor.

On the other hand, in order to estimate the minimum embedding dimension from a data we need to use the Cao's algorithm which based on tow functions: E1(d) and E2(d), where d denotes the dimension, E1(d) is the finish changing when d is superior than or equal to the embedding dimension, staying close to one and E2(d) is used for discrimination between deterministic signals and stochastic signals. For example if the signal is deterministic, there exist some d such that E2(d)! equal to one. But if the signal is stochastic, then E2(d) converge to one for all the values. In the current stady, the estimeted minimal embedding dimension value using Cao's algorithm is m=4.

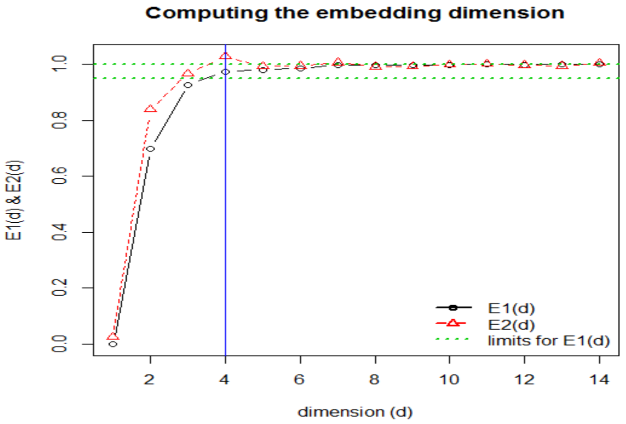

Fig. 5 Calculate the various values of the embedding dimension.

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[3] The Takens' theorem, provides that under certain conditions, it is still possible to immerse an N dimensional space in a space of embedding dimension m if m >2N. But as the size N is unknown, we can just replace it by the fractal dimension N_A of the strange attractor which characterizes the chaotic dynamical system during its embedding in k-dimensional Euclidian space.

Thus, the final phase space reconstruction of attractor can be obtained as following:

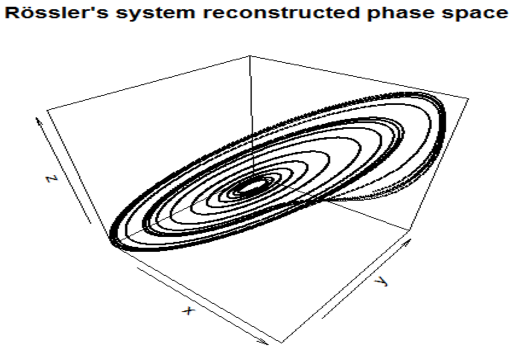

Fig. 6 Simulation the reconstructed phase space of the Rössler's attractor.

As shown in the Figures (6) and (2) above it is easy to conclude that the reconstructed and the initial phase space of Rösster attractor are different, but share similar topological features.

3) *Analytic analysis of stability and bifurcation of the Rössler system*

As shown in the Figue (2) above of the Rössler attractor, it has two types fixed points, one of these is the center of the attractor loop and the other type is situated comparatively just removed from the attractor. So, the system zeros are: the origin (0,0,0) as the obvious solution and the others fixed points exist if $c^2 \geq 4ab$, their coordinate are:

$$(\frac{c+\sqrt{c^2-4ab}}{2}, \frac{-c-\sqrt{c^2-4ab}}{2a}, \frac{c+\sqrt{c^2-4ab}}{2a})$$

$$(\frac{c-\sqrt{c^2-4ab}}{2}, \frac{-c+\sqrt{c^2-4ab}}{2a}, \frac{c-\sqrt{c^2-4ab}}{2a})$$

By determining respective eigenvalues and eigenvectors we can analyze the stability of each of these fixed points. So, the Jacobian matrix that represents this system can be calculated as:

$$J = \begin{pmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ z & 0 & x-c \end{pmatrix}$$

whose eigenvalues λ are the zeros of the following cubic polynomial characteristic:

$$-\lambda^3 + (a+x-c)\lambda^2 + (ac-ax-1-z)\lambda + x - c + az = 0$$

The critical points are stable when all eigenvalues accept negative real values. For λ = 0 we get b = 1 and the bifurcation fork associated with it. For a complex value λ the critical control parameter b changes to a Hopf bifurcation equation:

$$b_H = \frac{a(2-a^4+ca^3+2a^2-ca+c^2+(c-a)\sqrt{a^6-4a^4+2ca^3-4a^2+c^2})}{2(a^2+1)^2}$$

Then the two critical points are stable for:

$$b_H \leq b < \frac{c^2}{4a}$$

As illustrated in the accompanying diagram, for fixed points a=0.2 and c=5.7, as b converge to 0 the attractor converge to infinity.

4) *Important dimensions of Rössler system*

The presence of non-linearity in the dynamic system does not mean it is necessarily chaotic. The most important indication of the evidence of chaos is the presence of a strange attractor, which is characterized a low, fractional (non-integer) dimension. Henceforth, one of the main ways adopted to characterize the dynamics of a chaotic process is to quantify several types of dimensions associated with its strange attractor. To measures complexity need to indicate a low dimensional strange attractor among these dimensions we present the three most interesting dimensions of a statistically that information, embedding and the correlation [1].

