4ème Conférence Internationale des Energies Renouvelables (CIER-2016)
Proceedings of Engineering and Technology – PET
Vol.17 , pp. 60 - 65

A Robust Model Predictive Control for a DC Motor-Based Photovoltaic Pumping

Omar Hazil[#*1], Sofiane Bououden[**2], Akila Djoudi Gherbi[#3], Billel Taghezouit [#4], Yahia Bakelli[#5]

[#]*Centre de Développement des Energies Renouvelables CDER,*
BP 62 Route de l'Obsevatoire, Bouzaréah,16340 Alger, Algérie
[*]*Department of Electronics, Faculty of Technology,University of Constantine 1,*
Constantine, Algeria.
[1]omar-h94@hotmail.fr
[3]*dakila95@yahoo.fr*
[4]bi.taghezouit@cder.dz
[5]y.bakelli@cder.dz
[**]*Faculty of sciences and technology, University Abbes Laghrour,*
Khenchela, Algeria
[2]ss.bououden@gmail.com

***Abstract*—This paper describes the modeling and control of a PV Pumping System which comprises a PV generator, a buck DC-DC converter and a DC motor-pump. By using the state-space averaging method, the PV system model possesses nonlinear behavior, highly dependent on operation point and environmental variables which make it difficult to control if we need to obtain maximum power. A simple robust model predictive control (RMPC) using linear matrix inequality (LMI) is proposed to regulate terminal voltage of photovoltaic generator, always keeping the system to work at maximum power. Simulation results are presented to verify the proposed method.**

***Keywords*—buck DC-DC converter, PV generator, DC motor pump, linear matrix inequality (LMI), robust predictive controller.**

I. Introduction

The solar energy is gaining increased importance as renewable source and has assumed an increasing role in modern electric power production technologies. One of the most popular applications of the photovoltaic energy utilization is the water pumping system for irrigation and domestic water supplies in remote areas of developing countries[1-2]. PV water pumping systems have the advantages of: reliability, low maintenance, ease of installation and the matching between the powers generated and the water usage needs [3-4].

For a better optimization of the energy, PV water pumping systems have to operate at their maximum power point (MPP). This maximum power point varies largely in time according to temperatures and irradiation levels; it is difficult to maintain optimum matching at all set of climatic conditions. In order to avoid the energy losses, a DC-DC converter known as a maximum power point tracker (MPPT) is used to match continuously the output characteristics of a photovoltaic generator to the input characteristics of a motor pump [5-6].

In this paper, we present a PV water pumping systems which includes photovoltaic array generator, DC/DC converter and DC motor coupled to a centrifugal pump. A robust predictive controller [7] based on linear matrix inequalities (LMI) [8] is applied to keep the PV generator voltage at a reference value taking into account uncertainty in the PVG operation point.

The following sections will show the PV pumping system modeling with the state-space averaging method and will present the regulator design in details. Finally we will give some simulation results to test the robustness of the proposed control law.

II. Model Pumping System

The block diagram of the analyzed photovoltaic system is depicted in Fig 1. This system consists of PV generator, DC-DC converter and a DC motor coupled to a centrifugal pump.

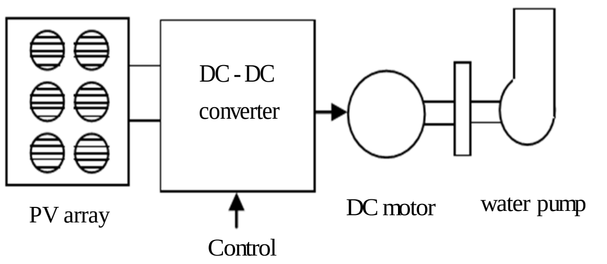

Fig. 1 Configuration of the PV pumping system

A. Photovoltaic Array Model

In order to appropriately represent the PVG, consider the equivalent circuit, shown in Fig 2, where the photovoltaic cell is represented by an electric current generator which is equivalent to a current source parallel to a diode, i_{PH} represent the current (photo-current) generated by solar radiation (G), R_{SH} and R_S are intrinsic shunt and series resistances of the module, respectively. Note, R_{SH} is irradiation dependent and R_S is constant.

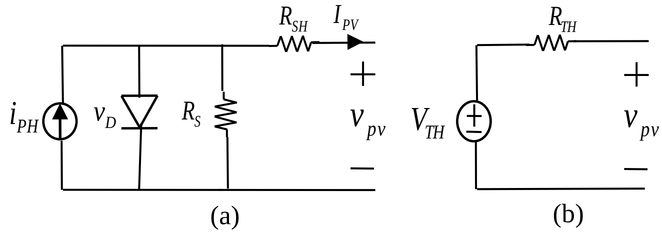

Fig. 2 Equivalent electrical scheme of the PVG: (a) Detailed, (b) Thévenin.

Photovoltaic generators are neither constant voltage sources nor current sources but in a real situation the array will be forced to operate at the boundaries of the constant current and constant voltage modes if a maximum power tracker is employed [9]. Consequently, the PV array may be represented by the simple Thévenin's equivalent circuit of Fig.3 with

$$R_{TH}=R_S+R_{SH}/\!/R_D \quad (1)$$

$$V_{TH}=I_{pv}R_{SH} \quad (2)$$

It can be observed that the Thévenin equivalent circuit parameters are both environmental variables and operating point dependent.

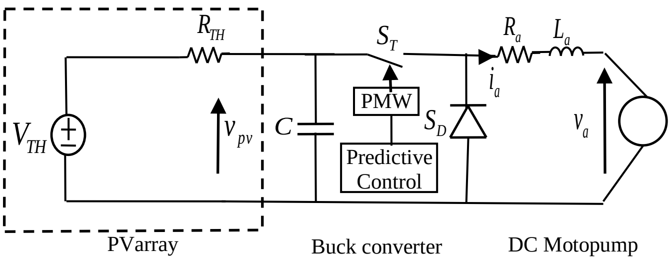

Fig. 3 Equivalent electrical scheme of the PV pumping system.

B. Buck Converter Average Model

Resultant average model of PV pumping system is shown in Fig. 2, where v_{pv} is the photovoltaic array voltage. This voltage must be controlled in order to keep the array operation at the maximum power point; the output voltage v_a (DC motor voltage) is related to the photovoltaic array voltage by (3):

$$v_a=dv_{pv} \quad (3)$$

Where d is the duty cycle of the switch S_T

$$0 \le d \le 1 \quad (4)$$

The equations that describe the system can be described as the following:

$$\begin{aligned} L_a\dot{i}_a(t) &= -R_a i_a(t) - k_b\omega(t) + v_{pv}(t)d(t) \\ J\dot{\omega}(t) &= k_b i_a(t) - (k_T+F)\omega(t) \\ C\dot{v}_{pv}(t) &= -d(t)i_a(t) + v_{pv}(t)/R_{TH} + V_{TH}/R_{TH} \end{aligned} \quad (5)$$

Where ω and J are respectively the rotation speed and the moment of inertia of the group, k_b is the constant of the electric couple, k_T is the strength's constant against electrometrical. R_a and L_a represent respectively the armature resistance and inductance. F is the viscous friction coefficients of the DC machine.