- The information dimension is then defined by:

$$D_{\inf} = \lim_{\varepsilon \to 0} \sum_i \frac{p_i(\varepsilon)\log[p_i(\varepsilon)]}{\log[\varepsilon]}$$

where ε is the scaling radius, pi(ε) is probability measure.

- The embedding dimension is defined as the length m of the used single vector "butter embedding space" that does can reconstruct the successive phase space of a process. This measure is usually calculated from initial observed data, respecting to the following condition of Takens' embedding theorem:

  $$m \geq 2d+1, \forall d \geq 0$$

  According that, the invariants $m \geq D_c$ will be estimated if the correlation dimension $D_c \leq d$ of the attractor is sufficient.

- The correlation dimension is originally proposed by [Grassberger and Procaccia 1983] and is defined by:

$$D_c = \lim_{\varepsilon \to 0} \frac{d\log[C_{\varepsilon,m}]}{d\log[\varepsilon]}$$

where the slope of the curve C (ε) with respect to ε, with, C (ε) is the correlation sum of the data set, calculated from actual observed data, according to the following formula:

$$C_{\varepsilon,m} = \frac{1}{N_p} \sum_{j=m}^{N} \sum_{k<j-w} \Theta(\varepsilon - |s_j - s_k|) \quad \varepsilon \to 0$$

where m is the embedding dimension, sj are m-dimensional delay vectors, N is the number of points in s, Np=(N-m+1)(N-m-w+1)/2 is the sufficiently large number of pairs of points covered by the sums, Θ is the Heaviside step function, Θ(s) =1 for s>0 and Θ(s)=0 for s≤0, and w will be discussed below.

The correlation dimension is the probability that two points in the game of data are separated by a Euclidian distance ε. Its role is to measure the difference between two points of the phase space trajectory after an infinitesimal time. The adequate reconstructed minimum time-delayed embedding of state space which guarantees the topology of the solution can be determined as the following. In Figure (5), are shown the curves of computing embedding dimensions.

If the embedding dimension m and also increases Dc continues to increase, then the system is stochastic. However, if the data is generated by a deterministic process (compatible with the chaotic behavior), then Dc reaches a saturation limit finished beyond certain relatively small value of m. The correlation dimension can therefore be used to distinguish the true stochastic process of the deterministic chaos (large or small dimensions). But a non-integer result of the correlation dimension indicates that the dynamic system data is probably fractals. In Figure (5), are shown the curves of correlation integrals required for calculating the correlation dimension.

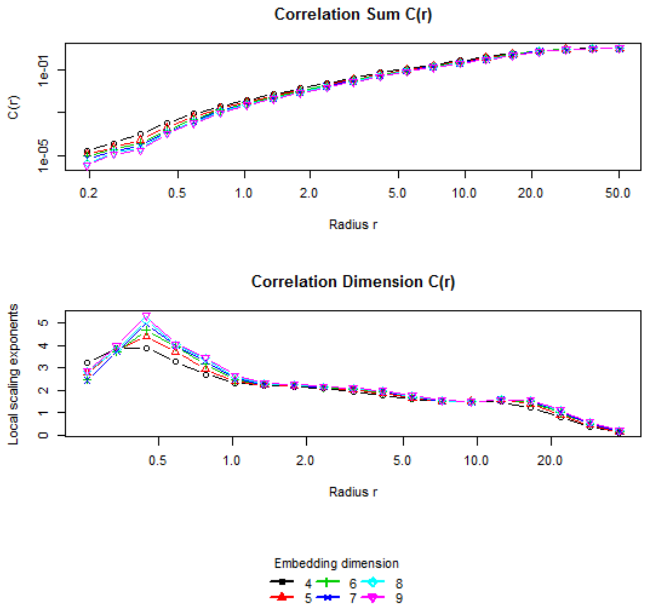

Fig. 5 Simulation of curves correlation integrals and correlation dimensions based on the various values of the embedding dimension.

5) *Lyapunov exponent of Lorenz system*

Chaotic systems are defined by the speed of initial conditions to diverge the close proximity to each other. One way to measure the rate of divergence is through Lyapunov exponents. Suppose we have two initial conditions given by (x1t, y1t, z1t) and (x2t, y2t, z2t). Then their difference with respect to time is given by:

$$d(t) = \sqrt{(x_{1t}-x_{2t})^2 + (y_{1t}-y_{2t})^2 + (z_{1t}-z_{2t})^2}$$

For t extremely small, we can approximate it by an exponential increase with time of d0eλt form. In this case, λ is the largest Lyapunov exponent that provide a measure of sensitive dependence on initial conditions which gives a characterization of the nature of an attractor is defined by,

$$\lambda_k = \lim_{t\to\infty}\lim_{\varepsilon_0\to 0}\frac{1}{t}\log\frac{\|\hat{\varepsilon}_t-\varepsilon_t\|}{\|\hat{\varepsilon}_0-\varepsilon_0\|}$$

where ε0 and εt are respectively the initial dynamical system state in phase space, and the image of those states after time t.