The expression (5) can be compacted in the following manner,

$$\dot{x} = f(x(t),u(t)) \quad (6)$$

The total instantaneous quantities can be presented as the sum of the DC and AC components,

$$\begin{aligned} i_a(t) &= \tilde{i}_a(t) + I_a & v_{pv}(t) &= \tilde{v}_{pv}(t) + V_{pv} \\ \omega(t) &= \tilde{\omega}(t) + \Omega & d(t) &= \tilde{d}(t) + D \end{aligned} \quad (7)$$

Substituting this into (5) a small-signal model can be derived as follows:

$$\begin{aligned} L_a\dot{\tilde{i}}_a(t) &= -R_a\tilde{i}_a(t) - k_b\tilde{\omega}(t) + D\tilde{v}_{pv}(t) + V_{pv}\tilde{d}(t) \\ J\dot{\tilde{\omega}}(t) &= k_b\tilde{i}_a(t) - (k_T+F)\tilde{\omega}(t) \\ C\dot{\tilde{v}}_{pv}(t) &= -D\tilde{i}_a(t) - I_a\tilde{d}(t) + \tilde{v}_{pv}(t)/R_{TH} \end{aligned} \quad (8)$$

linearized around an operating point given by

$$\begin{aligned} R_aI_a + k_b\Omega &= V_{pv}D \\ k_bI_a &= (k_T+F)\Omega \\ DI_a &= (V_{PV}+V_{TH})/R_{TH} \end{aligned} \quad (9)$$

We also introduce an added state variable to account for the integral of output regulation error. Let us define the new state variable as:

$$\dot{x}_e = v_{reff} - v_{pv} \quad (10)$$

The augmented averaged model of the PV system can be written as

$$\dot{x} = A\tilde{x} + B\tilde{u} + \tilde{f}(x(t),u(t)) \quad (11)$$

Where $\tilde{f}$ is a Lipschitz non-linearity, given by :

$$\tilde{f}(x(t),u(t)) = f(x(t),u(t)) - A\tilde{x} - B\tilde{u} \quad (12)$$

The nonlinear term is assumed to satisfy the Lipschitz condition as:

$$\left|\tilde{f}(X_1,t)-\tilde{f}(X_2,t)\right| \le M\left|X_1-X_2\right| \tag{13}$$

$$x=\begin{bmatrix} i_a \\ v_{pv} \\ \omega \\ x_e \end{bmatrix}, \quad A=\begin{bmatrix} -\frac{R_a}{L_a} & \frac{D}{L_a} & -\frac{k_b}{L_a} & 0 \\ -\frac{D}{C} & \frac{1}{R_{TH}C} & 0 & 0 \\ \frac{k_b}{J} & 0 & -\frac{k_T+F}{J} & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad B=\begin{bmatrix} \frac{V_{pv}}{L_a} \\ -\frac{I_a}{C} \\ 0 \\ 0 \end{bmatrix}$$

C. Uncertainty Model

We consider that the load R_{TH} at the operating point is uncertain or time-varying parameter. Then, matrices A and B depend on such uncertain which have been grouped in a vector p, and we can express (6) as a function of these parameter

$$\dot{x}=A(p)\tilde{x}+B\tilde{u}+\tilde{f}(x(t),u(t)) \tag{14}$$

In a general case, the vector p consists of N uncertain parameters $p=(p_1,\ldots,p_N)$, where each uncertain parameter p_i is bounded between a minimum and a maximum value $\overline{p}_i$ and $\underline{p}_i$

$$p_i \in \left[\underline{p}_i, \overline{p}_i\right] \tag{15}$$

The admissible values of vector p are constrained in an hyperrectangle in the parameter space R^N with $L=2^N$ Vertices $\{\upsilon_1,\ldots,\upsilon_L\}$. The images of the matrix $[A(p),B(p)]$ for each vertex υ_1 corresponds to a set $\{\varsigma_1,\ldots,\varsigma_L\}$. The components of the set $\{\varsigma_1,\ldots,\varsigma_L\}$are the extrema of a convex polytope, noted $Co\{\varsigma_1,\ldots,\varsigma_L\}$, which contains the images for all admissible values of p if the matrix $[A(p), B(p)]$ depends linearly on p, that is

$$[A(p),B(p)] \in Co\{\varsigma_1,\ldots,\varsigma_L\} = \left\{\sum_{i=1}^{L}\lambda_i\varsigma_i, \quad \lambda_i \ge 0, \quad \sum_{i=1}^{L}\lambda_i\right\} \tag{16}$$

In this context, we consider that $N=2$ and the parameter vector $p \in [1/R_{TH}]$ where:

$$1/R_{TH} \in \left[1/R_{TH\max}, 1/R_{TH\min}\right] \tag{17}$$

Since the PV system matrice A depend linearly on the uncertain parameter $1/R_{TH}$, we can define a polytope of $L=2$ Vertices that contains all the possible values of the uncertain matrices. The vertices of the polytopic model are:

$$A_1=\begin{bmatrix} -\frac{R_a}{L_a} & \frac{D}{L_a} & -\frac{k_b}{L_a} & 0 \\ -\frac{D}{C} & \frac{1}{R_{TH\max}C} & 0 & 0 \\ \frac{k_b}{J} & 0 & -\frac{k_T+F}{J} & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, A_2=\begin{bmatrix} -\frac{R_a}{L_a} & \frac{D}{L_a} & -\frac{k_b}{L_a} & 0 \\ -\frac{D}{C} & \frac{1}{R_{TH\min}C} & 0 & 0 \\ \frac{k_b}{J} & 0 & -\frac{k_T+F}{J} & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},$$

$$B_1=B_2=B$$

At maximum power transfer, the PV generator can be replaced by a voltage or a current source possessing high dynamic resistance in the former case and low dynamic resistance in the latter. In order to define range of changes for dynamic resistance, refer to PVG equivalent circuit of Fig. 3. At open circuit condition, R_D is low, dominating the parallel connection with R_{SH} [10,11].Thus we have:

$$R_{TH}\big|_{oc} \to R_S+R_D \tag{18}$$

bounded by

$$R_{TH}\big|_{oc} > R_{TH\min}=R_S \tag{19}$$

At short circuit and the reference condition, R_D is high, and R_{SH} dominates the parallel connection. Thus we have:

$$R_{TH}\big|_{sc} \to R_S+R_{SH} \tag{20}$$

Note that R_S is constant and R_{SH} is irradiation dependent [12]

$$\frac{R_{SH}}{R_{SH,ref}}=\frac{G}{G_{ref}} \tag{21}$$

$R_{SH,ref}$ is shunt resistance at STC (stands for Standard Test Conditions of *1* sun irradiation and 25^oC PVG temperature). Finally as an approximation,

$$R_{TH}\big|_{sc} < R_{TH\max}=R_{SH.STC} \tag{22}$$

III. ROBUST MODEL-BASED PREDICTIVE CONTROL USING LMIS

Consider the infinite horizon quadratic performance index as follows:

$$J(k)=\sum_{i=0}^{\infty}x(k+i|k)^T Qx(k+i|k)+u(k+i|k)^T Ru(k+i|k) \tag{23}$$

where $R(i)$, $Q(i)$ are two positive definite states and control weights respectively. Let us introduce a quadratic function $V(x)=x^TPx$, $P>0$ of the state $x(k|k)$ of the system (14), with $V(0)=0$. At sampling time k, suppose the following inequality is satisfied

$$V(k+i+1|k)-V(k+i|k) \ge -(x(k+i|k)^T Qx(k+i|k) + u(k+i|k)^T Ru(k+i|k)) \tag{24}$$

Summing (24) from $i=0$ to $i=\infty$, we have

$$x(\infty|k)^T Px(\infty|k)-x(k|k)^T Px(k|k) \ge -J \tag{25}$$

If the resulting closed-loop system for (14) is stable, $x(\infty|k)$ must be zero and result in

$$J \le x(k|k)^T P x(k|k) \le \gamma \quad (26)$$

where γ is a positive scalar and is regarded as an upper bound of the objective in (23)

$$\sum_{i=0}^{\infty} x(k+i|k)^T Q x(k+i|k) + u(k+i|k)^T R u(k+i|k) \le \gamma \quad (27)$$