- If λ<0, the dynamic system converge to a stable fixed point.
- If λ = 0, the dynamic system converge to a stable limit cycle.
- If λ > 0, the dynamic system is chaotic. Then the difference between two close trajectories increases exponentially with time, which corresponds to an unbounded behavior.

In our case, The Figure (6) shows an estimate of the maximum Lyapunov exponents according to the different values of the embedding dimension m=4;5;6;7.

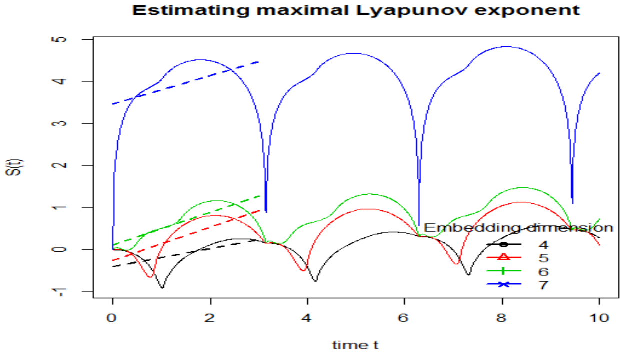

Fig. 6 Simulation of maximum Lyapunov exponents based on the various values of the embedding dimension.

When applied to the Lorenz system, the R algorithm suggests the use of an embedding dimension of 4. The final phase space reconstruction can be obtained as the result reported on the following figure 7:

Note that the reconstructed and the original phase space, although different, share similar topological features.

IV. EMPIRICAL FRAMEWORK

1) *Structure of the Tunisian industrial system*

According to statistics provided by [2], the structure of Tunisia's industrial sector reported on Table I shows that is composed of 5592 entreprises which having 10 or more employees, of which 2510 are intended for export entirety.

2) *Dynamics of industrial innovation in Tunisia*

The structure of the Tunisian industrial system in 2016, gives us an inhomogeneous state consisting of several branches interactions.

On the one hand, significant diversification in the industrial fabric not only allows a possible competition between the various companies operating in various sectors of industrial activity, but also a possible collaboration (strategic, technological, marketing, use, commercial, organizational) between these various producing entities. Therefore, heterogeneity ensuring the presence of interactions between the various sectors of industrial activity and subsequently among the various dynamic structures interactions, promotes industrial innovation or competitiveness caused by or through collaboration.

On the other hand, the inter and intra branches interdependencies can ensure not only the dynamism of the industrial system, but also guarantees its opening to its environment and especially strengthen the cooperation agreements at the level of research (private or public) between industrial enterprises and various higher education institutions and scientific research or research and development departments (R & D) related to some public enterprises. This new system allows dynamic industrial innovation generated by research cooperation between the industrial business and academic institutions.

All these ways of promoting industrial innovation play an indispensable role in creating added value and subsequently contribute positively to PIB. Which need to seek to strengthen the national system of industrial innovation since it contributes significantly to the economic development of Tunisia.

3) *Obstacles and consequences of industrial innovation in Tunisia*

However, despite the evolution of Tunisian industrial innovation system has a low added value. That is why its share in PIB remains far negligible compared to the contribution of the services sector's innovation system. In particular this weakness admits serious impact on employment policy. Indeed, its capacity is limited and unable to reduce unemployment. It is targeted towards the low-skilled labor and for hundreds of thousands of jobs and has only allowed recruiting total workforce very small young graduates. As [17], this suggests to us that the lack of a genuine complementarily between the university and the labor market is causing the lack of consistency between the structural changes in the economy and training of university graduates. This explains the current state of imbalance marked by involuntary unemployment.

V. CONCLUSION

Nowadays, it became a need to have a strong energy strategy to resist against the energy dependence and its high cost. In this way, we were confronted to the energy transition mechanism. The empirical evidences show that the dynamic of the renewable energy cycle transition in Tunisia has a nonlinear data generating process. Testing and modelling the presence of nonlinearity using the two regime univariate smooth transition autoregressive (STAR) model is proposed for the present study to investigate possible nonlinearity in average. All statistical tests can provide evidence of nonlinearity. Analysis shows that uses the STAR class of nonlinear models needs to have different functional choices between LSTAR and ESTAR type nonlinearities. Further empirical investigation shows that LSTAR model is more appropriate to capture observed asymmetries in our case. These results are important for our national policy makers in energy sector, as they offer improved explication for the nonlinear dynamism generating process of energy transition mechanism in Tunisia and give the necessary more accurate economic and social inferences. The results are also interesting for other perspectives open studies suggesting preference of nonlinear modelling with smooth transition when structural change is permitted.

Using the analysis of the nonlinear dynamics of industrial innovation system in Tunisia has enabled us to assess better representation of nonlinear dynamics of endogenous self-sustained fluctuations in industrial innovation cycle system in Tunisia with the Lorenz model.

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