Then, by substituting the state space equation (14) in the robust stability constraint (24), one has

$$\begin{aligned} &\left[Ax(k+i|k) + Bu(k+i|k) + \tilde{f}(x(k+i|k), u(k+i|k)\right]^T P \\ &\left[Ax_i(k+i|k) + Bu(k+i|k) + \tilde{f}(x(k+i|k), u(k+i|k)\right] \\ &- x(k+i|k)^T P x(k+i|k) + x(k+i|k)^T Q x(k+i|k) \\ &+ u(k+i|k)^T R u(k+i|k) \le 0 \end{aligned} \quad (28)$$

Suppose the terms involving of $\tilde{f}$ in this inequality satisfy the following condition:

$$\begin{aligned} &2\left[Ax(k+i|k) + Bu(k+i|k)\right]^T P\left[\tilde{f}(x(k+i|k), u(k+i|k)\right] \\ &+ \left[\tilde{f}(x(k+i|k), u(k+i|k)\right]^T P\left[\tilde{f}(x(k+i|k), u(k+i|k)\right] \\ &\le \mu x(k+i|k)^T W^T W x(k+i|k) \end{aligned} \quad (29)$$

where $\mu=\lambda_{max}(P)$ and W is the corresponding matrix of the quadratic bound which will be determined later in the next section. By replacing the condition (29) in the inequality (28), the following condition holds for all $i > 0$.

$$\begin{aligned} &\left[Ax(k+i|k) + Bu(k+i|k)\right]^T P\left[Ax(k+i|k) + Bu(k+i|k)\right] \\ &- x(k+i|k)^T P x(k+i|k) + x(k+i|k)^T Q x(k+i|k) \\ &+ u(k+i|k)^T R u(k+i|k) + \mu x(k+i|k)^T W^T W x(k+i|k) \le 0 \end{aligned} \quad (30)$$

the inequality can be expressed as:

$$P - (A_i + BK)^T P (A_i + BK) - Q - K^T R K - \mu W^T W \ge 0 \quad (31)$$

Define $P=\gamma G^{-1}$; $K=\gamma P^{-1}$ and $\alpha=\mu^{-1}\gamma$ and using Schur complement lemma twice, we have

$$\begin{bmatrix} G & (A_i G + BY)^T & (WG)^T & (Q^{1/2}G)^T & (R^{1/2}Y)^T \\ A_i G + BY & G & 0 & 0 & 0 \\ WG & 0 & \alpha I & 0 & 0 \\ Q^{1/2}G & 0 & 0 & \gamma I & 0 \\ R^{1/2}Y & 0 & 0 & 0 & \gamma I \end{bmatrix} \ge 0 \quad (32)$$

For robust constrained infinite horizon MPC, we incorporate both input constraint into the optimization problem. Then, the receding horizon state feedback gain K, which at the sampling time k minimizes the upper bound $V(x(k|k))$ on $J(k)$ and satisfies the specified input constraint, is given by $K= \gamma P^{-1}$, where $G>0$ and Y are the solutions to the following LMIs:

$$\min_{G,Y,\gamma,\alpha} \gamma \quad \text{subjecte to}$$

$$\begin{bmatrix} I & x(k|k)^T \\ x(k|k) & G \end{bmatrix} \ge 0$$

$$\begin{bmatrix} G & (A_i G + BY)^T & (WG)^T & (Q^{1/2}G)^T & (R^{1/2}Y)^T \\ A_i G + BY & G & 0 & 0 & 0 \\ WG & 0 & \alpha I & 0 & 0 \\ Q^{1/2}G & 0 & 0 & \gamma I & 0 \\ R^{1/2}Y & 0 & 0 & 0 & \gamma I \end{bmatrix} \ge 0 \quad (33)$$

$$\begin{bmatrix} u_{max}^2 I & Y \\ Y^T & G \end{bmatrix} \ge 0$$

$$G - \alpha I > 0$$

IV. Simulations and Results

The performances of the proposed control design are illustrated through simulations. The numerical parameter values used are given by:

- PV generator: R_{SH} *=13.5620*Ω, R_S *= 0.2670* Ω, *Ipv=19.2000A*, V_{TH}*= 260.3904 V*, R_{TH}*= 13.8290* Ω.
- Capacitor: $C=4000.10^{-6}F$.
- The permanent magnet DC motor-pump is characterized by a nominal operating point: $U_n=24V$ and $I_n=12A$, $W_n=3000$ *round/mn (rpm)* and a power $P_n=0.3hp$.
- The identified parameters of DC motor are in USI: $R_a=1.072$, $L_a=0.05$, *J=476.10-6*, $F=88.10^{-5}$, $k_T=14.10^{-4}$, $k_b=45.10^{-3}$.

Fig 4 shows the transient simulation of the PV pumping system under the dynamic resistance perturbations. The waveforms depicted in the Fig 4 are the duty-cycle d, PV voltage v_{pv}, PV power and motor speed ω. We can notice that the PV voltage settle to their desired value in *0.1 seconds*, the duty-cycle saturates at *45%*. These simulation results show that the control law RMPC is able to stabilize the system on the desired PV generator voltage in presence of dynamic resistance perturbations. The overshoots and long settling time seen in PV system responses are the result of more aggressive move in the manipulated variable.

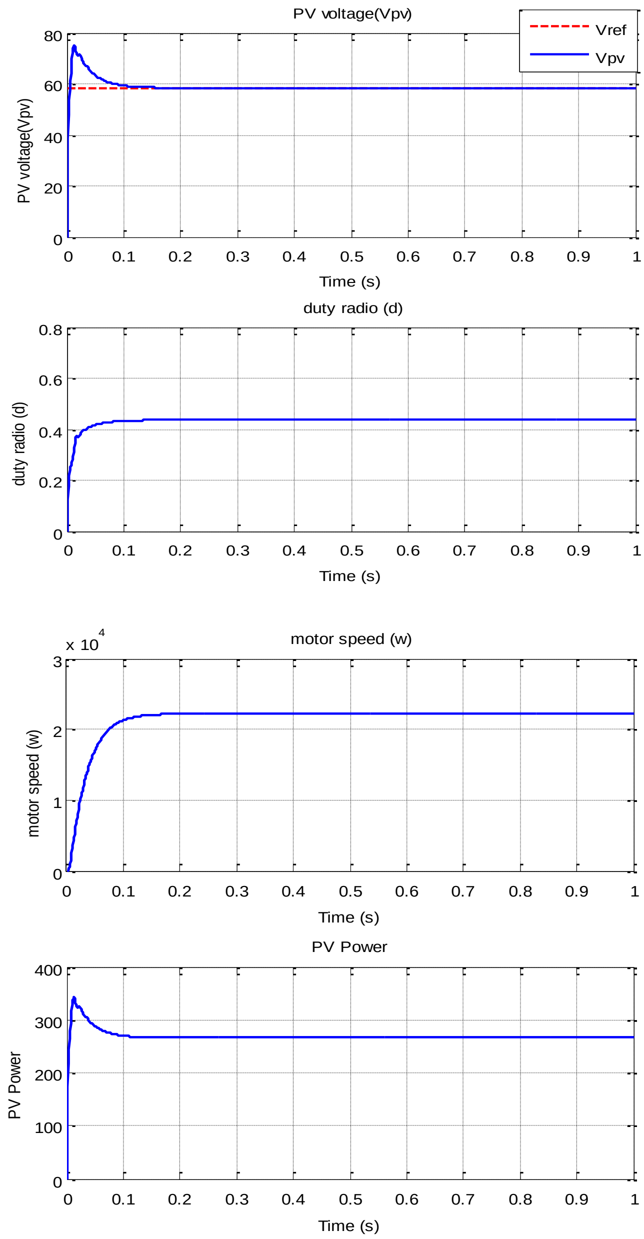

Fig. 4 the transient simulation of the PV system

V. CONCLUSIONS

This work has presented a robust predictive controller design framework based on LMIs for a photovoltaic pumping system. At first the non linear state space averaging model is generated and linearized around equilibrium point, this model take into account parametric uncertainty by means of a polytopic representation. Then, the state feedback control law is obtained by minimizing the upper bound of the infinite horizon cost function at each time instant. The stability condition of the closed-loop system is guaranteed over the whole uncertainty domain in the sense of Lyapunov. Finally, the obtained LMI-based RMPC controller has been applied to regulate the PV generator voltage in presence of dynamic resistance changes.

Simulation results show the efficiency and the robustness of the proposed approach. The RMPC algorithm of the PV pumping system is currently under experimental stage and in near future we will publish the first results if they are satisfactory.

REFERENCES

[1] A. Mokeddem, A. Midoun, D. Kadri, S. Hiadsi, and I. a. Raja, *"Performance of a directly-coupled PV water pumping system"*, Energy Convers. Manag, vol. 52, no. 10, pp. 3089–3095,Sep. 2011.

[2] Arun Ghosh, S G Malla and C N Bhende, *"Small-signal modelling and control of photovoltaic based water pumping system"*, ISA Transactions, Vol. 57, pp. 382-389, July 2015.

[3] J. Appelbaum, J. Bany. "*Analysis of a direct coupled DC motor and a photovoltaic converter*," In: 1st commission of European Community conference on photovoltaic solar energy, Luxembourg, September 27–30, Reidel, Dordrecht, The Netherlands, 1979.

[4] S. Singer, J. Appelbaum. "*Starting characteristics of direct current motors powered by solar cells*". IEEE Trans Energy Convers 1993; 8(1):47–53.

[5] C. Hua, C. Shen, "*Study of maximum power tracking techniques and control of DC/DC converters for photovoltaic power system."* In: 29th Annual IEEE PESC. IEEE Computer Society Press, New York, pp. 86–93. 1998.

[6] M.A.S. Masoum, H. Dehbonei, E.F. Fuchs, *"Theoretical and experimental analyses of photovoltaic systems with voltage- and current-based maximum power-point tracking*." IEEE Trans. Energy Conver. 17 (4) .2002.

[7] Kothare, M., V. Balakrishnan, and M. Morari, "*Robust Constrained Model Predictive Control Using Linear Matrix Inequalities,"* Automatica, Vol. 32, pp. 1361-1379 (1996)

[8] BOYD S., GHAOUI L.E., FERON E., BALAKRISHNAN V, *"Linear matrix inequalities in control theory"* SIAM, Philadelphia, 1994.

[9] F. Nakanishi, T. Ikegami, K. Ebihara, S. Kuriyama, Y. Shiota, *"Modeling and Operation of a 10 kW Photovoltaic Power Generator Using Equivalent Electric Circuit Method*", Proc. IEEE Photovoltaic Specialists Conference, 2000.

[10] M. Sitbon, , S. Schacham and A. Kuperman, *"Disturbance Observer Based Voltage Regulation of Current-Mode-Boost-Converter-Interfaced Photovoltaic Generator,"*, IEEE Transactions on Industrial Electronics. 0278-0046 (c) 2015.

[11] Mohammed A. Elgendy, Bashar Zahawi , and David J. Atkinson, *"Assessment of Perturb and Observe MPPT Algorithm Implementation Techniques for PV Pumping Applications*".IEEE Transactions on Sustainable Energy, vol. 3, no. 1, pp. 21-33, 2012

[12] H. Tian, F. Mancilla-David, K. Ellis, E. Muljadi and P. Jenkins, *"A cell-tomodule-to-array detailed model for photovoltaic panels*," Sol. Energy, vol. 86, no. 9, pp. 2695 – 2706, Sep. 2012.